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Hydrological behaviour of a drained agricultural peat catchment in the tropics. 2: Time series transfer function modelling approach

Ayob Katimon[1], Shamsuddin Shahid[1], Ahmad Khairi Abd Wahab[1] and Ani Shabri[2]

[1]*Department of Hydraulics and Hydrology, Faculty of Civil Engineering, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia*
ayobkatimon@utm.my

[2]*Department of Mathematics, Faculty of Science, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia*

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Abstract Transfer function models of the rainfall–runoff relationship with various complexities are developed to investigate the hydrological behaviour of a tropical peat catchment that has undergone continuous drainage for a long time. The study reveals that a linear transfer function model of order one and noise term of ARIMA (1,0,0) best represents the monthly rainfall–runoff relationship of a drained peat catchment. The best-fitted transfer function model is capable of illustrating the cumulative hydrological effects of the catchment when subjected to drainage. Transfer function models of daily rainfall–runoff relationships for each year of the period 1983–1993 are also developed to decipher the changes in hydrological behaviour of the catchment due to drainage. The results show that the amount of rain water temporarily stored in the peat soil decreased and the catchment has become more responsive to rainfall over the study period.

Key words drained peat catchment; rainfall–runoff; transfer function model; ARIMA

Comportement hydrologique d'un bassin versant agricole drainé en zone de tourbière tropicale. 2: Approche de modélisation de la fonction de transfert

Résumé Des modèles de fonction de transfert pluie-débit de complexités diverses ont été construits pour étudier le comportement hydrologique d′ un bassin versant en zone de tourbière tropicale ayant subi un drainage continu pendant une longue période. L′ étude a révélé qu'une fonction de transfert linéaire d'ordre un avec un terme de bruit ARIMA (1,0,0) est le modèle représentant au mieux la relations pluie-débit d′ un bassin versant drainé en zone de tourbière à l'échelle mensuelle. Le meilleur ajustement du modèle est capable d′ illustrer les effets hydrologiques cumulatifs sur le bassin versant lorsqu′ il est soumis à un drainage. Des fonctions de transfert de la relations pluie-débit à l'échelle journalière ont également été construites pour chaque année de la période 1983–1993, afin de mettre en évidence les modifications du comportement hydrologique du bassin en réponse au drainage. Les résultats montrent que la quantité d′ eau de pluie stockée temporairement dans la tourbe a diminué, et que le bassin est devenu plus sensible aux précipitations au cours de la période d′ étude.

Mots clefs bassin versant drainé en zone de tourbière; pluie–débit; modèle de fonction de transfert; ARIMA

INTRODUCTION

Drainage is widely used in peatlands for lowering of peat water tables (Holden *et al.* 2004), reclamation of land for agriculture and other purposes. Land drainage activities can alter the hydrological characteristics of a catchment which, in turn, may have potential impacts on peat ecosystems (Holden *et al.* 2006, Worrall *et al.* 2007, Langner and Siegert 2009). To evaluate changes in the hydrological behaviour of drained catchments, which are often found in agricultural peatland, a sound understanding of the hydrological functions is required. Traditionally, paired-catchment experiments are used to evaluate the effect of disturbances (Newson and Robinson 1983, Robinson 1990, Brown *et al.* 2005). However, this approach is not only time-consuming, but also requires a properly planned and instrumented catchment. The paired-catchment approach is also unable to evaluate the relative importance of various factors (Schellekens 2000). Another easier approach for evaluating the impacts of disturbances on the hydrological functions of a catchment is to compare the hydrological records before and after the catchment has been altered (Newson and Robinson 1983, Rulli and Rosso 2007). This requires pre- and post-drainage hydrological data. When neither of the aforementioned approaches is applicable, due to either the absence of paired catchments or the unavailability of pre-drainage hydrological records, as experienced in the present study, alternative approaches such as deterministic physically-based models or systems-based (black-box) models are used (Mutua and Al-Weshah 2005, Mkhandi and Kumambala 2006, Beven 2012). Deterministic physically-based hydrological models are based on the complex laws of physics, generally expressed as systems of nonlinear partial differential equations (Skaggs 1991, Beven 2012). They are basically parameter-rich models that require intensive quantitative knowledge of the physical characteristics of the catchment at the spatial level (Zhang *et al.* 2009).

Systems-based models rely heavily on systems theory developed in other branches of engineering sciences and make little or no attempt to simulate the individual constituents of hydrological processes (Mkhandi and Kumambala 2006). The essence of these models is the empirical discovery of transfer functions (TF) which inter-relate the input (usually rainfall) and the output (usually discharge) in the time domain (Mutua and Al-Weshah 2005, Romanowicz *et al.* 2010). In comparison to physically-based models, the transfer function time series modelling approach has several advantages (Ali and Dechemi 2004, Young 2006). Physically-based hydrological models require parameterization and are based on the predetermined theory of hydrology, whereas a transfer function model is essentially a "black box" (Hipel and McLeod 1994, Lohani *et al.* 2011). The transfer function modelling approach does not require any theory to link the input and output series. In places where hydrological processes are not clearly defined, such as in drained peatlands (Katimon *et al.* 2002), time series transfer function modelling approaches are found to be appropriate (Mutua and Al-Weshah 2005).

This is the second of two papers describing the hydrological behaviour of a drained agricultural peat catchment in the tropics (Johor, Malaysia). In Part 1 (Katimon *et al.* 2013), the hydrological data of the catchment were analysed through conventional quantitative hydrological approaches to characterize the hydrological behaviour of the catchment, as well as changes in behaviour due to continuous drainage over a long period. In the present paper, dynamic transfer function models of the rainfall–runoff relationship with various complexities are developed to understand the changes in the hydrological behaviour of a tropical peat catchment that has undergone continuous drainage for a long time. Long-term rainfall–streamflow records obtained from a 184-ha drained agricultural catchment are used for the study.

STUDY AREA

The study area, located in the peat area of Parit Madirono in Johor, Malaysia (latitude: 01°42′35″ N; longitude: 103°16′15″ E) in the southwest of Peninsular Malaysia, and known as Madirono catchment, is described in detail in Part 1 (Katimon *et al.* 2013). A hydrological monitoring programme in this catchment between 1981 and 1996 has provided reliable long-term hydrological records for the catchment for use in this study.

METHODOLOGY

Transfer functions (TF) are linear models with which an output variable can be forecast as a linear weighted combination of past outputs (stream flow) and inputs (rainfall). Any residual model error can be represented through a noise model, which is generally an autoregressive integrated moving average (ARIMA) model (Bell *et al.* 2001, Yuan *et al.* 2009). The basic structure of the TF model and the algorithm used to develop the models in the present study are discussed below.

Model structure

A single linear TF model representing the relationship between input and output time series can be expressed as:

$$Y_t = C + v(B)X_t + N_t \tag{1}$$

where Y_t is the output series or exogenous variables; X_t is the input series or endogenous variables; C is the constant term; $v(B)$ is the dynamic component, or impulse response function of the model; N_t is the stochastic noise; and B is the backshift operator. The stochastic noise N_t may be autocorrelated and assumed to be independent of X_t. Because the dynamic term $v(B)$ in equation (1) represents the dynamic behaviour of serial correlations of X_t at different time lags, it can be written in polynomial form as (Makridakis *et al.* 1998):

$$v(B) = v_0 + v_1 B + v_2 B^2 + \cdots + v_k B^k \quad (2)$$

where $v_0, \ldots, v_k$ are transfer function weights, or impulse response weights. Thus, equation (1) becomes (Makridakis *et al.* 1998):

$$Y_t = C + (v_0 + v_1 B + v_2 B^2 + \cdots + v_k B^k)X_t + N_t \quad (3)$$

where k is the order of the transfer function, i.e. the longest lag in input series X_t.

Parsimonious model structure

The degree of model complexity, as indicated by the number of parameters, is fundamental to model developers and model users. An important criterion of a good model is its simplicity or parsimony. A parsimonious model contains the least number of coefficients, but adequately explains the behaviour of the observed data (Ledolter and Abraham 1981, Box *et al.* 1994, Chappell *et al.* 1999). Astrup *et al.* (2008) conducted a study to find the appropriate level of complexity for a simulation model, and concluded that the simplest and the most complex growth functions had the poorest predictive ability, while functions of intermediate complexity had the best predictive ability. According to Wagener *et al.* (2001), careful consideration must be given in using parsimonious models to ensure that the model does not omit one or more hydrological processes important for a particular problem. Beven (1989) suggests that, in spite of the dozens of parameters normally included in watershed models, three to five parameters should be sufficient to reproduce most of the information in a hydrological record. Jakeman and Hornberger (1993) and others have drawn similar conclusions. Following the above-mentioned suggestions, parsimonious transfer function models are developed in the present study.

In equation (3), the term $v(B)$ could have a large number of weights, v (thus a large number of time lags). This can present serious estimation problems, since the size of the sample is always limited. By reducing the number of parameters, a parsimonious model can be developed. Thus, the term $v(B)$ in equation (2) is rewritten in a simpler form as (Makridakis *et al.* 1998):

$$v(B) = \frac{\omega(B)}{\delta(B)} X_{t-b} + N_t \quad (4)$$

and the parsimonious form of equation (3) becomes:

$$Y_t = C + \frac{\omega(B)}{\delta(B)} X_{t-b} + N_t \quad (5)$$

where $\omega(B) = \omega_0 - \omega_1 B - \omega_2 B^2, \ldots, -\omega_s B^s$ and $\delta(B) = 1 - \delta_1 B - \delta_2 B^2, \ldots, -\delta_r B^r$, and b, s and r are constants. Constant b is a delay factor, i.e. a period of delay, b, before X_t begins to influence Y_t. The constant r is the decaying factor of the impulse response weights, and s is the "dead time" factor.

Feedback checking

The possibility of feedback arises when the inputs are stochastic, such as in rainfall events (Pankratz 1991, Box *et al.* 1994). Although it is unlikely that streamflow (output) may affect rainfall (input), a standard feedback test is desirable. Therefore, to ensure that there is no feedback from earlier values of the Y_t series to current values of the X_t series, the input series is regressed on its own past, and on the past of the output series (Granger and Newbold 1986). Decomposing the B terms into: $B = X_{t-1}$, the regression-lag model of equation (3) of order k becomes:

$$Y_t = C + v_0 X_t + v_1 X_{t-1} + v_2 X_{t-2} + \ldots + v_k X_{t-k} + N_t \quad (6)$$

To check the feedback effect of the Y_t series on the X_t series, the following equation can be estimated:

$$X_t = C + b_1 X_{t-1} + b_2 X_{t-2} + \cdots + b_k X_{t-k} + c_1 Y_{t-1} + c_2 Y_{t-2} + \cdots + c_k Y_{t-k} + N_t \quad (7)$$

Using a multiple regression approach, coefficients $c_1, c_2, \ldots, c_k$ can be computed and their significance can be estimated.

Modelling algorithm

The algorithm proposed by Pankratz (1991) is used for the development of the TF models. The algorithm is summarized in the flow chart presented in Fig. 1.

In Step 1, the free-form distributed-lag model equation of order k can be written, from equation (1), as:

$$Y_t = C + v_0 X_t + v_1 X_{t-1} + v_2 X_{t-2} + \dots + v_k X_{t-k} + N_t \quad (8)$$

where $v_0, v_1, \dots, v_k$ are impulse response weights or TF coefficients and N_t is the noise series. The order of $v(B)$ is chosen arbitrarily according to their significance levels, and the response weight values are estimated by using the multiple regression approach.

A proxy ARIMA model for the noise series is used in the TF model (Step 2). The noise series produced by the distributed-lag model is compared to that of the proxy model in terms of stationarity. The best-fitted ARIMA model for the mean monthly flow series was in the form ARIMA (1,0,0). Thus, ARIMA (1,0,0) is the proxy noise model used for the development of a TF model of the rainfall–runoff relationship, and can be written as:

$$(1 - \phi_1 B)\, Y_t = C + a_t \quad (9)$$

Considering only the noise terms, equation (9) becomes:

$$Y_t = \frac{1}{1 - \phi_1 B} a_t \quad (10)$$

where ϕ_1 is the AR(1) parameter and a_t is the error series.

A TF model of order k is thus a combination of its distributed-lag model (equation (6)) and the ARIMA model of the disturbance series (equation (10)), and can be written as:

$$Y_t = c + v_0 X_t + v_1 X_{t-1} + v_2 X_{t-2} + \dots + v_k X_{t-k} + \frac{1}{1 - \phi_1 B} a_t \quad (11)$$

Daily rainfall and streamflow records collected by the Water Resources Division of the Department of Irrigation and Drainage, Malaysia, over the period 1983–1993 are used to develop the transfer function models and simulate the hydrological processes of the catchment.

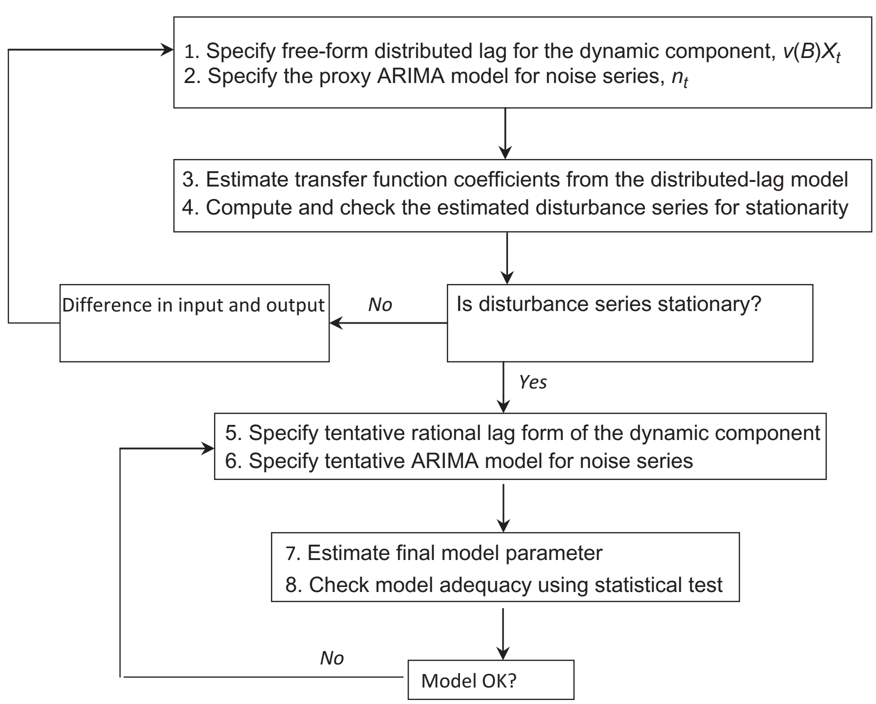

Fig. 1 Flow chart of the transfer function algorithm.

RESULTS AND DISCUSSION

Transfer function model of the study catchment

Development of the TF model of the mean monthly rainfall–runoff relationship for the experimental catchment is discussed below in detail. Figure 2 shows the monthly rainfall–streamflow series of the study catchment. The rainfall series, denoted by P_t, is the input variable, while the streamflow series, Q_t, is the output variable. The rainfall series is the known factor that affects the runoff series. Assuming the serial relationship between P_t and Q_t of order k, this can be written as:

$$Q_t = c + v_0 P_t + v_1 P_{t-1} + v_2 P_{t-2} + \ldots + v_k P_{t-k} + N_t \quad (12)$$

Thus, the serial relationship between the past time-lag series of the present input series and the past output series can be written as:

$$P_t = C + b_1 P_{t-1} + b_2 P_{t-2} + \ldots b_k P_{t-k} + c_1 Q_{t-1} + c_1 Q_{t-1} + c_2 Q_{t-2} + \cdots + c_k Q_{t-k} + N_t \quad (13)$$

To check the feedback effect of the output series on the input series, a multiple regression technique is used to estimate the values of $c_1, c_2, \ldots, c_k$. Estimated c_i values up to the order of 3 are given in Table 1. It can be seen from Table 3 that, except for constant C, all the corresponding t values are small and not significant at the 95% level of confidence. Therefore, it is very clear that there is no feedback effect from the past of the output (streamflow) to the input series (rainfall).

Table 1 Statistical output of feedback effect analysis of the mean monthly rainfall–streamflow time series

Parameter	σ	CV	t-test	Significance, p
C	28.42		5.896	0.000
c_1	0.079	0.386	0.611	0.542
c_2	0.086	−0.034	0.409	0.683
c_3	0.085	0.010	0.123	0.903

Note: σ, standard deviation; CV, coefficient of variation.

A multiple regression model is fitted to the monthly rainfall–runoff time series to obtain lagged values. During model fitting, it is assumed that the noise series N_t belong to an ARIMA (1,0,0) model and the error series of this proxy model is stationary. The fitted multiple regression model to the mean monthly rainfall–runoff data of lagged values up to x_{t-3} is:

$$Q_t = 191.68 + 0.137 P_t - 0.0211 P_{t-1} - 0.059 P_{t-2} - 0.165 P_{t-3} + N_t \quad (14)$$

The statistical evidence of equation (14) is presented in Table 2 and the plot of the TF coefficients against their lags is shown in Fig. 3. It is clear from Fig. 3 that a non-exponential pattern of the decaying factor exists. Using identification rules proposed by

Table 2 Estimates of TF coefficients and their statistics

Parameter	CV	Std error	t-test	Significance, p
C	191.684	47.682	4.020	0.000
v_0	0.137	0.073	1.888	0.061
v_1	−0.02114	0.073	−0.290	0.772
v_2	−0.05906	0.073	−0.809	0.419
v_3	−0.165	0.072	−2.278	0.024

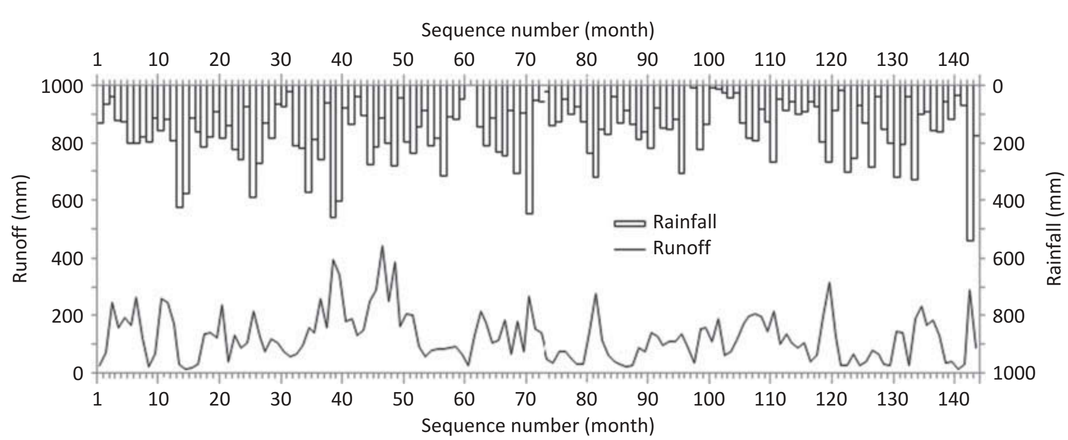

Fig. 2 Mean monthly rainfall–runoff time series of the study catchment.

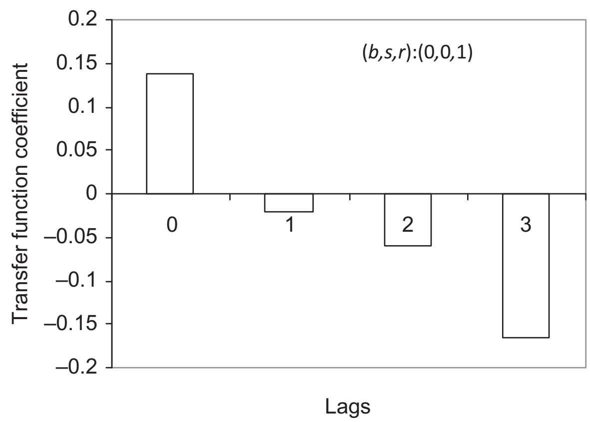

Fig. 3 Estimates of TF coefficient showing a decaying pattern.

Pankratz (1991) and Makridakis *et al.* (1998), the following (b,s,r) model order is identified:

- From the p values of Table 2, it is clear that there is no delay: the first significant coefficient is at lag 0. Therefore, the model constant $b = 0$. This is expected in the case of the mean monthly rainfall–runoff relationship in hydrology.
- The decay pattern of the coefficient (indicated by the dotted line) follows a simple exponential decay. Thus, $r = 1$.
- Figure 3 shows that the coefficient began to decay at lag 0. Thus, $s = 0$.

An error series is thus obtained as:

$$N_t = Q_t - 191.68 - 0.137P_t + 0.0211P_{t-1} + 0.059P_{t-2} + 0.165P_{t-3} \quad (15)$$

Figure 4(a), (b) and (c), respectively, shows the regression errors, the auto-correlation-function (ACF) plot and the partial-auto-correlation-function (PACF) plot of the model of equation (15). It can be seen from the N_t series ACF and PACF plots that the significant spikes are at lags 1 and 3. This suggests that AR(1), MA(1), AR(3) or MA(3), or a combination of ARIMA models could be the best-fitted model. Nevertheless, using the Aikaike Information Criterion (AIC) (Makridakis *et al.* 1998), the AR(1) model has the smallest AIC value. Therefore, the ARIMA (1,0,0) model is adopted as the best-fitted error series for the mean monthly rainfall–runoff relationship.

With a zero dead time ($s = 0$), the general form of the parsimonious model of the full model of equation (12) can be written as:

$$Q_t = C + \frac{\omega(B)}{\delta(B)} P_{t-b} + N_t \quad (16)$$

where $\omega(B) = \omega_0 - \omega_1(B)$ and $\delta(B) = 1 - \delta_1 B$.

Thus:

$$Q_t = C + \frac{\omega_0 - \omega_1(B)}{1 - \delta_1(B)} P_t + N_t \quad (17)$$

where $N_t = 1/(1 - \phi_1 B)a_t$ and a_t is the error series.

Parameters ω_1 and δ_1, and constant C are estimated from the initial values of ω_0 and ϕ_1 using the ordinary least square (OLS) method.

Parameter estimation using OLS method

As the objective is to find the best model parameters, ω_0, ω_1, δ_1, ϕ_1 and C, a best-fitting model is used to present the input–output relationship. First a preliminary estimate is chosen and then a computer program is used to refine the estimate iteratively until the sum of square errors (SSE) is below a threshold level. For a regression model with one independent variable, the estimator can be presented as (Gujarati 1988):

$$b_1 = \frac{\sum_{i=1}^{n} (X_{1,i} - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_{1,i} - \overline{X})^2} \quad (18)$$

where $\overline{X}$ and $\overline{Y}$ are the sample means of X_i and Y_i.

Following practical rules (Makridakis *et al.* 1998), the initial or range values of ω_0 and ϕ_1 are taken from the regression-lag model (equation (15)). Thus, the final model of the mean monthly rainfall–runoff relationship becomes:

$$Q_t = C + \frac{\omega_0 - \omega_1 B}{1 - \delta_1 B} P_t + \frac{1}{(1 - \phi_1 B)} a_t \quad (19)$$

Confidence intervals at 95% can be estimated from the noise parameters as:

$$\left[\hat{Q}_t - 1.96\sqrt{\sigma_a^2/\left(1 - \phi^2\right)},\ \hat{Q}_t + 1.96\sqrt{\sigma_a^2/\left(1 - \phi^2\right)}\right] \quad (20)$$

where $\hat{Q}_t$ is the simulated streamflow and σ_a^2 is the variance of the noise series, a_t. The model parameters, ω_0, ω_1, δ_1, ϕ_1 and C are estimated iteratively using

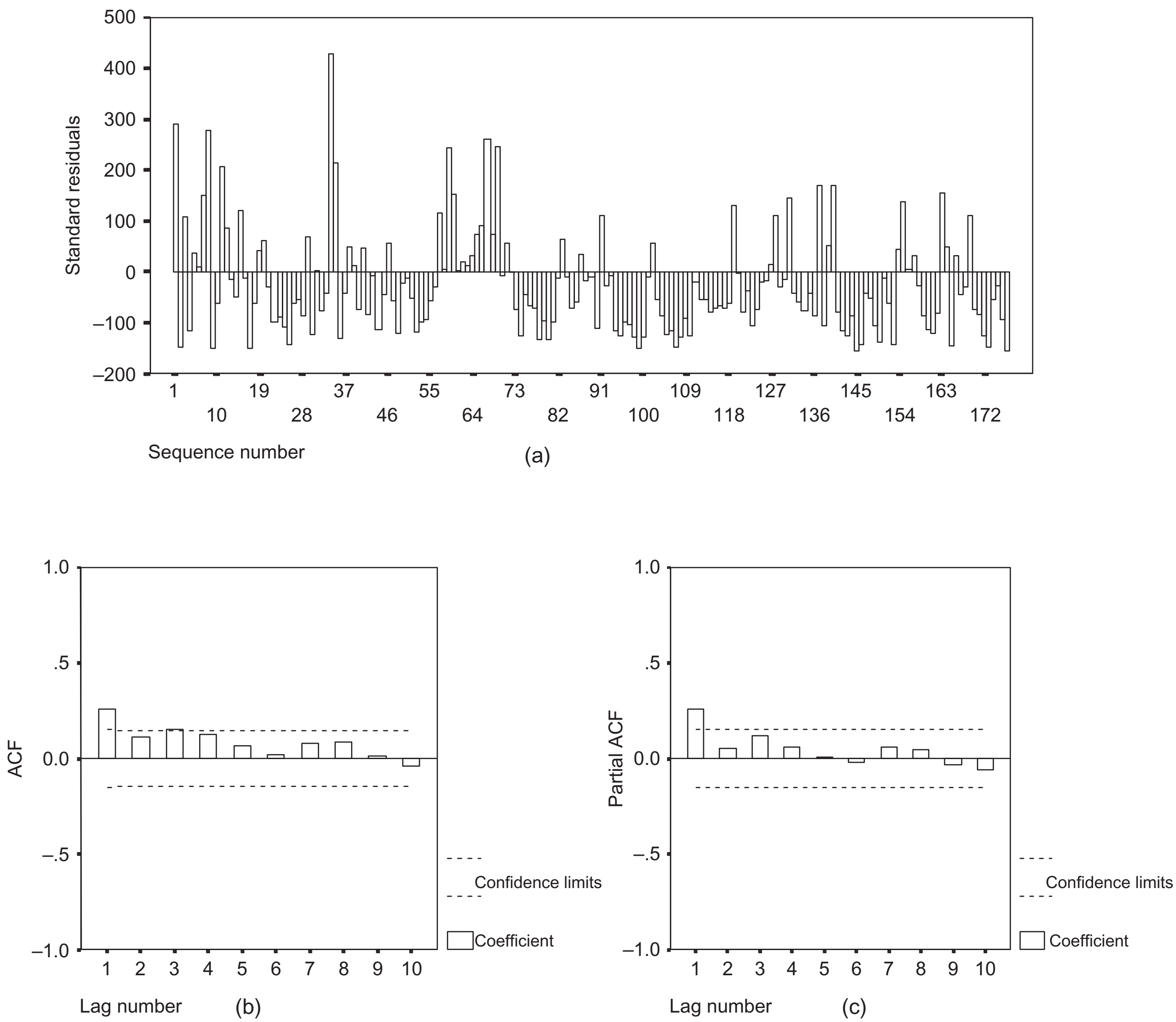

Fig. 4 (a) Regression errors from equation (15); (b) ACF plot; and (c) PACF plot.

the ordinary least square (LS) method. Equation (19) cannot be solved analytically, because it involves non-linear functions. In the present study, the parameters are estimated iteratively by using a program written in MATLAB. The following values of model parameters are obtained:

$C = 159.53$
$\omega_0 = 0.1773$
$\omega_1 = 0.0010$
$\delta_1 = 0.3030$
$\phi_1 = 0.2348$

where $\omega(B)$ is of order zero ($s = 0$), $\delta(B)$ is of order one ($r = 1$) and the noise term is ARIMA (1,0,0). Therefore, the final model can now be written as:

$$Q_t = 159.53 + \frac{0.1773P_t}{(1 - 0.3030B)} + \frac{1}{(1 - 0.2348B)}a_t \tag{21}$$

where $B = P_{t\text{-}1}$ and a_t is the error series.

The SSE in the prediction by the model is 37.45. The model coefficient (δ_1) also satisfies $|\delta_1| < 1$, a criterion used to check the stability of a first-order model (Box *et al.* 1994). Therefore, it can be considered as a reasonable model.

Finally, the error series of the model is checked. Figure 5 shows the histogram plot of the residual series a_t from the model of equation (20). As shown

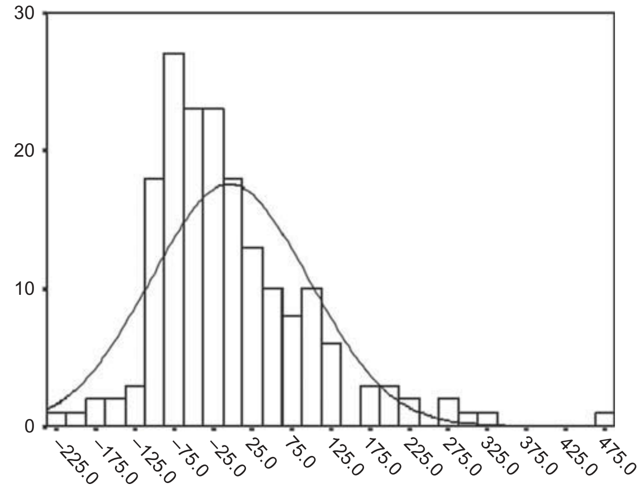

Fig. 5 Histogram of residuals from the TF model.

in Fig. 5, the residuals are roughly symmetrical and, therefore, it can be stated that the error is normally distributed.

Model interpretation: monthly rainfall–streamflow relationship

The TF model of the mean monthly rainfall–streamflow relationship shows that, when rainfall rises by one unit, runoff responds immediately ($b = 0$). Runoff rises (ω_0 is positive) initially by 0.177 units ($\omega_0 = 0.177$). With subsequent time lags, runoff rises gradually, but with a decaying amount according to the first-order exponential decay pattern, with a decay coefficient, $\delta_1 = 0.3030$. The constant term ($C = 159.53$) indicates that the flow series rises by 159.53 units in each time period in addition to any other movements dictated by the TF or disturbance of ARIMA pattern.

Evaluation of model performance

The root mean square error (RMSE) and Nash-Sutcliffe efficiency (NSE) (Nash and Sutcliffe 1970) are used to examine the model performance. The RMSE and NSE measure the goodness of fit and are defined as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} (Y_m - Y_o)^2}{N}} \tag{22}$$

$$\text{NSE} = 1 - \frac{\sum_{i=1}^{N} (Y_o - Y_m)^2}{\sum_{i=1}^{N} \left(Y_o - Y_{\text{avg}}\right)^2} \tag{23}$$

where Y_m is the model predicted discharge, Y_o is the observed discharge, Y_{avg} is the average observed discharge, and N is the number of data points.

For the TF model (equation (20)) of the mean monthly rainfall–streamflow relationship, the RMSE is 31.88 mm. This value is reasonably small. Figure 6(a) compares the simulated flow obtained by using the model with the observed flow and Fig. 6(b) presents a scatter plot of the simulated series *vs* the observed series. Figure 6(a) shows that the relationship between the mean monthly rainfall and the mean daily rainfall is fairly represented by the TF model of equation (20). Nevertheless, the scatter diagram in Fig. 6(b) shows that the runoff is under-predicted by 2% with NSE of 0.98. Overall, it can be concluded that the TF model of the rainfall–runoff relationship is capable of showing the hydrological dynamics of the catchment by means of its steady-state function.

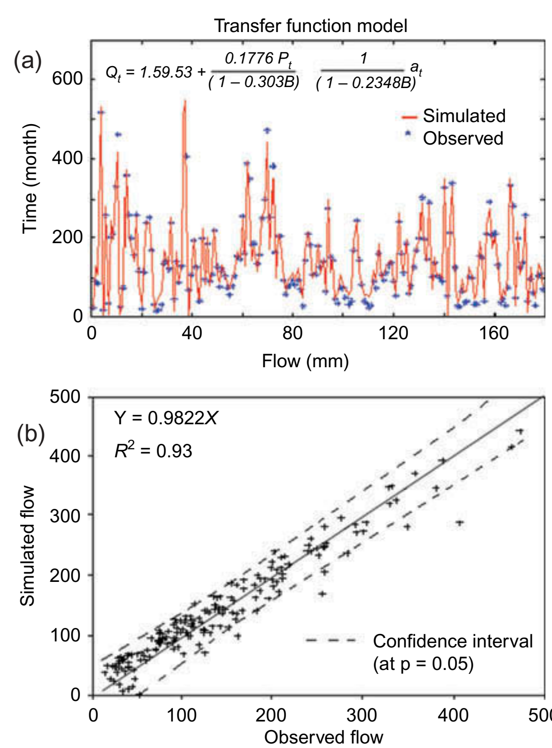

Fig. 6 (a) Plot of observed and modelled monthly streamflow series; (b) scatter plot of modelled monthly streamflow series *vs* observed series.

Transfer function models of the daily rainfall–streamflow relationship

The general TF model of equation (20) is also applied to simulate daily streamflow from runoff. Transfer function models of daily rainfall–runoff relationships for different hydrological years are developed to examine the changes in the model parameters over the study period. The TF coefficients or impulse response functions of the models are regarded as the output or response at times $j \geq 0$ to a unit pulse input at time 0. According to Box *et al.* (1994), when there is no immediate response, one or more of the initial v values will be equal to zero.

The estimated model order and model parameters for individual years are tabulated in Table 3 and graphically presented in Fig. 7. Employing the estimated model parameters and the distribution lags of

Table 3 Transfer function model parameters in the different time series

Year	Model order			Transfer function parameters				
	b	s	r	ω_0	ω_1	δ_1	C	σ
1983	0	1	2	0.0040	0.031	0.243	5.5597	5.59
1984	0	1	2	−0.0254	0.001	0.203	3.7313	3.049
1985	0	0	1	0.0916	−0.109	−0.017	2.2413	0.768
1986	0	1	1	0.0982	−0.459	−0.007	4.4199	3.546
1987	0	1	1	0.0547	−0.079	0.003	2.1535	1.874
1988	0	1	1	0.2210	−0.349	−0.207	1.8381	0.727
1989	0	0	1	0.1538	−0.239	−0.067	1.4077	1.963
1990	0	1	0	0.0682	−0.139	0.003	1.3478	1.809
1991	0	0	1	0.1780	−0.179	0.013	0.8519	1.220
1992	0	1	1	0.3353	−0.499	−0.217	1.7458	1.865
1993	0	1	1	0.0150	−0.109	−0.027	1.3843	1.68

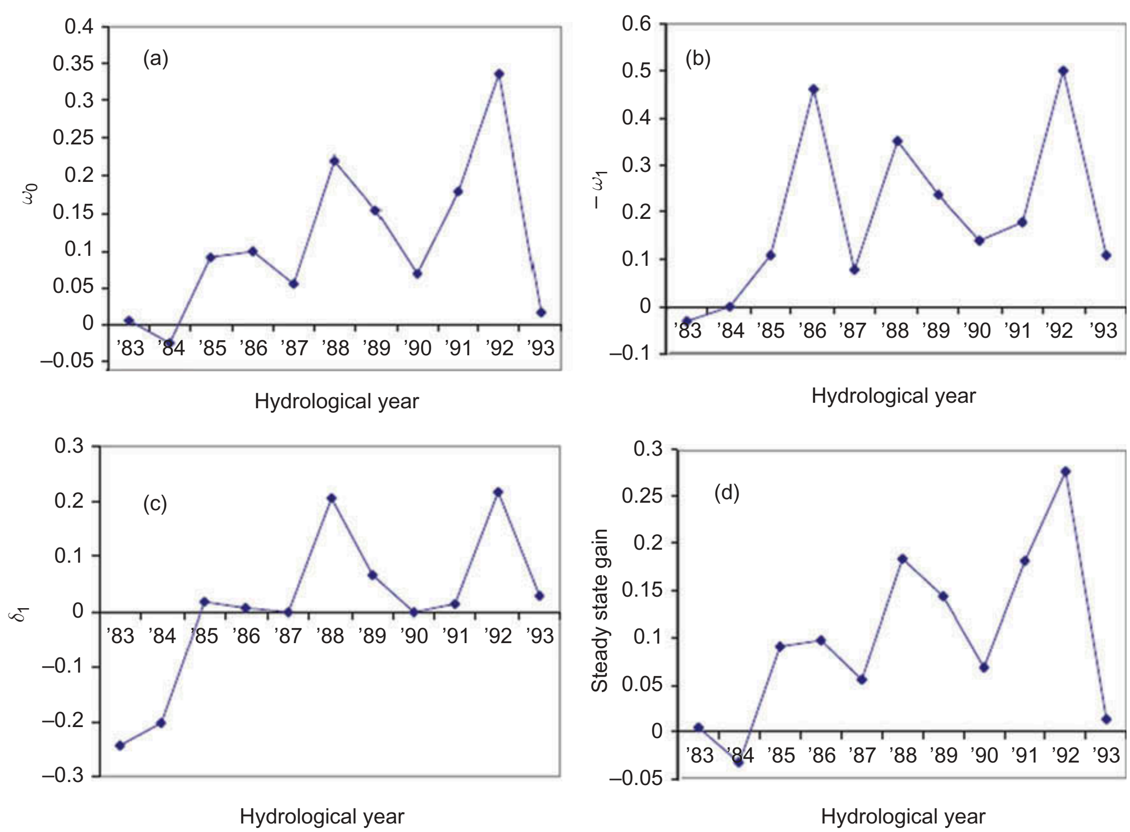

Fig. 7 Variation of model parameters with time: (a) ω_0, (b) ω_1 and (c) δ_1. (d) Steady-state gain, g, of the transfer function model.

the impulse response coefficients, the complete TF model for each hydrological year is developed. For example, daily rainfall–runoff series relationships for the years 1983 and 1992, respectively, are best represented by the TF models given in equations (24) and (25):

(b,s,r) : (0,1,2), ARIMA (3,1,1)

$$Y_t = 5.59 + \frac{(0.004 - 0.03B)}{1 - 0.243B - \delta_2 B^2} X_t + \frac{1}{\left(1 - 0.77B - 0.22B^2 - 0.16B^3\right)} a_t \tag{24}$$

(b,s,r) : (0,1,1), ARIMA (2,0,0)

$$Y_t = 1.75 + \frac{(0.34 - 0.5B)}{1 - 0.22B} X_t + \frac{1}{\left(1 - 0.85B + 0.14B^2\right)} a_t \tag{25}$$

The computer-generated plots of simulated series *vs* observed series for individual hydrological years are presented in Fig. 8. Corresponding scatter plots of simulated values *vs* observed values for different hydrological years at 1:1 scale are also given in Fig. 8. The simulated results demonstrate that the developed TF models have under-predicted the runoff in almost all hydrological years, by 4% to 30%, with a NSE of between 0.5 and 0.98. Nevertheless, the RMSE values are reasonably small, ranging from 0.7 to 1.8 mm. Therefore, the TF models can be regarded quite acceptable. Errors in daily rainfall–runoff models are found higher compared to that in the monthly rainfall–runoff model. This is due to the smoothness in monthly data. Usually, monthly streamflow time series are smoother than daily streamflow time series, and hydrological models can fit the monthly time series with less error compared to daily time series.

To test the stability of the models, the values of model parameters are checked (Pankratz 1991, Box *et al.* 1994, Veloce 1996). For a first-order TF model, if the parameter δ_1 ranges between –1 and 1, it can be regarded as stable model. For the second-order model, parameters δ_1 and δ_2 should satisfy the following criteria: $\delta_2 + \delta_1 < 1$; $\delta_2 - \delta_1 < 1$; and $-1 < \delta_2 < 1$.

As indicated by the order (r) of the models (Table 3), it is obvious that most of the TF models developed for daily rainfall–runoff belong to first-order dynamic models, except for the years 1983, 1984 and 1990. For these three hydrological years, the second-order model seems to be more suitable. However, as the present modelling effort is focused on development of a parsimonious model, only first-order models are considered to show the changes in model parameters with time. Figure 7(c) shows that δ_1 values of all the models satisfy the condition of stability. On this count, it can be stated that the models are capable of predicting daily runoff from rainfall.

Interpretation of the daily rainfall–streamflow relationship models

A TF model of an input–output series of a catchment system can be represented by a simple block diagram, as shown in Fig. 9. With rainfall as the input series and streamflow as the output series, the dynamic components of the figure can collectively represent the physical behaviour of the catchment, such as the rheological properties (e.g. specific yield and water transmissivity) of the peat materials, as well as the geometric properties of the basin. The geometric aspects of the basin that govern the rainfall–runoff processes include the depth of unsaturated profile, the peat deposit, and the ground surface configuration.

Thus, the coefficients, ω_0, ω_1 and δ_1 of the TF models shown in Fig. 9 represent the physical properties of the system. Analogous to discrete signal processing theory in control engineering, the impulse response pattern of the dynamic system represents the inertia or resistance of the system. Intuitively, the deviation of the output series, Q_t can be regarded as a linear aggregate of a series of superimposed impulse response functions scaled by the deviation of the input series, P_t (Box *et al.* 1994).

For a first-order discrete TF model, the model is said to be more stable when $|\delta_1|$ is close to null. To interpret the physical meaning of the model input–output relationship, a steady-state gain (SSG) function, g is introduced. The SSG of a model is a measure of sensitivity of the equilibrium level of the output series to one unit change in the input series. In other words, the SSG of a dynamic system is defined as the change in output series divided by the change in input series when the rate of change in the output series has reached equilibrium stage. For a stable model, the SSG can be expressed in terms of model parameters and is defined as (Pankratz 1991, Box *et al.*1994, Hipel and McLeod 1994):

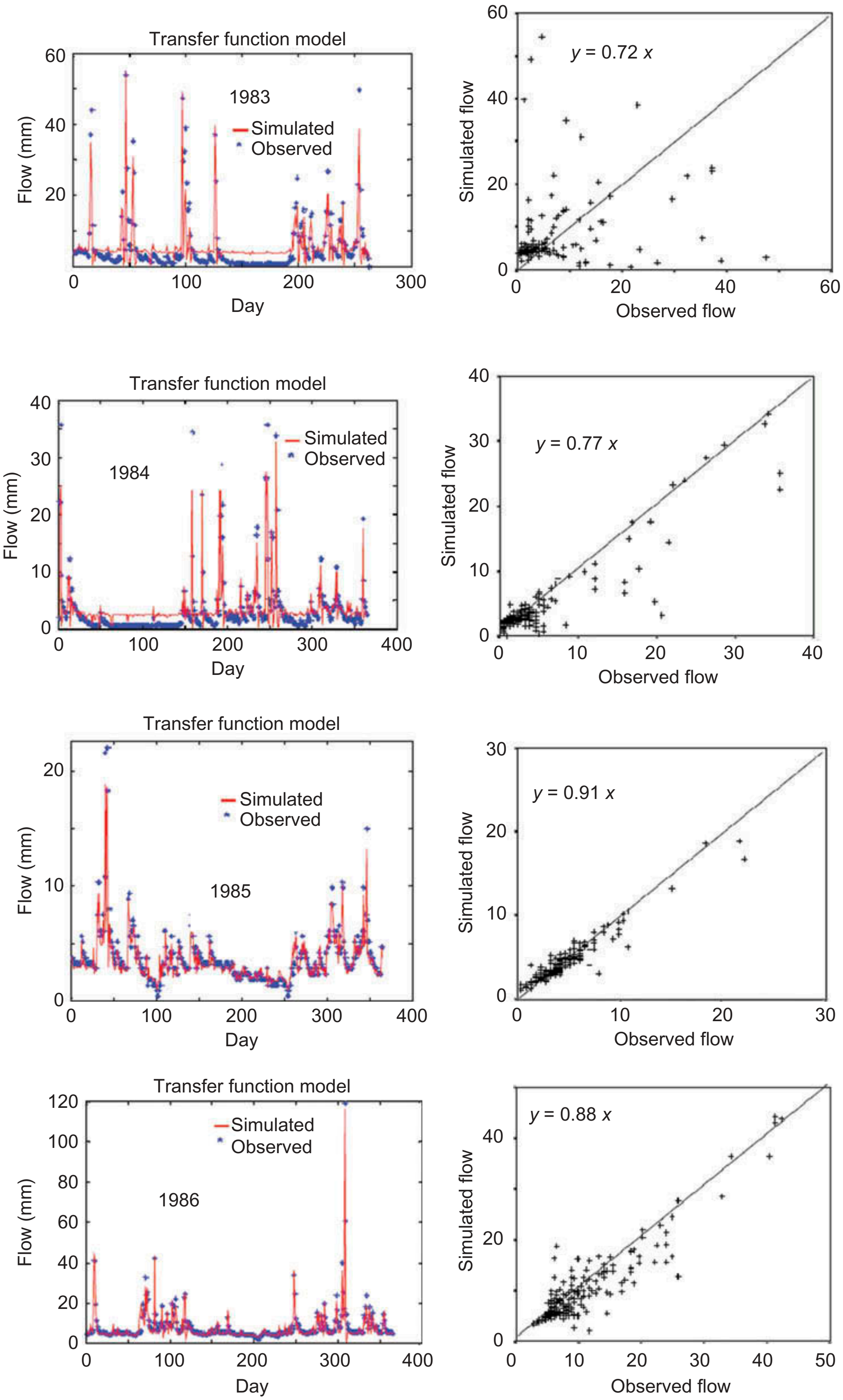

Fig. 8 Computer-generated plots of simulated *vs* observed series (TF model) for individual hydrological years (left) and corresponding scatter plots of simulated *vs* observed at 1:1 scale (right).

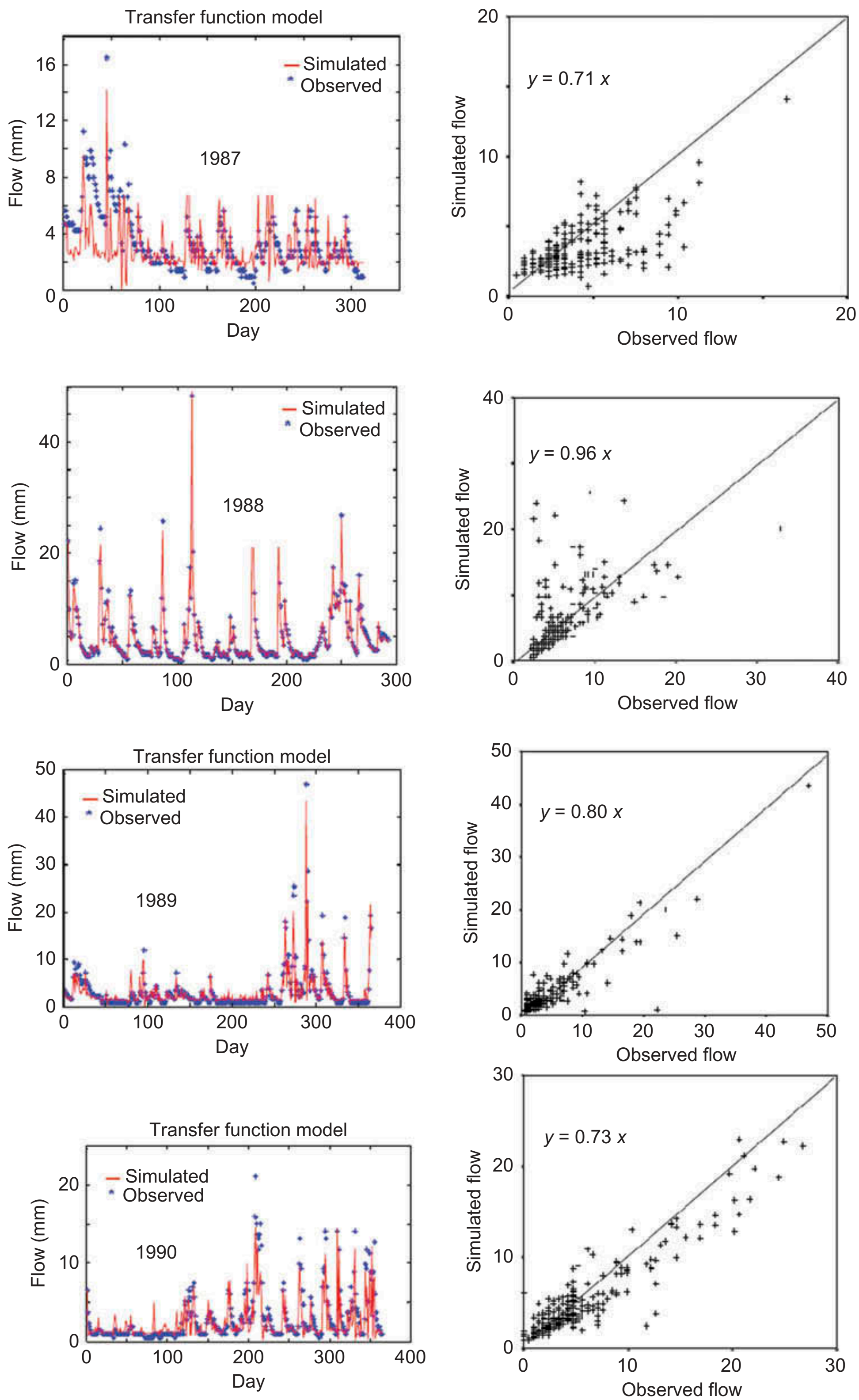

Fig. 8 (Continued).

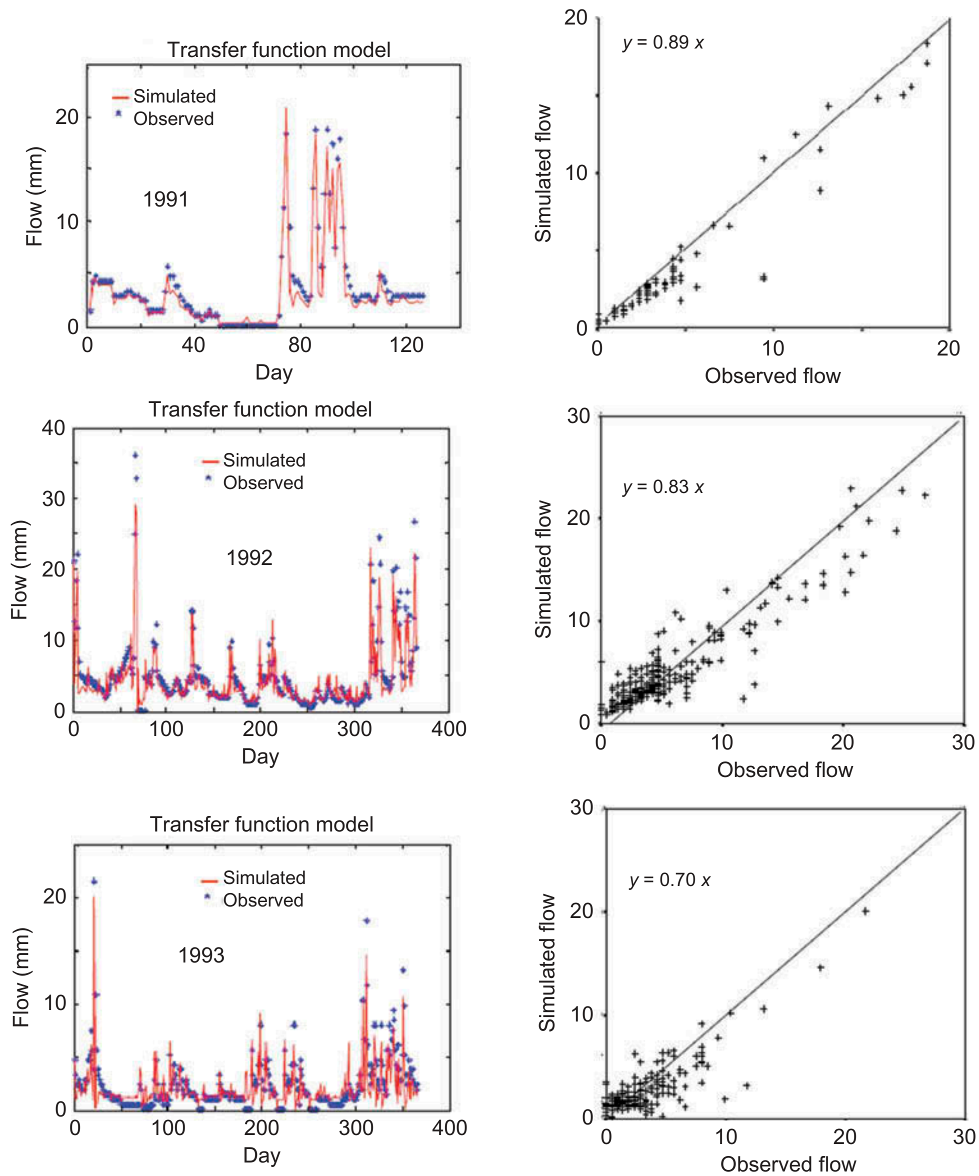

Fig. 8 (Continued).

$$g = \sum_{i=0}^{\infty} v_i = \frac{\omega(1)}{\delta(1)} \tag{26}$$

For a first-order model:

$$\begin{aligned} v(B) &= \frac{\omega_0 B}{(1 - \delta_1 B)} \\ &= \omega_0 \left(1 + \delta_1 B + \delta_1^2 B^2 + \ \ldots\right) B \end{aligned} \tag{27}$$

Considering B as an ordinary algebraic variable, with $B = 1$, and substituting it into the above equation, we can get the simple form of steady-state gain as:

$$g = \frac{\omega_0}{1 - \delta_1} \tag{28}$$

And, for example, the steady-state gain for the model of equation (21) is: $g = 0.1771/(1 - 0.3030) = 0.25$. Thus, for this particular example, one unit rise in rainfall (P_t) will lead to an eventual equilibrium rise in runoff (Q_t) by 0.25 units. The variation in SSG of the transfer function model in this study in different years is presented in Fig. 7(d). The variation is almost identical to ω_0 (Fig. 7(a)). This is expected, because most of the models are first-order models.

To assess the impacts of rainfall over the model parameters, the relationships between annual rainfall and model parameters are analysed. The variation

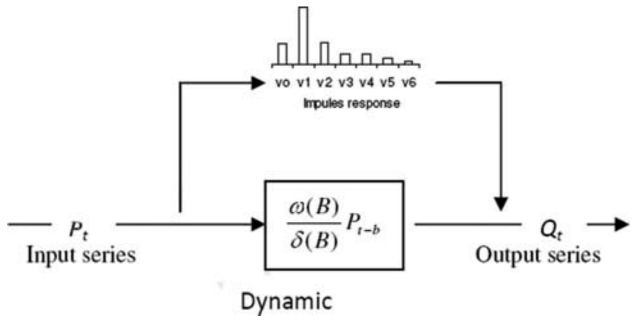

Fig. 9 Block diagram of an input–output transfer function model.

of annual rainfall over the study catchment during the period 1983–1993 is shown in Fig. 10(a) and scatter plots of annual rainfall with four model parameters are shown in Fig. 10(b), (c), (d) and (e). Correlation analysis using the non-parametric Kendall-tau method reveals no significant relationship between rainfall and model parameters. The hydrological response of a catchment to rainfall depends on many factors, including, for example, rainfall amount, rainfall intensity, rainfall distribution and soil moisture condition. Therefore, the above result does not cancel the influence of rainfall on model parameters. However, the analysis of rainfall, runoff and groundwater level data carried out in the first part of this study (Katimon *et al.* 2013) indicates that drainage has changed the hydrological behaviour of the catchment. Therefore, it can be stated that the changes in model parameters may be due to drainage, as well as to variations in rainfall and other factors.

It can be seen from Fig. 7(d) that SSG varies from year to year; it more or less follows a positive exponential trend, at least until the year 1992. This can be interpreted as: (a) the catchment becoming more responsive to rainfall; and (b) the time delay of the streamflow reaching equilibrium stage is increasing. When related to the storage capacity of the catchment, this means that the amount of rain water temporarily stored in the soil is reducing with time.

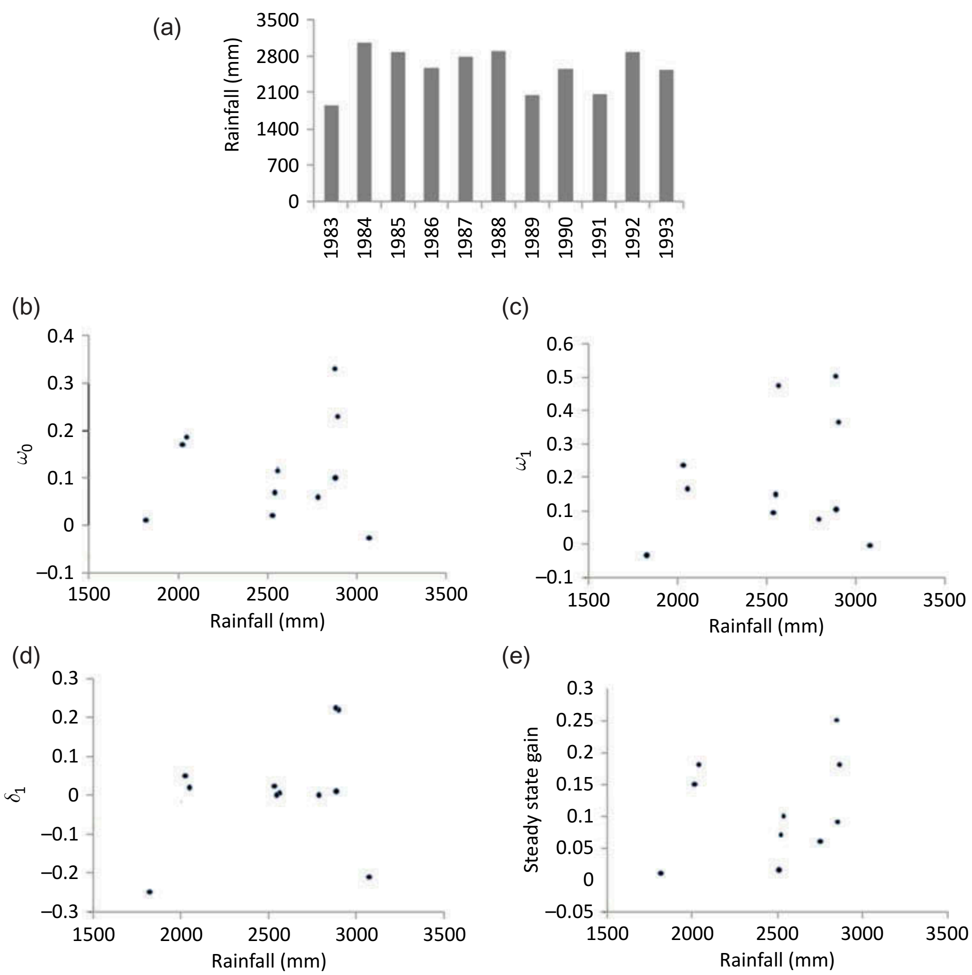

Fig. 10 (a) Variation of annual rainfall over the study catchment with time (1983–1993); and the relationships between rainfall and parameters: (b) ω_0, (c) ω_1, (d) δ_1, and (e) steady-state gain, *g*, of the transfer function model.

CONCLUSIONS

The hydrological behaviour of a catchment depends on rainfall amount and intensity, and the distribution of rainfall, antecedent soil moisture condition, land cover and many other factors. It is not possible remove the influence of all these factors to clearly show the impacts of drainage on catchment hydrological behaviour. Usually, a paired-catchment method or pre- and post-drainage data are required to understand the impact of drainage on the hydrological behaviour of a catchment. As it is not possible in the present study area, due to the absence of paired catchments, and unavailability of pre-drainage hydrological records, empirical transfer function models were developed to investigate the changes in the dynamic relationships between rainfall and streamflow (runoff) of a drained tropical peat catchment. It was found that the mean monthly rainfall–runoff relationship of the catchment is best represented by a first-order transfer function model. The model can reasonably predict streamflow of the peat catchment from rainfall with a minor difference in terms of timing and magnitude of the responses. Transfer function models of daily rainfall–runoff relationships for each year over the period 1983–1993 were also developed to investigate the changes in hydrological parameters due to continuous drainage. Differences in the number of parameters and parameter values in different years may be due to the difference in climate in individual years. However, continuous changes in a few hydrological variables have been observed. It is not possible to come to a concrete decision about the impacts of drainage on peat hydrological behaviour with the data available for the study area. However, quantitative analysis of storm hydrographs and their relationships with rainfall and water table levels presented in the first part of this study (Katimon *et al.* 2013) indicates that continuous drainage over a long period has changed the hydrological behaviour of the catchment. The present study also indicates that the catchment has become more responsive to rainfall, the time delay of the streamflow to reach equilibrium has become longer, and the amount of rain water temporarily stored in the soil has reduced.

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