RESEARCH ARTICLE

Advanced Science Letters
Vol. 24, 3536–3543, 2018

Mathematical Models for Mitigating Ebola Virus Disease Transmission: A Review

Chinyere Ogochukwu Dike[1,2,3], Zaitul Marlizawati. Zainuddin[3,*], and Ikeme John Dike[4]

[1] *Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia*
[2] *Federal College of Education P.M.B 2042, Yola, Adamawa State, Nigeria*
[3] *UTM Centre for Industrial and Applied Mathematics, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia*
[4] *Department of Statistics and Operations Research, Modibbo Adama University of Technology, P.M.B 2076, Yola, Adamawa State, Nigeria*

The recent intricate and unmatched Ebola Virus Disease (EVD) frenzied action in West Africa has emphasized the requirement to evaluate the features of epidemics of EVD critically. The need to eradicate the EVD menace necessitated the review of the spread dynamics and measures to contain the virus. This paper reviewed recent EVD rampage in West African nations. The review was conducted on the basis of available proposed Ebola epidemic models, and applied mathematical models. Observation indicated the need to adopt a waiting time technique for describing EVD dynamics and control measures; since EVD spread is a function of time, and transmits by contact. The paper proposed the use of various models of queueing theory and showed their applicability to resolve EVD problems. Current challenges that need research attention were suggested.

Keywords: Ebola, Epidemics, Queueing Theory, Transmission, Control Measure.

1. INTRODUCTION

The recent unprecedented Ebola Virus Disease (EVD) which began in December, 2013 has resisted numerous months of determination, alleviation and control. WHO[1] and Baize et al.[2] disclosed that southern Guinea was the first probable place the EVD started. Though, five types of genus Ebola virus (*Filoviridae* family) exist[2–6] namely: Reston Ebola Virus (REBOV), *Sudan Ebola Virus, Tai Forest Ebola Virus, Zaire Ebola* Virus (ZEBOV) and Bundibugyo Ebola Virus; Siettos[7] associated the Zaire Ebola Virus (ZEBOV) as the contributing factor to the unparalleled epidemic.

EVD is transmitted from contaminated persons to fellow human by direct interaction, mostly by means of slimy tissues or cracked outer protective layer of the body. Avenues for transmission and spread of EVD abound. Healthcare workers might get infected while attending to patients that have confirmed or suspected EVD due to proximate interaction with patients once contagion control precaution are not stringently practiced.[8] EVD can also be transmitted during burial ceremony where grievers have close contact with the body of the dead individual. Those infected continue to be infectious as the following contain the virus—blood, breast milk, body fluids and semen. One month and three weeks later before recovery, those recovering from Ebola disease can transmit the illness from their sperm.

*Author to whom correspondence should be addressed.

Nevertheless, individuals are not transmissible or contagious till they advance in symptoms of Ebola. The development epoch is beginning from 2 to 21 days (epoch during virus infection to the time the signs becomes obvious). Abrupt initial phase of increase in temperature, tiredness, headache, muscle pain and sore throat are the initial signs. Internal and external bleeding in the stools and release from flesh around the teeth, frequent bowel excretion, symptoms of weakened kidney and liver roles, and nausea are the subsequent signs. Enlarged liver enzymes, white blood cell becomes low and platelet counts are symptoms from laboratory findings.[9–11]

There have been cases of EVD outbreak in the past, but the recent outbreak caused unparalleled rampage. For instance, in 1976, the former EVD epidemic was in isolated areas of Democratic Republic of Congo and Central Africa. Although 2014 Ebola outbreak in West Africa is the current and prevalent epidemic up till date has stretched to prominent municipal in West Africa like Guinea, Liberia, Sierra Leone, Senegal and Nigeria.[8,12–15] WHO[16] stated that from 29 August, 2014, there were 15 confirmed cases and 6 confirmed deaths in Nigeria which started on 20 July, 2014 through an infected traveller Mr. Sawyer coming from Liberia. In Senegal, as at 18 September, 2014, there was 1 confirmed case and 0 confirmed death; while according to WHO,[17] in Guinea as at 23 December, 2015, there were 3351 confirmed cases and 2083 confirmed deaths; in Liberia as of 9 May, 2015, there were 10666 total cases and 4806 total deaths;

 1936-6612/2018/24/3536/008 doi:10.1166/asl.2018.11432

also in Sierra Leone as of 7 November, 2015, there were 8704 confirmed cases and 3589 confirmed deaths. The statistics above speak volumes on the death caused by the disease. Therefore, it is essential for a review of the illness and issues surrounding it.

Although researchers like Chowell and Nishiura[18] have reviewed the transmission dynamics and control of EVD, their submission may be considered obsolete bearing in mind the recent EVD rampage in West Africa. In this paper, the current EVD rampage in West African nations was discussed. The available related literatures on EVD based on general epidemic model were also reviewed. Recent works on the application of applied mathematical models for solving EVD problems were also examined. Therefore, this paper proposes the use of various models of queueing theory and showed their applicability to resolve EVD problems. Finally, current challenges that need research attention were suggested.

2. EVD RAMPAGE IN WEST AFRICAN NATIONS

There existed, in some West African nations, EVD epidemic incidence. The affected nations are Guinea, Liberia, Sierra Leone, Senegal and Nigeria. In December 2013 in Guinea, the epidemic started and afterwards moves into nearby nations.[19–21] It led to substantial death of human and social interruption with specified incidence death rates of about 70%.[19,22,23] WHO[24] stated that precisely 57–59% of those infected are hospitalized. There have also been reported cases of death of some healthcare providers that take care of the affected patients.[25] On the other hand, as of 2016, the epidemic is under control, abrupt epidemics of the illness are possible to linger for some time.[26] Nevertheless, the outbreak has been associated to certain havocs other than loss of life within the affected nations and the environs. Such havocs include economic effects, food shortage, socio-cultural impediments, ecological problems and religious practice instability.

The economy of the EVD-rampaged countries and the world at large has been affected.[27] World Bank[28] indicated that a sum of $1.62 billion was marshalled by December 2015 for financing Ebola outbreak. Some governments allocated huge amount of their currency for setting up special hospitals and medical equipment for the treatment of Ebola.[10] Nigeria spent eight billion naira (₦ 8 billion) on the limited Ebola outbreak.[29–31] Hotel business in Nigeria slumped by 75% as at April 2014 due to the dreads of the outbreak.[32] The outbreaks caused massive economic consequences in affected West African nations and to other African nations that were not affected. It led to closing of borders, flight cancellations, affected foreign investments and tourism activities.[33] International Monetary Fund (IMF) proposed that the economic effect of Ebola epidemic could lead to the death of additional individuals than the named disease.[34]

The epidemic could endanger yield and food security in West Africa as warned by Food and Agricultural Organization (FAO) department of the United Nations (UN).[35,36] It is the consequence of people moving away from the distributed agricultural affected areas.[37] They reported that West African countries began to gain knowledge of human agricultural insufficient labour as the loss or damage of EVD increased. They continued that many fields of crops were not given required attention as a result of restrictions placed on movement of people in order to prevent the spread of EVD. This made it impossible to achieve food security. Subsequently, with little to no goods moving into these countries, the prices of food shot up and led to food shortages. Furthermore, the prohibition of bush meat like chimpanzee, antelope, and buffalo to avoid additional infection deprived many households of a vital source of protein.[38–40]

The transmission of EVD has affected the socio-cultural relationship of most West African countries. African culture has the traditional unanimity of showing kindness to their loved ones who are ill, which is a risk when an Ebola patient is involved.[41,42] The sincere effect that, comforting tactile touch of handshake and hugging produce is important for social relationship, social harmony, fitness, existence, safety, and also for communicating feelings.[43] Unfortunately, these essential cultural elements have been affected as a result of Ebola epidemic.[10] The culture of touch, hold, hug and kiss was stated to have led to the wiping away of entire families making the survivors to hands off from their loved ones who are sick.[44,45] The epidemic has led to the fear of human contacts making markets and shops to be closed which makes producers and traders to lose their income.[46,47] In the affected countries, tourism was directly impacted.[32] Again, there are countries in Africa and other continents such as Gambia, Ghana, Zimbabwe, Tanzania, United States of America and Kenya that remained untouched by EVD epidemic, likewise reported unfavourable effects on tourism.[30,31,48–56] The incident impedes socio-cultural related issues in the affected nations. Furthermore, Nwaoga et al.[10] stated that Ebola outbreak has an effect on religious worshippers. They noted that churches like Catholic and Anglican in Nigeria implemented new mode of worship so as to avert the transmission of Ebola.

Ebola Virus Disease also has an ecological effect by adversely affecting the natural state of biodiversity; in that human activity has made the subtle ecological stability to become destabilized.[57] Human mismanagement of natural environment creates avenue for birth and spread of epidemics which are currently affecting all and sundry including wild and domestic animals.[57,58] Ebola has a high death rate among primates and domestic animals such as gorillas, chimpanzee, pigs, dogs and so on.[59–64]

3. EBOLA EPIDEMIC MODELS

The most extensively used Ebola epidemic model is the SIR (Susceptible, Infected, Removed/Recovered) model which was presented in 1932 by Kermack and Mckendrick as specified by Bashar et al.[65] SIR model can be explained as follows: S (Susceptible) meaning persons that are not infected with the virus from interaction through an infected person; I (Infected) meaning persons that have the disease and can infect people that are in susceptible state; and R (Removed/Recovered) meaning those who recuperate or die from the illness. Moreover, virtually all existing literature on Ebola outbreak forecast are centred on the amendment of the rudimentary SIR model, while some still maintained the basic SIR model.

Chowell et al.[66] and Lekone and Finkenstadt[67] modelled the course of Ebola epidemics in 1995 at Congo and 2000 at Uganda using SEIR (Susceptible, Exposed, Infected, Removed) model. Legrand et al.[68] also modelled Ebola epidemics in 1995 at Congo and 2000 at Uganda using SEIHFR (Susceptible, Exposed, Infected, Hospitalized, Funeral, Removed) model. Rivers et al.[69] expanded the basic model of kermack and Mckendrick in kermack and Mckendrick[70] and the work of Chowell et al.[66] by applying the same SEIHFR as in Legrand et al.[68] in Liberia and

Adv. Sci. Lett. 24, 3536–3543, 2018

Sierra Leone, to model the influence of intrusions on the epidemic of Ebola. To gain additional understanding in the epidemic underlying forces, they divided the infectious phase into three stages namely: community infection location represented by (I), hospital infection location represented by (H), and traditional funeral infection represented by (F).

Jaime[71] used SIR and SEIR models to simulate two Ebola outbreaks: the 1976 outbreak in Yambuku, Zaire and the 1995 outbreak in Kikwit, Zaire. They reported that the R_o (basic reproduction number) for Yambuku, Zaire outbreak in 1976 ranged from 2.6 to 8.6, while that of Kikwit, Zaire in 1995 which is $1.57 \leq R_o \leq 5.03$ was a little lower. Kiskowski[72] used SEIR model to demonstrate the effects of community mixing in Guinea and Liberia.

Stadler et al.[73] used the following model: BD (Birth Death), BD_{sa} (Birth Death sampled ancestor), BDSIR (Birth Death Susceptible Infected Removed), coalSEIR (coalescent Susceptible Exposed Infected, Removed), BD_{ss} (Birth Death sampled superspreaders), BDEI (Birth Death Exposed Infected). The result from the model showed that for the continuing Ebola virus epidemic, having more sequencing data available on the commencement of the recent outbreaks will raise rapid understanding of the underlying forces of the pathogen.

Siettos et al.[7] used $SEID_bD_IR$ (Susceptible, Exposed, Dead buried, Dead Infected, Removed) model which is the modification of SEIR model. They observed that epidemiological characteristics associated with the transmission further to the biological characteristics of Ebola virus including the ones observed in the past epidemics are alike.

Gomes et al.[74] applied SEIHFR model in measuring the international spreading menace associated with West African 2014 Ebola epidemic. It was found that in African countries the lingering of the epidemic is more probable to occur which will escalate the danger on a longer time scale of international distribution. Athaus[75] applied SEIR model. Their model specified no decrease in R_e (effective reproduction number) towards August 2014 ending in Liberia, as well as a decrease in R_e to about unity for Sierra Leone, which would show the termination of the epidemic, by July 2014 ending.

4. APPLIED MATHEMATICAL MODEL FOR EBOLA VIRUS DISEASE

Applied mathematical models are essential in measuring the prospective influence interventions might have towards transmission control, and in giving direction as to the forthcoming forecasts of such significant on-going civic health calamities.[76] Various applied mathematical models have been utilized by researchers.

Chowell et al.[66] used SEIR epidemic model and applied the method of differential equation to study the Ebola outbreak in Congo in 1995 and Uganda in 2000. They opined that class S, which is the Susceptible persons that have not yet caught the disease from interaction with an infected person, move into the exposed class E. Class E are the infected individuals that are not infectious at the per-capita rate $\beta I/N$, where β, N and I/N are transmission proportion for each individual for each day, size of total population and the likelihood that an infected individual made a contact (i.e., assuming uniform mixing) respectively. An incubation period of $1/k$ days was assumed asymptomatic and uninfectious for exposed individuals to go to the infectious class I. Similarly, at per-capita rate γ, infectious individual then, go to class R which is death or recovered. Therefore, ordinary differential equations were proposed to model the problem:

$$\dot{S}(t) = -\frac{\beta S(t) I(t)}{N} \tag{1}$$

$$\dot{E}(t) = \frac{\beta S(t) I(t)}{N} - kE(t) \tag{2}$$

$$\dot{I}(t) = kE(t) - \gamma I(t) \tag{3}$$

$$\dot{R}(t) = \gamma I(t) \tag{4}$$

$$\dot{C}(t) = kE(t) \tag{5}$$

Where $\dot{S}(t)$, $\dot{E}(t)$, $\dot{I}(t)$, then $\dot{R}(t)$ signify the number of susceptible, exposed, infected, and removed individuals at time t and the dot represents the derivatives. $C(t)$ helps to retain path of the aggregate number of Ebola cases from the time of inception of indications, although it is not an epidemiological state. It was found that education, contact tracing and quarantine which were adopted as the control procedures decreased the transmission of the outbreak.

Rivers et al.[69] used SEIHFR epidemic model and applied method of differential equation to model the effect of intrusions on an epidemic of Ebola in Sierra Leone and Liberia. They projected the following differential equations to model the problem:

$$\frac{dS}{dt} = -\frac{\beta_I SI + B_H SH + \beta_F SH}{N} \tag{6}$$

$$\frac{dE}{dt} = \frac{\beta_I SI + \beta_H SH + \beta_F SF}{N} - \alpha E \tag{7}$$

$$\frac{dI}{dt} = \alpha E - [\gamma_H \iota + \gamma_I (I - \iota)(I - \delta_1) + \gamma_D (I - \iota)\delta_1] I \tag{8}$$

$$\frac{dH}{dt} = \gamma_H \iota I - [\gamma_{DH}\delta_2 + \gamma_{IH}(1 - \delta_2)] H \tag{9}$$

$$\frac{dF}{dt} = \gamma_D (1 - \iota)\delta_1 I + \gamma_{DH}\delta_2 H - \gamma_F F \tag{10}$$

$$\frac{dR}{dt} = \gamma_I (1 - \iota)(1 - \delta_1) I + \gamma_{IH}(1 - \delta_2) H + \gamma_F F \tag{11}$$

Where β_I is the proportion of contact in the community, β_H is the proportion of contact in the hospital, β_F is the proportion of contact during funeral, $1/\alpha$ is the incubation period, $1/\gamma_H$ is the time before hospitalization, $1/\gamma_{DH}$ is hospitalization to death time, $1/\gamma_F$ is traditional funeral duration, $1/\gamma_I$ is infection duration, $1/\gamma_D$ is infection to death time, $1/\gamma_{IH}$ is hospitalization to recovery time, ι is the probability of a hospitalized case, δ_1 is un-hospitalized proportion of death case, and δ_2 is hospitalized case fatality rate. They found from their result that protracted obligation of resources and backing will be essential to combat the epidemic.

Gomes et al.[74] applied the method of differential equation and used SEIHFR epidemic model in measuring the international spreading menace associated with West African 2014 Ebola epidemic. To model the problem, they proposed the following differential equation:

$$R_o = R_I + R_H + R_F \tag{12}$$

$$R_I = \frac{\beta_I}{\Delta} \tag{13}$$

$$R_H = \frac{\theta\beta_H}{\gamma_{dh} + \gamma_{ih}(1-\delta_2)} \quad (14)$$

$$R_F = \frac{\delta\beta_F}{\gamma f} \quad (15)$$

where $\Delta = \gamma_h\theta_1 + \gamma_d\delta_1(1-\theta_1) + \gamma_i(1-\theta_1)(1-\delta_1)$, R_o denote the basic reproduction number, R_I signify the spreads in the community, R_H represent the spreads in the hospital, R_F is the transmissions from dead persons, β_1, β_H, β_F is the contact rate in the community, hospital and during funeral, θ is the ratio of hospitalized cases, θ_1 is the rate of hospitalization for the infectious compartment l, δ_1 is the fatality rate of infected persons that are not hospitalized, δ_2 is the fatality rate of infected persons that are hospitalized, δ is the same as the ratio of transitions from H to F, γ_h, γ_d, γ_i, γ_{dh}, γ_{ih}, γ_f are the time from inception of signs to hospitalization, death, infectiousness, hospitalization to death, hospitalization to the completion of infectiousness for survivors and funeral. They obtained the symbol for R_o from the work of Diekmann and Heesterbeek[77] and, Driessche and Watmough.[78] Legrand et al.[68] indicated that R_o can be expressed as the addition of three terms for the model, thus $R_o = R_I + R_H + R_F$, where R_1 represents the community transmission, R_H means the hospital transmission and R_F indicates transmission from dead individuals.

5. AVAILABLE QUEUEING THEORY MODELS

Queueing theory encompasses waiting which is one of the nastiest experiences of life. Models of the queuing theory have so many practices in actual life circumstances like happenings in petrol station, post office, toll booths, telephone exchange, computer systems, data communication, production system, parking place, assembly of printed circuit boards, call centre of an insurance company, traffic lights, supermarket, transmission of disease[79, 80] etc. Sztrik, Cooper, Winston and Ross[80–83] stated that queueing theory can be categorised into infinite-source model and finite-source model according to the cardinality of their sources.

5.1. Infinite-Source Model

The arrivals in the system in infinite-source models of the number of customers are independent, which result in a mathematically manageable model. Infinite-source model includes M/M/1 queue, M/M/1 queue with balking customers, priority M/M/1 queue, M/M/1/K queue (Systems with finite capacity), M/M/∞ queue, M/M/n/n queue (Erlang-Loss System), M/M/n queue, M/M/c/K queue (Multi-server, Finite-Capacity systems), and M/G/1 queue. The models are discussed below.

5.1.1. M/M/1 Queue Model

M/M/1 queueing model is the fundamental queueing model. It signifies a system whose arrival is Poisson with rate λ. The implication is that the inter-arrival times (M) are random variables that are independent and exponentially distributed with parameter λ; the service time (M) is also independent and exponentially distributed with parameter μ; and a single server (1). M/M/1 queueing model was used to derive the mean number of customer in the system, mean number of waiting customer (mean queue length), variance, server utilization, response time distribution of a customer, waiting time distribution which is the main performance measures of the system.

5.1.2. M/M/1 Queue with Balking Customers Model

M/M/1 queue with balking customers is a system where customers are disheartened when there are many lines and may change their minds not to join the line. The number of customers follows law of Poisson with parameter ρ. M/M/1 queueing model with balking was used to derive performance measures such as mean, variance, distribution of the response and waiting time.

5.1.3. Priority M/M/1 Queue Model

In M/M/1 queueing model with priorities there are two classes of customers; each class follows Poisson process with parameter λ_1, and λ_2. The procedures are independent to each other. Also, for each class, the service times are expected to be exponentially distributed with parameter, μ. Therefore, $\rho_1 + \rho_2 < 1$, if the system is stable.

There are two types of Priority M/M/1 queue models such as Preemptive Priority and Non-Preemptive Priority. It is expected that class A has priority over class B. In the Preemptive Priority, considering the service discipline, if there is customer in the system belonging to class A, the service of a customer belonging to class B will certainly not be done. It shows class A preempts class B. In order words, even if class B customers are being serviced, the service discontinues as soon as the class A customer's request arrives. The discontinuous service is continued merely when class A customer is not in the system. Preemptive Priority M/M/1 Queue Model was used to derive the mean number of all classes (A, B) customers in the system and response time of all classes (A, B) requests. While in Non-preemptive Priority, the arrival of class A customer does not discontinue the request for service of class B. The discipline is called Head of the Line (HOP). Non-preemptive Priority M/M/1 Queue Model was used to derive the mean number of all classes (A, B) customers in the system and response time of all classes (A, B) requests.

5.1.4. M/M/1/K Queue Model (Systems with Finite Capacity)

M/M/1/K Queue Model (Systems with Finite Capacity) denotes a system in which the arrival is Poisson, the service time is exponential with one server and K (the highest number of customer in the system together with the one in service) is the capacity of an M/M/1 system. Birth-death process is the number of customers in the system with arrival rates $\lambda_k = \lambda, k = 0, \ldots, K-1$ and service rates $\mu_k = \mu, k = 1, \ldots, K$. However, for a fixed K and for any $\rho > 0$, the system is stable. Then, the stability condition is $\rho < 1$ if $K \to \infty$ since M/M/1/K distribution converges to M/M/1 distribution.

5.1.5. M/M/∞ Queue Model

The process $[N(t), t \geq o]$ which is the number of customers in the system is a birth-death process with rates:

$\lambda_k = \lambda, k = 0, 1, \ldots;\ \mu_k = k\mu, k = 1, 2, \ldots;\ P_k = (\rho^k/k!)P_0$, where $P_0^{-1} = \sum_{k=0}^{\infty} \rho^k/k! = e^e$ is the steady-state distribution. That is $P_k = (\rho^k/k!)e^{-\rho}$, showing that N follows Poisson distribution with parameter ρ which was used to derive performance measures.

5.1.6. M/M/n/n Queue Model (Erlang-Loss System)

In queueing theory, the model is the ancient and well-known system. The basis of theory of traffic or theory of congestion began through the inquiry of this system and Erlang was the leading person that got the well-recognised formulas. The arrival of customers follows a Poisson process and the service times are exponentially distributed by assumptions, with n servers. As soon as a new customer arrives the customer will be missed because the system is filled, if n servers are all busy. $[N(t), t \geq o]$ process is said to be in state k if k servers are engaged, which is k customers being in the system is the same. $[N(t), t \geq o]$ is a birth-death process which have rates

$$\lambda_k = \begin{cases} \lambda, & \text{if } k < n, \\ 0, & \text{if } k \geq n, \end{cases} \quad \mu_k = k\mu, \ k = 1, 2, \ldots, n \tag{16}$$

which was used to derive the most important measure and performance measure of the system that was initiated by Erlang and is denoted to as Erlang's B-formula, or loss formula and commonly represented by $B(n, \lambda/\mu)$.

5.1.7. M/M/n Queue Model

By assumption, the disparity of the classical queue is that the service is delivered by n servers functioning individually of one another. If the average arrival rate is more than the service rate, the modification is natural because the number of servers should be increased making the system not to be steady. First service distribution completion is necessary and parallel services are procured. Therefore, the arrivals follow a Poisson distribution. At random moments the distribution is like the moments distribution of the system at arrival. The likelihood that a customer has to wait on arrival was obtained, which is often used in diverse practical problems such as call centers, telephone systems etc. Erlang's C formula or Erlang's delay formula represented by $C(n, \lambda/\mu)$ is a well-known formula which was used in the derivation of main performance measures of the system.

5.1.8. M/M/c/K Queue Model (Multi-Server, Finite-Capacity Systems)

M/M/c/K Queue is a disparity of a multi-server system; that is, only greatest K customers which are death-birth process are permitted to stay in the system. M/M/c/K Queue model was used to derive the main performance of the system. Specifically, L'Hopital's rule should be used twice if $a = 1$, which is the ratio of traffic intensity to multi-server is equal to 1. Laplace-transform of the waiting and response times can be derived by using the law of total Laplace-transforms.

5.1.9. M/G/1 Queue Model

In the models above, their service times are exponentially distributed, while M/G/1 queue model arrival rate is Poisson as above but its service times is general. However, in practical problems service times are not exponentially distributed, so queueing systems which follow general distributed service times are natural. M/G/1 was used to derive the main performance measure.

5.2. Finite-Source Model

The arrival strength of the demand in finite model relies on the state of the system which makes the computations more complex. Finite-source models are significant from practical point of view and the operation times and service times are assumed to be independent random variables with given distribution function. Finite-source models include M/M/r/r/n queue, M/M/1/n/n queue, Heterogeneous queues: $\vec{M}/\vec{M}$/1/n/n/PS (Processor Sharing) queue, M/M/r/n/n queue, M/M/r/K/n queue, M/G/1/n/n/PS (Processor Sharing) queue and $\vec{G}$/M/r/n/n/FIF queue.

5.2.1. M/M/r/r/n Queue Model (Engset-Loss System)

A customer may find the system full by relying on the system capacity r in an M/M/r/r/n queue. In the finite-source model, the demand returns to the source and stays there for an exponentially distributed time as against the infinite-source model where the customer's request is missed. Birth-death process is the number of customers in the system as all the random variables are supposed to be exponentially distributed with the following rates $\lambda_k = (n-k)\lambda$, $0 \leq k < r$, then $\mu_k = k\mu$, $1 \leq k \leq r$. This gives the distribution called truncated binomial or Engset distribution is known as the distribution of a finite-source loss or Engset system. Particularly, if $r = n$ it means that there is no loss and each customer has its own server called binomial distribution which was also used to derive performance measures. By applying Bayes's law and bearing in mind the distribution of the system at an instant when an arriving customer arrives into the system, gives Little's formula for the finite-source loss system.

5.2.2. M/M/1/n/n Queue Model

The M/M/1/n/n queue model is the traditional machine interference problem where queue discipline FIFO (First In, First Out) order is applied and the broken machines have to wait until the single repairman fixes the failed machines. All the random variables are supposed to be independent of each other, on assumption that the operating times follow exponential distribution with parameter λ and μ repair rate. The steady-state distribution exists at all times, since the state space is finite, nevertheless if $\rho > 1$ extra repairmen are required. The M/M/1/n/n queue model was used to derive performance measures.

5.2.3. $\vec{M}/\vec{M}$/1/n/n/PS (Processor Sharing) Queue Model

In heterogeneous queues, bearing in mind n heterogeneous machines which operating and repair time follow exponential distribution with parameters $\lambda_k > 0$ and $\mu_k > 0$, for machine k, $k = 1, \ldots, n$. Single repairman repairs the failures according to Preemptive Priority disciplines and Processor Sharing, FIFO. The number of machines that failed at time t is represented as $N(t)$. The information is not sufficient to depict the behaviour of the system; owing to the heterogeneity of the machines since, there is the need to be aware which machine is under service. $N(t)$ dimensional vector with components $(x_{1(t)}, \ldots, x_{v(t)}(t))$ showing the indexes of the machines that failed. Therefore, machine with index $x_1(t)$ is under service for $N(t) > 0$ using FIFO discipline. When all machines are serviced, under Processor Sharing discipline, through a proportional service rate, that is, if $N(t) = k$, then the proportion is $1/k$, the order of indexes $(x_{1(t)}, \ldots, x_{n(t)}(t))$ is not significant, but $(x_{1(t)}, \ldots, x_{v(t)}(t))$ is ordered in a logical treatment. In Preemptive Priority case, assuming that smaller index and higher priority are the same, therefore the same ordering was used as before referring that the machine with the first index in this case is under service, since it has the maximum priority among machines that failed. The process $X(t)(t) = (v(t); x_1(t), \ldots, x_{v(t)}(t))$, $(t \geq 0)$, is a continuous-time Markov owing to exponential distribution where the ordering of $(x_{1(t)}, \ldots, x_{v(t)}(t))$ rely on the service discipline. Since a finite state Markov chain is ergodic therefore if the parameters λ_k, μ_k, $(1 \leq k \leq n)$ are all positive, there exists steady-state distribution which heavily relies on the service discipline.

The distribution of the Markov chain, the ordering of the indexes, the steady state solution with normalizing condition in M/M/1/n/n/PS Queue was derived including the performance measures.

5.2.4. M/M/r/n/n Queue Model

In M/M/r/n/n queue model finite-source homogenous model with r, $r \leq n$ servers that are independent are deliberated. The number of customers in the system at time t is denoted by $N(t)$, which, is a birth-death process with rates.

$$\lambda_k = (n-k)\lambda, \quad 0 \leq k \leq n-1$$

$$\mu_k = \begin{cases} k\mu, & 1 \leq k \leq r, \\ r\mu, & r < k \leq n \end{cases}$$

The steady-state distribution solution with normalizing condition and the main performance measures are derived using M/M/r/n/n queue model.

5.2.5. M/M/r/K/n Queue Model

M/M/r/K/n queue model is a combination of the finite-source systems deliberated above and for the most general system $K = r$. Therefore, from above the M/M/r/r/n queue model (Engest-Loss System) was treated for $r = 1$, M/M/1/n/n queue model for $K = n$, the system of M/M/r/n/n queue model was obtained. Therefore, delay loss system occurs for the value $r < K < n$, which means that customers must go back to the source since the system is filled, that is, customers may arrive into the system until the number of customers in the system is $K-1$. Birth-death process is the number of customers in the systems with rates:

$$\lambda_k = (n-k), \quad 0 \leq k < K,$$

$$\mu_k = \begin{cases} k\mu, & 0 \leq k \leq r, \\ r\mu, & r < k \leq K \end{cases}$$

where $1 \leq r \leq n$, $r \leq K \leq n$. In M/M/r/K/n queue model $P_0(n, r, k)$ is the normalizing constant which satisfies normalizing condition and the model is used to derive performance measures.

5.2.6. M/G/1/n/n/PS Queue Model

M/G/1/n/n/PS queue model is the generalization of $\overrightarrow{M}/\overrightarrow{M}$/1/n/n/FIFO queue model above. The distribution of service time and service discipline are the necessary differences. The number of customers as a stochastic process is not a Markov chain since the service times do not follow exponential distribution. The requests arrive from a finite-source with parameter λ where they spend an exponentially distributed time. The service time needed which is denoted as S has a generally distributed random variable with $E(S) < \infty$. Assuming that $(G(0^+) = 0)$, $G(x)$ and $g(x)$ are its distribution function and density function. Processor Sharing is the service discipline meaning that those being served are all customers in the service facility, but service rate is relative to the number of customers. M/G/1/n/n/PS queue model is used to derive the performance measures.

5.2.7. $\overrightarrow{G}$/M/r/n/n/FIFO Queue Model

$\overrightarrow{G}$/M/r/n/n/FIFO queue model is a generalized queue and a finite-source model that has multiple servers where customers source times are generally distributed heterogeneously and possess service times that are homogeneous exponential distribution. The service discipline is FIFO meaning they are served in accordance with the directive of their arrivals. The arrival of customers is from a finite source of size n and they are served by one of $r(r \leq n)$ servers according to a FIFO discipline at a service facility. A waiting line is formed if there is no idle server and the units are delayed. The service times of the units are assumed to be random variables which are identically and exponentially distributed with parameter μ. Customer with index i returns to the source after completing service and remain there for a random time τ_i having general distribution function $F_i(x)$ with density $f_i(x)$. All random variables are supposed to be independent of one another. G/M/r/n/n/FIFO queue model was used to derive performance measures.

6. THE RELEVANCE OF THE QUEUEING MODELS IN EVD TRANSMISSION

The spread of EVD after any outbreak is a function of time. The disease transmits by contact. Therefore, it is pertinent to opine that everyone within the disease outbreak region is considered exposed. The exposed individuals find themselves in a system and under a waiting time state. Each individual has the probability of being infected over a period of time. The eventual infected individuals move from the system of the exposed state to that of the infected state, yet under another waiting time state.

To establish the applicability of the queuing models in EVD transmission pattern, the queueing model parameters namely: arrival rate and service rate could be assumed to signify, infection rate and recovery/death rate respectively. Thus, the parameters can be integrated into the queueing model to explain the EVD transmission dynamics.

7. CONCLUSION AND RECOMMENDATION

Ebola Virus Disease has caused serious loss of life, waste of economic and material resources in West African nations like Nigeria, Sierra Leone, Guinea, Liberia, and Senegal (WHO, 2015). This paper recaps the recent West African nations EVD outbreak. Related literatures on EVD based on general epidemic model and the utilization of applied mathematical models for EVD problems were reviewed. The various models of queueing theory were discussed. The place of the queueing model in EVD transmission pattern was revealed.

Although, a lot of Mathematical models such as agent-based models and models based on ordinary differential equation, epidemic models based on SIS and SIR and their extensions, including assessment studies and intervention measures, have been proposed by several researchers on ways to handle outbreak of the disease;[18] there still exists current challenges that require research attention. Such challenges include.

The ordinary differential equation based approach describes reality with varying accuracy.[84] Again, equation free algorithms are generally not clear; the agent based approach comes with higher computational cost.[85] Furthermore, it is difficult to use stochastic individual based simulation for complex data.[85] Therefore, it is vital to propose an approach that is capable of solving the above problems with little or no difficulty. The queueing theory approach is a promising alternative.

Researchers have developed various epidemic based model of Ebola virus, mainly for Sierra Leone and Liberia. Their models were derived from the original SIS and SIR models proposed in 1932 as cited in Bashar et al.[65] Jaime et al.[71] used the SIS model to explain EVD transmission pattern. Other researchers such as Stadler et al.;[73] Chowell et al.;[66] Lewnard et al.;[86] and Kiskowski[72] extended the model to accommodate latent feature of the virus causing Ebola during incubation period. But, there are still some loopholes since all the EVD transmission phases have not been covered. Therefore, the extension of the basic SIS and SIR epidemic models that cover all EVD transmission phases is a welcome idea.

Finally, the way Ebola virus disease invades countries differ due to socio-cultural factor differences and population behavioural factor differences. Also, epidemic of EVD happens in batches sometimes, thereby generating a queue for each outbreak. Hence, there is a need to develop models to explain each individual queue due to an outbreak as well network of EVD queues.

Acknowledgments: The authors would like to thank the Malaysian Ministry of Higher Education for their financial funding through Research University Grant (RUG) vote number Q.J130000.2526.14H11. This support is gratefully acknowledged. The researchers would also like to express thanks to the Department of Mathematical Sciences, Faculty of science, UTM and UTM Centre for Industrial and Applied Mathematics (UTM-CIAM) for the aid in conducting this research work. Finally, the authors would like to show appreciation to the Management of Federal College of Education Yola, Nigeria and the Management of Tertiary Education Trust Fund (TETFUND), Nigeria.

References and Notes

1. World Health Organisation, Ebola virus disease in Guinea, Epidemic pandemic alert and response, World Health Organization regional office for Africa **(2014)**, http://www.afro.who.int/en/clusters-a-programmes/dpc/epidemic-a-pandemic-alert-and-response/outbreak-news/4063- ebola-hemorrhagic-fever-in-guinea.html, Date accessed 2/10/2014.
2. S. Baize, D. Pannetier, L. Oestereich, T. Rieger, L. Koivogui, N. Magassouba, B. Soropogui, M. S. Sow, S. Keïta, H. D. Clerck, A. Tiffany, G. Dominguez, M. Loua, A. Traore, M. Kolie, E. R. Malano, E. Heleze, A. Bocquin, S. Mely, H. Raoul, V. Caro, D. Cadar, M. Gabriel, M. Pahlmann, D. Tappe, J. Schmidt-Chanasit, B. Impouma, A. K. Diallo, P. Formenty, M. V. Herp, and S. Günther, *N Engl. J. Med.* 371, 1418 **(2014)**.
3. E. M. Leroy, B. Kumulungui, X. Pourrut, P. Rouquet, A. Hassanin, P. Yaba, A. Delicat, J. T. Paweska, J. P. Gonzalez, and R. Swanepoel, *Nature* 438, 575 **(2005)**.
4. H. Feldmann and T. W. Geisbert, *Lancet* 377, 849 **(2011)**.
5. S. K. Gire, A. Goba, K. G. Andersen, R. S. Sealfon, D. J. Park, L. Kanneh, S. Jalloh, M. Momoh, M. Fullah, G. Dudas, S. Wohl, L. M. Moses, N. L. Yozwiak, S. Winnicki, C. B. Matranga, C. M. Malboeuf, J. Qu, A. D. Gladden, S. F. Schaffner, X. Yang, P. P. Jiang, M. Nekoui, A. Colubri, M. R. Coomber, M. Fonnie, A. Moigboi, M. Gbakie, F. K. Kamara, V. Tucker, E. Konuwa, S. Saffa, J. Sellu, A. A. Jalloh, A. Kovoma, J. Koninga, I. Mustapha, K. Kargbo, M. Foday, M. Yillah, F. Kanneh, W. Robert, J. L. Massally, S. B. Chapman, J. Bochicchio, C. Murphy, C. Nusbaum, S. Young, B. W. Birren, D. S. Grant, J. S. Scheiffelin, E. S. Lander, C. Happi, S. M. Gevao, A. Gnirke, A. Rambaut, R. F. Garry, S. H. Khan, and P. C. Sabeti, *Science* 345, 1369 **(2014)**.
6. European Food Safety Authority, *EFSA Journal* 12, 3884 **(2014)**.
7. C. Siettos, C. Anastassopoulou, L. Russo, C. Grigoras, and E. Mylonakis, Modeling the 2014 ebola virus epidemic-agent-based simulations, temporal analysis and future predictions for Liberia and Sierra Leone, PLOS Current Outbreaks **(2015)**.
8. World Health Organisation, *Weekly Epidemiological Record* 89, 205 **(2014)**.
9. WHO, WHO statement on the meeting of the international health regulations emergency committee regarding the 2014 ebola outbreak in West Africa, August **(2014)**, http://www.who.int/mediacentre/news/tatements/2014/ebola-20140808/en/, Date accessed 10/10/2014.
10. C. T. Nwaoga, G. C. Nche, and F. U. Nnadi, *IOSR Journal of Humanities and Social Science (IOSR-JHSS)* 19, 69 **(2014)**.
11. CDC, Review of human-to-human transmission of Ebola Virus. Center for Disease Control and Prevention, MMWR, Morbidity and Mortality, 2014; Weekly Report, Ebola.
12. WHO, Ebola haemorrhagic fever in Zaire, 1976 **(1978)**, 56, 271-93, PubMed PMID:307456.
13. M. Chan, *N. Engl. J. Med.* **(2014)**, PubMed PMID:25140856.
14. S. Briand, E. Bertherat, P. Cox, P. Formenty, M. Kieny, J. K. Myhre, C. Roth, B. Chir, N. Shindo, and C. Dye, *New England Journal of Medicine* **(2014)**, DOI: 10.1056/NEJMp1409858.
15. D. G. Bausch and L. Schwaiz, *PLOS Negl. Trop. Dis.* 8, e3056 **(2014)**.
16. World Health Organisation, Database statistics **(2015)**, Date accessed 12/05/2015.
17. World Health Organisation, Database statistics **(2015)**, Date accessed 5/02/2016.
18. G. Chowell and H. Nishiura, *BMC Med.* 12, 196 **(2014)**.
19. WHO Ebola Response Team, *New England Journal of Medicine* **(2014)**, DOI: 10.1056/NEJMoa1411100.
20. UPDATE 1-Mali confirms new case of ebola, locks down Bamako clinic, Reuters, Reuters, November **(2014)**, http://in.reuters.com/article/2014/11/12/health-ebola-mali -idin16n0t15cn20141112, Date accessed 5/03/2016.
21. Ebola situation report (PDF), World Health Organization, January **(2015)**, http://apps.who.int/iris/bitstream/10665/149314/1/ roadmapsitrep_21jan2015_eng.pdf/ua=1, Date accessed 5/03/2016.
22. Case Fatality Rate for ebolavirus, Ebola data and statistics **(2015)**, Epidemic.bio.ed.ac.uk/ebolavirus_fatality_rate, Date accessed 5/03/2016.
23. Ebola response roadmap—situation report, December 2014 (PDF), World Health organization, December **(2014)**, http://apps.who.int/ebolaweb/sitreps/20141231/20141231.pdf, Date accessed 5/03/2016.
24. Ebola Situation report, Ebola data and statistics, World Health Organization, January **(2015)**, http://www.who.int/csr/diseas/ebola/situation-reports/en/, Date accessed 5/03/2016.
25. Ebola (Ebola Virus Disease), Ebola virus disease Information for Clinicians in U.S. Healthcare Settings, CDC, http://www.cdc.gov/vhf/ebola/hcp/clinician-information-us-healthcare-setting, Date accessed 5/03/2016.
26. WHO—WHO Director-General addresses the Executive Board World Health Organization, http://www.who.int/dg/speeches/2016/executive-board-138/en/, Date accessed 5/03/2016.
27. West African economies feeling ripple effects of Ebola, says UN (Press release), United Nations Development Programme, March **(2015)**, Date accessed 5/03/2016.
28. World Bank Group Ebola Response Fact Sheet, World Bank, December **(2015)**, http://www.worldbank.org/en/topic/health/brief/world-bank-group-ebola-fact-sheet, Date accessed 5/03/2016.
29. Impact of Ebola on e-commerce in Nigeria, CNBC Africa, http://www.cnbcafrica.com/video/bctid=3807923516001, Date accessed 5/03/2016.
30. allAfrica.com: Nigeria: Lagos Hotels Lose N8 Billion Over Ebola Outbreak, allAfrica.com, Date accessed 5/03/2016.
31. Ebola Reduced Lagos Hotel Patronage by 75% in 2014, Articles-Thisday live, thisdaylive.com, http://www.thisdaylive.com/articles/ebola-reduced-lagos-hotel-patronage-by-75-in-2014/207538/, Date accessed 5/03/2016.
32. Ebola Virus Eats Into West Africa's Economic Growth, AFKInsider, 2014-08-21, http://afkinsider.com/69855/ebola-virus-eats-west-africas-economic-growth/, Date accessed 5/03/2016.
33. West African economies feeling ripple effects of Ebola, says UN (Press release), United Nations Development Programme, March **(2015)**, Date accessed 5/03/2016.
34. IMF (International Monetary Fund) says Ebola impacted West African nations may need economic assistance, September **(2014)**, http://www.australiannews.net/index.php/sid/225466201, Australian News.Net, Date accessed 5/03/2016.
35. S. Lomax, Liberia: Ebola outbreak affects Agriculture in Lofa County, (2014-08-20), allAfrica.com, Date accessed 5/03/2016.
36. First WHO worker infected by ebola, DR Congo reports cases, Rappler.com, 2014-08-15, Date accessed 5/03/2016.
37. Ebola threatens food security in West Africa: FAO (Food and Agricultural Organization), Reuters, September **(2014)**, http://uk.reuters.com/article/2014/09/02/us-health-ebola-food-idukkbn0gx0hb20140902, Date accessed 5/03/2016.
38. How the Ebola Virus impacts food security in West Africa, September **(2014)**, Foodtank.com/news/2014/09/how-ebola-virus-impacts-food-security, Date accessed 8/04/2016.

39. Bringing science and development together through original news and analysis, Ebola worsens food crisis in West Africa, January **(2015)**, www.scidev.net/global/news/ebola-food-crisis-west-africa.html, Date accessed 8/04/2016.
40. Food security after ebola, July **(2015)**, foodtank.com/news/2015/07/addressing-food-security-in-west-africa, Date accessed 8/04/2016.
41. Ebola virus disease, West Africa-update July **(2014)**, WHO, htt://www.afro.who.int/en/clusters-a-programmes/dpc/epidemic-a-pandemic-alert-and-response/outbreak-news/4216-ebola-virus-disease-west-africa-3-july-2014.html, Date accessed 6/04/2016.
42. West African cultural norms/tradition that may impact ebola transmission (PDF), HCA, Archived from the original (PDF) on January **(2015)**, http://wesleymc.com/util/documents/traditionsthatmayimpactebolatransmission.pdf, Date accessed 6/04/2016.
43. J. Navarro, The art of handshaking **(2013)**, www.psychologytoday.com, Date accessed 5/05/2015.
44. S. Gassel, How cultural norms and practices impact ebola outbreak, Tufts University, Tufts University, http://publichealth.tufts.edu/about-us/public-health-rounds-newsletter/how-cultural-norms-and-practices-impact-ebola-outbreakpage, Date accessed 6/04/2016.
45. H. Cooper, Ebola's cultural casualty: Hugs in hands-on Liberia, The New York Times, October **(2014)**, http://www.nytimes.com/2014/10/05/world/africa/ebolas- cultural-casualty-hugs-in-hands-on-liberia.html, Date accessed 6/04/2016.
46. Mark Anderson and agencies, Ebola: Airlines cancel more flights to affected countries, The Guardian, August **(2014)**, http://www.theguardian.com/society/2014/aug/22/ebola-airlines-cancel-flights-guinea-liberia-sierra-leone, Date accessed 5/03/2016.
47. Ebola wreaks economic woe In West Africa, NPR. 2014-07-17, http://www.npr.org/2014/07/17/332351578/ebola-wreaks-economic-woe-in-west-africa, Date accessed 5/03/2016.
48. J. Brock, Ebola fears slowing tourist flow to Africa, Reuters, August **(2014)**, http://uk.reuters.com/article/2014/08/20/us-health-ebola-africatourism-idukkbn0gk1gg20140820, Date accessed 5/03/2016.
49. Video: Impact of ebola on e-commerce in Nigeria-CNBC Africa, CNBCAfrica.com, http://www.cnbcafrica.com/video/bctid=3807923516001, Date accessed 5/03/2016.
50. Mixed grill for businesses as ebola impacts economy, Vanguard News, http://www.vanguarngr.com/2014/09/mixed-grill-businesses-ebola-impacts-economy/, Date accessed 16/10/2014.
51. BBC News—the Gambia's tourist industry reels from ebola fallout, BBC News, January **(2015)**, http://www.bbc.com/news/business-30495101, Date accessed 5/03/2016.
52. Ghana News Agency, Ebola scares tourist from Cape Coast Castle in 2014, spyghana.com, February **(2014)**, http://wwwspyghana.com/ebola-scares-tourist-from-cape-coast-castle-in-2014/, Date accessed 5/03/2016.
53. M. Tom, Tourists abandon Ghana despite its successes against ebola, May **(2015)**, Humanosphere, http://www.humanosphere.org/social-business/2015/05/tourists-abandon-ghana-despite-its-successes-against-ebola/, Date accessed 5/03/2016.
54. Standard reporter, Standard digital news: Business—Kenya tourism board releases Sh140 million for tourism campaigns, March **(2015)**, Standard Digital News, http:/www.standardmedia.co.ke/business/article/2000154024/ktb-releases-sh140-million-for-tourism-campaigns, Date accessed 5/03/2016.
55. Tourism receipts down 3%, The standard, March **(2015)**, http://www.thestandard.co.zw/2015/03/29/tourism-receipts-down-3/, Date accessed 5/03/2016.
56. S. Gilbert-Rivett, Africa Travel week focuses on sustainable tourism, The M & G online, April **(2015)**, http://mg.co.za/article/2015-04-15-africa-travel-week-focusses-on-sustainable-tourism, Date accessed 5/03/2016.
57. J. Chiahemen, Virus refocuses World attention on Zaire, May **(1995)**, Reuters North American Wire.
58. J. H. Choi and M. A. Croyle, *BioDrugs* 27, 565 **(2013)**.
59. Ebola kills over 5,000 gorillas, BBC, December **(2006)**, http://news.bbc.co.uk/2/hi/science/nature/6220122.stm, Date accessed 6/04/2016.
60. E. M. Leroy, P. Rouquet, P. Formenty, S. Souquiere, A. Kilbourne, J. M. Froment, M. Bermejo, S. Smit, W. Karesh, R. Swanepoel, S. R. Zaki, and P. E. Rollin, *Science* 303, 387 **(2004)**.
61. P. Formenty, C. Boesch, M. Wyers, C. Steiner, F. Donati, F. Dind, F. Walker, and B. Le Guenno, *The Journal of Infectious Diseases* 179(Suppl. 1), S120 **(1999)**.
62. H. M. Weingartl, C. Nfon, and G. Kobinger, Review *Dev. Biol. (Basel)*. 135, 211 **(2013)**.
63. H. M. Weingartl, C. Embury-Hyatt, C. Nfon, A. Leung, G. Smith, and G. Kobinger, *Sci. Rep.* 2, 811 **(2012)**.
64. L. Allela, O. Boury, R. Pouillot, A. Délicat, P. Yaba, B. Kumulungui, P. Rouquet, J. P. Gonzalez, and E. M. Leroy, *Emerging Infect. Dis.* 11, 385 **(2015)**.
65. S. Bashar, M. Percy, and R. Singhai, Predicting the 2014 ebola outbreak in West Africa using network analysis **(2015)**, News, updatephotos.com/ebola_breakout_in_africa.
66. G. Chowell, N. W. Hengartner, C. Castillo-Chavez, P. W. Fenimore, and J. M. Hyman, *Journal of Theoretical Biology* 229, 119 **(2004)**.
67. P. E. Lekone and B. F. Finkenstadt, *Biometrics* 62, 1170 **(2006)**.
68. J. Legrand, R. F. Grais, P. Y. Boelle, A. J. Valleron, and A. Flahault, *Epidemiol. Infect.* 135, 610 **(2007)**.
69. C. M. Rivers, E. T. Lofgren, M. Marathe, S. Eubank, and B. L. Lewis, Modelling the impact of interventions on an epidemic of ebola in Sierra Leone and Liberia, PLOS Current Outbreaks, 2014 October, Edition 1 DOI: 10.1371/currents.outbreaks.fd38dd8507856540b0be3fcd78f5ccf.
70. W. Kermack and A. McKendrick, *Proceedings of the Royal Society of London, Series A* 141, 94 **(1933)**.
71. A. Jaime, B. Delmar, G. Milton, M. Josue, R. Francisco, and V. Noe, Mathematical models to study the outbreaks of ebola **(1996)**, https://dspace.library.com, Date accessed 10/11/2014.
72. M. A. Kiskowski, A three-scale network model for the early growth dynamics of 2014 West Africa ebola epidemic, PLOS Current Outbreaks **(2014)**, DOI: 10.1371/currents.outbreaks.c6efe8274dc55274f05cbcb62bbe6070.
73. T. Stadler, D. Kuhnert, D. A. Rasmussen, and L. Du Plessis, Insights into the early epidemic spread of ebola in Sierra Leone provided by viral sequence data, PLOS Currents Outbreaks **(2014)**, October; Edition 1, DOI: 10.1371/current.outbreaks.02bc6d927ecee7bbd33532ec8ba6a25f.
74. M. F. C. Gomes, A. P. Piontti, L. Rossi, D. Chao, I. Longini, M. E. Halloran, and A. Vespignani, Assessing the international spreading risk associated with the 2014 West African ebola outbreak, PLOS Currents Outbreaks **(2014)**, DOI: 10.1371/currents.cd818f63d40e24aef769dda7df9e0da5.
75. C. L. Althaus, Estimating the reproduction number of ebola virus (EBOV) during the 2014 outbreak in West Africa, PLOS Currents Outbreaks **(2014)**, DOI: 10.1371/currents.outbreaks.91afb5e0f279e7f29e7056095255b288.
76. C. I. Siettos and L. Russo, *Virulence* 4, 295 **(2013)**.
77. O. Diekmann and J. Heesterbeek, Mathematical epidemiology of infectious disease: Model building, analysis and interpretation, Chichester, England: Wiley **(2000)**.
78. V. P. Driessche and J. Watmough, *Mathematical Biosciences* 180, 29 **(2002)**.
79. O. J. Okoro, *GlobalJournal of Science Frontier Research Mathematics and Decision Sciences* 13, 2249 **(2013)**.
80. J. Sztrik, Basic queueing theory, University of Debrecen, Faculty of Informatics **(2012)**, http://irh.inf.unideb.hu/user/jsztrik.
81. R. B. Cooper, Introduction to queueing theory, Second edn., Computer Systems and Management Science Florida, Florida Atlantic University, Boca Raton, Florida, Elsevier North Holland, Inc., New York, Oxford **(1981)**.
82. W. L. Winston, Operations research applications and algorithms, Fourth edn. **(2004)**, www.math.washington.edu/~billey/class/480/course.notes/Winston.OperationsResearch.pdf, Date accessed 20/10/2014.
83. S. M. Ross, Introduction to probability models, Ninth edn., Academic Press **(2007)**, www.books.elsevier.com. ISBN-13:978-0-12-598062-3, ISBN-10:0-12-598062-0, Elsevier Inc.
84. E. Isberg, Are differential equations the proper tool to describe reality? Retrieved from http://www.researchgate.net, Date accessed: 19/05/2015.
85. J. K. Kelso and G. J. Miline, Stochastic individual-based modelling of influenza spread for the assessment of public health interventions, *19th International Congress on Modelling and Simulation*, Perth, Australia, December **(2011)**, http://mssanz.org.au/modsim2011, Date accessed 20/05/2015.
86. J. A. Lewnard, M. Ndeffo, J. A. Alfaro-Murillo, F. L. Altice, L. Bawo, T. G. Nyenswah, and A. P. Galvani, *Lancet Infect. Dis.* 14, 1189 **(2014)**.

Received: 20 June 2017. Accepted: 22 September 2017.