AN EVALUATION OF THE DEPTH OF SIMILARITY BETWEEN LINEAR AND NON-LINEAR WAVE THEORIES

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Abstract

An evaluation of the wave behavior and the effects of kinematics and dynamic properties of wave particles on the submerged members of a jacket platform have been accomplished. The Stokes' fifth-order and Airy wave theories have been employed to evaluate the kinematics and dynamic properties of wave particles. SAP2000 software was used to determine and evaluate the wave properties. The aim of this study is to model both theories and determining a depth of water, in which both theories indicate the same behavior. The study has found that the non-linear Stokes' fifth-order theory shows similar behavior with the linear Airy wave theory from *10* meters below the *SWL* toward the sea floor with 99% of similarity.

Keywords: *kinematics and dynamic wave properties, offshore structures, Airy wave, fifth order Stokes theory.*

1 Introduction

The offshore structures have interactions with waves, currents and winds, hence, complex fluid loadings are produced, which sometimes are quite difficult to evaluate. However, practical techniques have been used for evaluating wave loads such as, diffraction theory for the large structures and *Morison* equation for slender bodies. The solution of the *Morison's* equation requires the determination of drag and inertia coefficients and information about the fluid kinematics properties.

Usually, in determination of the wave kinematics properties, deterministic design wave or a random wave approach are applied. The random wave can be obtained by the energy spectra method for designing the sea state, while for designing waves, which is characterized by period and wave height, statistical methods as the most probable of highest waves for a given return period are employed. Irregular waves for estimating the kinematics properties developed by Gudmestad (1990), which was based on his previous studies (Gudmestad and Connor, 1986). Also, the wave kinematics design can be determined by several wave theories including, stream function, *Stokes* second, third and fifth-order, *cnoidal*, and *Airy* (Dalrymple and Dean, 1991).

In case of non-linear waves several theories have been introduced. The *Stokes* fifth-order theory could be applied as the deterministic theory, which is suitable for deep-water regions. In this case, to model the non-linear and linear wave theories, a finite element default software, named SAP2000 has been used.

Fenton (1985) explained the Stokes wave theory as the solution of wave problem. The specified design parameters are obtained in terms of the four quantities of the wave height *H*, period *T*, mean water depth *d*, and wavelength *L*.

There are two essential features of stokes theory: the Fourier series represent all variation in the direction of propagation, and the coefficients can be written as perturbation expansions. The accuracy of the theory was examined by Fenton (1990) for waves with height up to *97.5%* of the highest possible height. The horizontal velocity distribution under the crest was chosen for the criterion of the accuracy. The results of Fenton (1990) showed that fifth-order *Stokes* theory is very accurate within its validity region. Then, Hedges *et al*. (1995) extended the argument with an expression for the region of applicability of *Stokes* theory.

The lateral loads on the offshore structures generated by surface waves are important in the phase of designing the offshore structures. However, in deep-water regions, the behavior of surface waves in deeper levels of water could be evaluated by linear wave theory. With regard to the complicated behavior of fluid around the structures, simplifying the calculations of the kinematics and dynamic properties of the water particles is desirable, subject to the adequate considerations of wave behaviors.

Chakrabarti (2005) argued that in modeling the waves by fifth-order theory, away from the SWL (downward), the behavior of waves are similar with the linear wave theory, and it can simply be designed by a linear theory. However, this behavior should directly influence the properties of the water particle. On the other hand, the exact depth of water, in which the behavior of both theories are alike is unknown. Therefore, determining a depth of water, in which both theories yield the same result (hereinafter it is called *the depth of similarity*) is the aim of this study.

The computation of the Airy wave theory is easy with simple iteration for computing the kinematics and dynamic properties compared with the Stokes fifth-order theory. Therefore, finding a depth, in which the properties of both theories are exactly identical is desirable for the designers who design structures for the depth of water under the depth of similarity, such as deploying pipelines to transfer hydrocarbon from the wellheads.

The significant wave height used in this study has been estimated by the data observed in the Federal Territory of Labuan off the coast of Sabah, Malaysia. The Labuan data was evaluated and used for the first time by Center for Coastal & Ocean Engineering (2009) as a report of extreme wave analysis for the Marine Department of Malaysia. The authors estimated the significant wave height, $H_S = 4.90\ m$, for a return period of *100* years, which is used as the wave height in this study.

There are four sections in this paper. Following this introduction, Section 2 provides summaries as the procedure of work and background of methods. The discussion about the obtained accelerations and velocities and pressure of the loads on the offshore structure are presented in Section 3. Finally, Section 4 contains the conclusions drawn from this study.

2 Background and Procedure of Work

2.1 Fifth-Order Stokes Theory

The fifth-order theory includes five components in a series form. Each component is generally placed in an order of magnitude, which is smaller than the previous one in succession. The horizontal velocity represents the following form with a five-term series as:

$$u = \sum_{n=1}^{5} u_n \cosh nks \cos n(kx - \omega t) \tag{1}$$

Where k is the wave number, $s = a + d$, in which a, is the amplitude of wave, d is the depth of water, ω is the frequency, and t, stands as time of wave propagation, for more details readers are referred to Chakrabarti (1987). It should be note that the frequencies with the higher components can be considered as the multiples of the fundamental wave frequency. Hence, the higher components decay faster with taking distance from *SWL* (Chakrabarti, 2005).

2.2 Airy wave theory

Airy wave theory is the most useful wave theory among all wave theories. It is also known as small amplitude wave theory, or sinusoidal wave theory. The assumption for applying the theory is based on the small wave height compared with the water depth or wave length. The dynamic pressure, p, takes on the first-order form, it can be calculated by

$$p = \rho g \frac{H}{2} \frac{\cosh ks}{\cosh kd} cos\Theta \tag{2}$$

where g is the gravity acceleration m/s^2, k is the number of wave, ρ is the density of the water kg/m^3, $\Theta = k(x - ct)$ the parameters of c, x and t stand for wave celerity, horizontal distance and time of wave propagation, respectively (Chakrabarti, 1987).

2.3 *Morison* Equation

When the wave flow is assumed unsteady, linear, with a simple harmonic motion, a more complex flow around the pile can be observed compared with the steady flows. If we assume a simple oscillatory flow over one cycle, it will change the wake region (low-pressure) immediately behind the pile every half cycle (Patel, 2013). The empirical formula known as *Morison* equation is used to compute the effects of velocity and acceleration of water particles on the structure members. The force obtained by this formula is for unit length of the pile,

$$f = \rho C_M \frac{\pi D^2}{4} \dot{u} + \frac{1}{2} C_D \rho |u| u \qquad (2)$$

where *f* is the horizontal force per unit length, $\dot{u}$ and u are acceleration and velocity of wave particles in the horizontal direction, respectively, and C_M , C_D empirical constants are hydrodynamic coefficients (Wilson, 2003).

2.4 Procedure of Work

The procedure of conducting this study is summarized as the following steps:

1. Determining the environmental specifications with their values such as wave, current, seismic, wind, faults, (in this study we examined the wave height only).
2. Determining the mean depth of sea water where the offshore structure is placed.
3. Determining the dominant wave height.
4. Determining the apparent wave period.
5. Determining the most suitable wave theory which in this study, fifth-order Stokes theory was chosen based on *API* standard recommendation.
6. Modeling two wave theories (Airy wave and fifth-order Stokes theory) to evaluate the kinematics and dynamic properties of the wave particles and compare the models, then imposing the loads generated by Stokes fifth-order theory on the structure's members.

3 Results and Discussion

In the present study, the idealized three dimensional fixed jacket platform with fixed constrained foundation is examined (Fig.1). The height of the structure is *70 m*, the width and length *35.29 m* (x and y-direction), in the bottom, with the leg batter of 10 m (for each *7 m* rise, *1 m* run). The structure is made by steel.

As discussed earlier, the significant wave height (H_S) was estimated *4.90 m* for the Labuan wave height data set.

In this research for determining the apparent period, two procedures were carried out; NORSAK recommendation formula (Standard, 2004), and historical observation followed by engineering judgment. Therefore, based on the results, *T = 9 Sec* is concluded as the true apparent period for the considered wave height.

Based on RP2A-WSD [pg.14](2007) recommendation chart, the fifth-order Stokes' theory is selected as a suitable wave theory. The chart was adopted from a report of Atkins Engineering Services (Barltrop *et al*., 1990). Although there are formulas to compute and determine the type of water depth, which are classified in deep, intermediate and shallow water, however, the recommended chart has the capability to classify the type of water depth.

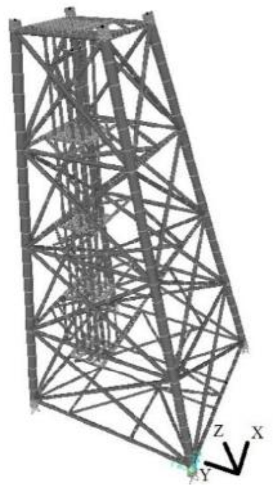

Figure 1: An idealized three-dimensional offshore platform

In this case, Based on RP2A-WSD (2007) recommendation the hydrodynamic coefficients C_M is ranged in *1.5* to *2* and C_D is *0.6* to *1,* to use in Morison equation for computing the wave pressure on the submerged elements of jacket platform. The marine growth on the members generally have a roughness of e > 10^{-3}. The API guideline recommends to use *0.04 m* for depth from *0* to *45.5 m*.

In the subsequent pages, several graphs obtained by the modeling of wave theories are shown and explained for developing the discussion. Fig.(2) represents the arrow-selection graph, which displays the results of velocity by fifth-order Stokes' theory. On the bottom of the figure the wavelength computed by the program (*L* = *128 m*). The maximum velocity is *2.8853 m/s* under the crests, away from the *SWL*, downward, the magnitude of velocities decline gradually.

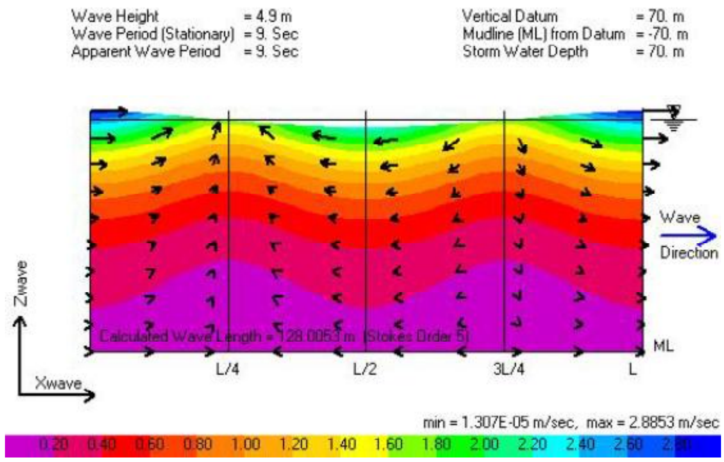

Figure 2: Arrow-velocity graph for fifth-order Stokes theory.

In Fig.(3) also the arrow-selection graph displays the velocity of water particles for the wave computed by Airy wave theory. On the bottom of the figure the wavelength computed by the program (*L* = *126.2 m*). The maximum velocity is *2.9034 m/s* under the crests, which is higher than the velocity computed by non-linear theory in Fig.(2). Although in both Figs.(2) and (3) the wavelengths and the maximum velocities are different, however, by taking distance from *SWL* the shape of waves and the velocities are approximately identical.

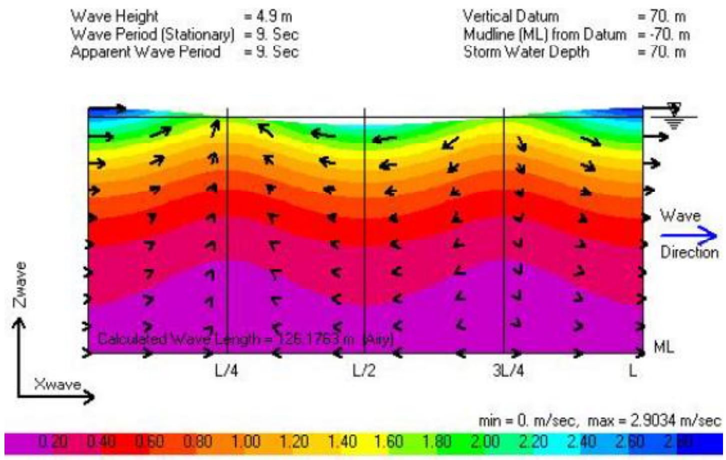

Figure 3: Arrow velocity graph for Airy wave theory.

Fig.(4) indicates the contour-graph of the acceleration particles of the considered wave generated by the fifth-order *Stokes'* theory. The computed amplitude of waves is ±*2.38 m* at crests and troughs from to the *SWL*. The maximum acceleration is *1.7684* m/s^2 under the crests.

Fig.(5) shows a contour-graph of the acceleration particles of the considered wave obtained by the *Airy* wave theory. There are some areas in depth with 0 acceleration in Fig.(5) and (4), which represent that the waves have no effects in the very deep regions. Means that in both theories water particle accelerations and velocities gradually decrease with taking distance from the *SWL*, downward. Therefore, the drag and inertia forces on the jacket platform are declined away from the surface of the water.

In addition, away from SWL, the behavior of waves in fifth-order Stokes' theory are more similar with the linear wave theory, which proves the arguments of Chakrabarti (2005). The author also argued that under the effect of these properties many submerged structures could simply be designed with the linear theory. However, the author has not specified the exact depth with similar properties.

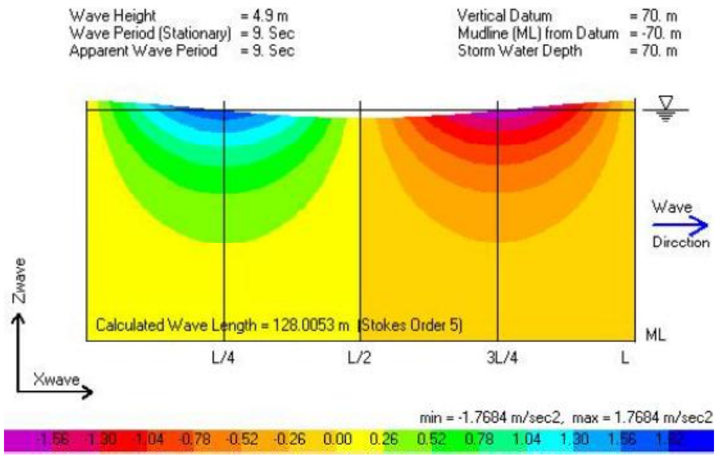

Figure 4: Acceleration contours graph for fifth-order Stokes theory.

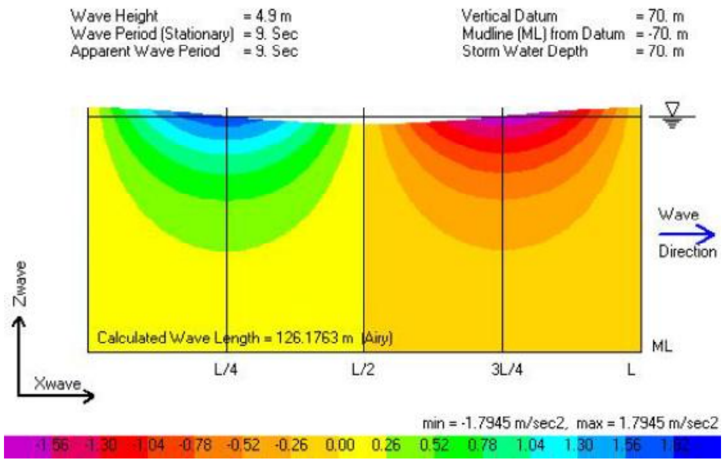

Figure 5: Acceleration contour graph for Airy wave theory

The aim of this study is to determine a depth of water below the *SWL*, in which both theories show the same behavior. To this purpose, the horizontal pressure of both theories are listed and normalized in Table (1). In the first and fifth columns, the random elevations from the crest to the sea floor for both theories are listed. The second and sixth columns represent the horizontal pressure corresponding to the same elevation for both theories. In the third and seventh columns the normalized dynamic pressure are computed. And in the last column, the percentage of similarity between the two theory are computed. The values of $({}^{S}/_{d})$, in the fourth column is the non-dimensional depth $(s = y + d)$, in which d is the depth of water and y stands for the amplitude of wave oscillation.

The normalized columns show that the values differ until around *10 m* below the *SWL*. After which downward to the bottom, both theories show the same values with 100% of similarity. The results indicate that the depth of $-10\ m$ is *the depth of similarity* for both models; however, with regard to the input data in both models, the depth of similarity can be changed from one region to another. Means that the ocean water properties such as density, viscosity and temperature, and wave parameters such as wave height, period and depth of water influence the depth of similarity.

Table 1: Comparison table for Airy and Stokes fifth order theory.

Stokes - Elevation (m)	Pressure (ton.f/m2)	P/H (ton.f/m3)	s/d	Airy - Elevation (m)	Pressure (ton.f/m2)	P/H (ton.f/m3)	Depth of Similarity (%)
2.38	0.13	0.03	1.03	2.38	0.31	0.06	96
1.98	0.49	0.10	1.03	1.98	0.66	0.14	96
1.59	0.85	0.18	1.02	1.59	1.01	0.21	97
1.19	1.21	0.25	1.02	1.19	1.37	0.29	97
0.79	1.58	0.33	1.01	0.79	1.73	0.36	97
0.40	1.94	0.41	1.01	0.40	2.08	0.44	97
0.00	2.30	0.48	1.00	0.00	2.44	0.51	97
-0.40	2.67	0.56	0.99	-0.40	2.80	0.59	97
-0.79	3.03	0.64	0.99	-0.79	3.16	0.66	97
-1.19	3.40	0.71	0.98	-1.19	3.52	0.74	97
-1.59	3.77	0.79	0.98	-1.59	3.88	0.82	98
-1.98	4.13	0.87	0.97	-1.98	4.24	0.89	98
-2.38	4.50	0.95	0.97	-2.38	4.61	0.97	98
-8.02	9.80	2.06	0.89	-8.02	9.86	2.07	99
-13.65	**15.21**	**3.20**	**0.81**	**-13.65**	**15.23**	**3.20**	**100**
-19.29	20.70	4.35	0.72	-19.29	20.71	4.35	100
-24.92	26.26	5.52	0.64	-24.92	26.26	5.52	100
-30.56	31.87	6.70	0.56	-30.56	31.87	6.70	100
-36.19	37.52	7.88	0.48	-36.19	37.52	7.88	100
-41.83	43.21	9.08	0.40	-41.83	43.20	9.08	100
-47.46	48.92	10.28	0.32	-47.46	48.91	10.28	100
-53.10	54.65	11.48	0.24	-53.10	54.64	11.48	100
-58.73	60.39	12.69	0.16	-58.73	60.38	12.69	100
-64.37	66.15	13.90	0.08	-64.37	66.14	13.90	100
-70.00	71.92	15.11	0.00	-70.00	71.91	15.11	100

The values in Table (1) are plotted in Fig.(6) to see the comparison of the two theories. Due to the proximity of the normalized pressure values for both theories in vicinity of the seafloor, the graph shows a conformation of both theories until the last four values at the upper part of the graph, which are not matched properly, indicating the difference of the values in the normalized pressures for both theories.

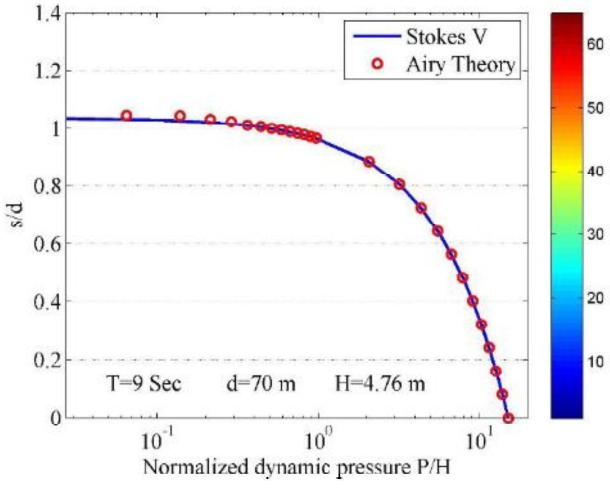

Figure 6: Comparison graph by normalized dynamic pressures

The wave loads on the submerged members are presented in Fig.(7). The loads on the structure's legs are greater at the areas closer to the *SWL*, which is a result of higher velocity at *SWL*. The loads in the bottom are generated by hydrostatic pressure in deep water; however, the overturning loads by surface waves make strong loads on entire structure parts. Due to the lack of dead and live loads on the structure, uplifting loads on the braces and beams show its dominancy.

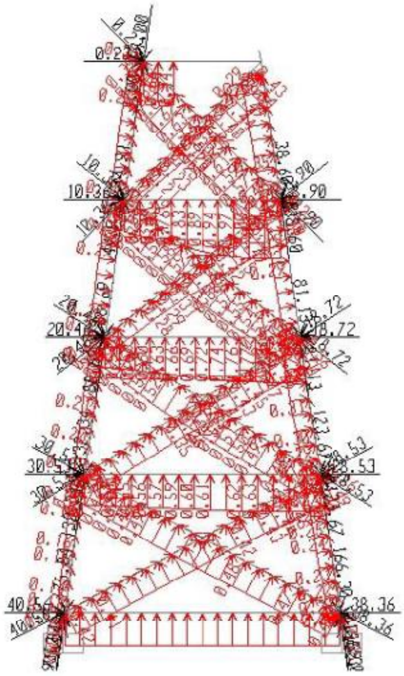

Figure 7: The wave loads applied on the structure

4 Conclusion

Two wave theories have been modeled to determine the behavior of wave in different levels of water. Airy wave theory is the most useful wave theory among all wave theories. It is easy to model and its formulas obtain the values of velocity, acceleration and pressure with a simple iteration. The Stokes fifth-order wave theory, on the other hand, is usually prescribed for the deep-water regions. It is a non-linear theory with high level of accuracy compared with other theories to obtain the kinematics and dynamic properties of waves. The results of developing the two theories indicate that the wave parameters, such as wave period, wave height and depth of water have significant effects on the results of both wave theories. In addition, both wave theories show identical behavior below the *SWL*. The depth of similarity initiates from $-10\ m$ ($10\ m$ below the *SWL*) toward the bottom with 99% of similarity between the linear and non-linear theories for the Labuan offshore area in the continental shelf of Malaysia. However, the depth of similarity could be different from one region to another, due to the effects of ocean water properties and wave parameters, which contribute in the analysis directly.

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