On the Effectiveness and Relevancy of Engineering-Mathematics Teaching

Colin Gordon Black

Department of Mechanical Engineering,
Faculty of Engineering & Technology, Asian University,
Banglamung, Chon Buri, 20150, Thailand.

Abstract

It goes without saying that mathematics is an indispensable part of the curriculum of any engineering course. But, as anyone who has been involved in the teaching of the subject can testify, making it relevant to the needs of undergraduate engineers is not always the easiest of tasks. Moreover, students often poorly relate the mathematics they are taught in one course to that used in their other core engineering subjects. It may also be the case now, in the era of computer-algebra and sophisticated simulation software that both the knowledge and the types of mathematical skills actually required by engineers may have changed even though the curricula and teaching practice of many engineering-mathematics courses may not yet reflect this new reality. This paper first takes a brief look at the history of the emergence of mathematical thinking and how it eventually became an indispensable part of engineering. It then explores issues relating to the effective teaching (and learning) mathematics for engineering, including: course content, teaching methods as well as possible ways of inspiring young engineers to see mathematics as a useful friend rather than a necessary, but somewhat annoying, companion. It also examines questions such as: what mathematical skills are now essential in order to be a competent engineer? When does the mathematics presented need be proved rigorously or when will heuristic or less formal proofs suffice? In addition to presenting some insights, based on the author's experience of teaching many undergraduate mathematics, engineering and physics courses, the paper also attempts to address some of the difficulties, challenges and shortcomings of the traditional teaching of the subject and offers some new perspectives and recommendations.

Keywords: engineering; mathematics; education

1. Introduction

In many parts of the world there is a move from elite to mass higher education. In other words, more students than ever are going to university to study, and this is putting ever greater pressure on those who provide service mathematics courses [1]. In addition, there is often a considerable range in the mathematical abilities and knowledge of students arriving at universities to study engineering courses. Most engineering degree courses use the first year to ensure that all students have a firm grounding in basic subjects, especially mathematics and physics, and also to rectify, as best as possible, shortcomings or gaps in the students' knowledge as a result of their disparate educational backgrounds. But some students begin engineering and science courses with a less-than-required level of mathematical skills for their intended level of study. In 1995, the London Mathematical Society reported that in the UK there is "the perception of a marked change in mathematical preparedness even amongst the very best applicants" [2]. The reasons for this are not necessarily because of any innate lack of ability on the part of the students, but because of a variety of factors largely outside their control. For example, the previous reference notes the shift away in English schools from core mathematical techniques to more time-consuming activities such as data surveys and investigations. A reduction in the amount of time (around 20%) spent teaching mathematics in schools, as well as a delay in the teaching or even the removal of "difficult" topics was also noted. Other factors at play include changes in school mathematics curricula without the necessary changes in university curricula [3], as well as "grade inflation" [4] (a euphemism for the lowering of standards). The fact that students may not have been always taught the subject by a qualified expert in the subject, especially at critical stages in their education is surely also an issue. The shortage of qualified or competent mathematics (and physics) teachers in schools in comparison to other subjects is a widespread problem [e.g. 5].

Whilst it may not be entirely correct to discuss problems, and possible causes, originating in one part of the world and presume that they apply elsewhere, some, if not all, of the issues mentioned above will have a global resonance. Certainly, from my recent experience in teaching engineering mathematics and physics in Thailand, there is also a problem in the level of mathematical preparedness of some students entering university to study engineering. In fact, there often seems to be a problem with mathematics per se on a much deeper level within many societies. In short, mathematics has something of an image problem. And apart from being seen as difficult (which it often is) it is also sometimes perceived as being practically useless (which it most definitely is not!). In this context, I can recall recently having a conversation with an experienced engineer who, surprisingly to me, was also quite skeptical about the significance of the role of mathematics in engineering. With this apparent widespread "resistance" to mathematics, it will perhaps provide some insight into the problem to re-examine briefly how mathematical thinking in general emerged and to note especially the history of the relationship between practical applied engineering and mathematics.

2: Abstract thinking: from prehistory to Maxwell's equations

According to most accepted sources within the scientific community, modern humans, or *Homo sapiens*, emerged some 150,000 to 200,000 years ago in Africa [e.g. 6]. From that time, and during the development of human civilization over an almost unimaginably vast stretch of time, it is generally accepted that in terms of its intrinsic abilities and structure the human mind has not changed at all. In other words, a human being of average intelligence living 50,000 years ago, had the opportunity been available, could in principle have been taught the basics of something like differential calculus. I say 50,000 years, rather than 150,000 because, according to the fossil record, evidence of the faculty of human speech doesn't appear until after this time [7]. Moreover, our hypothetical prehistoric student would have presumably wanted also to ask some questions whilst learning differential calculus! Human speech is essentially a representation of thoughts, images, observations and intentions by the use of regulated sound. In other words it is, like mathematics, a form of abstraction: a way of symbolically representing various aspects of reality. Music (one human activity inherently connected with mathematics) is another way of analogizing, especially with regard to human experience and emotion, and appears to have had a similar chronology to human speech [8]. But solid physical art, the representation of real (or imaginary) objects and forms using materials, represents another and unique strand of man's desire to abstract or recreate aspects of the world around him. Solid prehistoric art produced by *Homo Sapiens*, in the form of carvings, decorated stones, or jewelry, dates back some 70,000 years [9]. However, the creation of images on surfaces, as for example in cave paintings, appears to have first happened very much later than solid art. The earliest examples (e.g. Lascaux in France) have been dated back to relatively recent times, somewhere between 20,000 and 30,000 years ago [Ibid].

Mathematical-type thinking, as with all the abstracting intellectual activities discussed above, also finally emerged because of humankind's general desire to understand and model aspects of the world around him. Attempts to represent time and the seasons can be seen in prehistoric artifacts from 35,000 years ago [10]. There is also evidence that early hunters had the concept of "zero", as well as of "one" or "many" animals in their flock. And although it had required a considerable passage of time in order to develop, in comparison to the various forms of art mentioned earlier, the concept of a number, central to mathematical thinking, was finally born. The construction of monolithic monuments also gives testimony to the fact that prehistoric peoples also had some knowledge of geometry. This type of activity represents perhaps the earliest example in human history of the coupling of mathematical thinking and engineering; Stonehenge (see Fig. 1) in England [11], constructed over 5000 years ago, is one such example of this. The Egyptian pyramids, the first of which appeared in approximately the same era, provide another striking example of this.

Fig 1. Stonehenge, England: an ancient example of the use of engineering mathematics?

Mathematics also began to help humans understand the world, not to mention also the heavens above, in a profoundly more comprehensive way. The Babylonians, Egyptians, Greeks, Indians, Chinese, Arabs and the Islamic world (notably the Persian mathematician, Muhammad bin Mūsā al-Khwārizmī) in short, many of the cultures of the ancient and medieval worlds contributed, at one time or another, to the flowering of early mathematical thought. It is worth noting that the Islamic world's contribution to the development of symbolic mathematics and geometry might at least in part have been influenced by perceived religious restrictions or prohibitions on the literal representation of living things, ironically reminiscent of much earlier times when herdsmen used each scratch on a rock to represent one head of cattle.

Now let us fast forward in human history to the scientific revolution that took place in seventeenth-century Europe. It is here where, perhaps, the idea of an engineer in the modern sense of the word began to emerge. By this we mean a "calculating" engineer rather than the empirical or experiential engineers of the prehistoric and ancient worlds (though as noted earlier, these ancient engineers still used some degree of mathematical thinking). This "modern" engineer could be said to have developed from a merging of the traditional artisans of the Middle Ages (e.g. blacksmiths, tanners, joiners, bellows-makers, and so on) with people willing and able to apply the new rationalism of the scientific revolution, pioneered by scientists like Galileo. And it was Galileo who, more than anyone else at his time or previously, successfully applied a "mechanical-mathematical mode of analysis" to a wide range of mechanical engineering problems [12]. In other words, mathematics plus engineering was shown to equal progress: significant progress at that. This juncture, I would suggest, represents the first documented encounter of world-changing significance in human history between engineering and mathematics. It also proved to be the intellectual engine that, from this point, propelled intellectual development forward with astonishing speed. Fig. 2 shows a page from one of Galileo's notebooks [13] in which he investigated projectile motion using mathematical analysis.

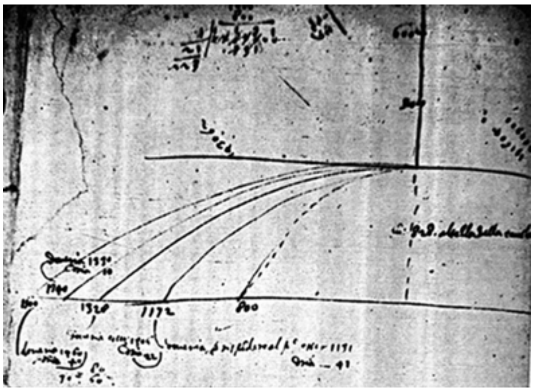


Fig 2: Page from one of Galileo's notebooks (source: [13]).

The scientific revolution led to the industrial revolution, resulting initially in the industrialization of Europe in the eighteenth and nineteenth centuries. Throughout this period, the worlds of mathematics and engineering continued to progress rapidly. It is fair to say, however, that these two fields did not (and have not) always seen eye-to-eye. But when the separate fields of engineering and mathematics do join forces and work together, the effect is decidedly synergistic. Some might say that the effect is magical; in fact, in the case of James Clerk Maxwell, we could say that the effect was literally "electrifying". James Clerk Maxwell was both a gifted engineer as well as being an exceptionally gifted mathematician and scientist. In 1865, Maxwell published a paper entitled: "*A Dynamical Theory of the Electromagnetic Field*". This paper contained a set of equations describing, for the first time ever, one of the most fundamental aspects of our universe: the interplay between electricity and magnetism; that is, electromagnetism. Maxwell's equations represent an astonishing pinnacle in human achievement, and combine profundity, brevity and beauty. Using the vector notation of Oliver Heaviside (another outstanding example of a gifted engineer and practical mathematician), these equations were finally represented as shown in Fig. 3.

$$\nabla \cdot D = \rho$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Fig. 3: Maxwell's Equations (in vector notation, as expressed by Heaviside).

The profound influence of Maxwell's equations on physics, engineering and ultimately on all aspects of the modern world cannot be underestimated, ranging from modern physics to mobile phone technology. But like all abstractions, Maxwell's equations do not give the whole picture (and perhaps this is a general point that we need to stress when talking about mathematical models). Whilst adequate and verifiable for most practical purposes, some incorrect conclusions following from the equations led to the development of quantum mechanics. In turn, quantum mechanics is often referred to as the most conclusive and verified theory of nature ever produced.

But what does all of this have to do with the teaching of engineering mathematics? It might appear odd, even inappropriate, that I should present a potted history of mathematical thinking and its relevancy to engineering in a paper ostensibly about engineering-mathematics teaching. In brief, my reasons for doing this are as follows. Firstly, to emphasize that whenever engineering and mathematics have worked together effectively in human history, there has been a synergy leading to dramatic progress and development. Surely, this fact, not to say also this effect should also be reflected in the teaching of engineering mathematics. After all, this observation serves to emphasize how really vital mathematics is to engineering (and science). Secondly, I have also implied that mathematical-type thinking is, at least in some sense, a human activity, an art, involving the abstraction or modeling of the world, or indeed the universe, that surrounds us. I know that this is not the whole picture, especially with regard to so-called pure mathematics, but at least in the current context of applying mathematics to the physical world it is a valid definition. In this sense, mathematics has similarities and parallels with all the other forms of art and modes of representation that have gradually developed and emerged from humankind's collective psyche over the aeons. Taking into account its relatively recent emergence as a human activity, as shown by the chronology presented earlier, this might indicate one reason why so many people find thinking mathematically more difficult than all the other forms of creative and abstracting intellectual activities usually labeled as "art". This naturally leads us to consider the best ways to effectively teach (and learn) this relatively new intellectual tool. But before addressing this question in detail, it is interesting to note at this point how far humankind has traveled from the abstractions found in cave paintings, probably inspired by the shamanistic visions of an alternative reality, to the awesome virtual reality simulations of today where art, engineering and mathematics are combined in ways that would have been unimaginable even a few decades ago.

3. How best to teach engineering mathematics?

It is surely not indisputable to say that mathematics for engineers and scientists should be taught in a way that is as integrated as far as possible with the subject or subjects to which it is to be applied. Traditionally, however, the bulk of mathematics for engineering courses has been taught separately as part of service mathematics courses, in some cases as common service courses for engineering students following a variety of disciplines such as electrical, mechanical, civil, software and chemical engineering. This is usually concentrated into the first two years of study (a fact that may itself cause some problems) though some specialized mathematics courses may be taught in the latter years of an engineering degree course. For example, at Asian University, we have a third-year electrical engineering-mathematics course including topics such as Z-transforms and Fourier Transforms and other mathematical tools that are a priority for electrical engineers but not so much for mechanical engineers. Clearly, whilst there will be some common needs, there will also be significantly different needs from each of these disciplines.

In addition to the issue of teaching aspects of mathematics that may not be immediately relevant, there is also the problem of the time interval between exposure to some relevant mathematics and its application to a particular engineering discipline. It is also not uncommon to be confronted with a situation where the "horse has been put before the cart", and students are exposed to an engineering concept without yet having been taught the mathematics. An example that springs to mind from personal experience is when I have tried to use some concepts involving vectors in mechanics to students who have not yet formally met them in their service mathematics courses. Being taught the same thing twice is the inevitable result, though this is not always necessarily a bad thing for some students, even if it does not appear to be the most efficient use of teaching time on paper. In and ideal world, the necessary mathematics would be taught "just in time", but this is not a realistic or practicable prospect.

Various solutions have been suggested to address some of the concerns given above. Discipline-specific supplements to a core common engineering mathematics course are one possible way to solve, or at least ameliorate, the situation [14]. A greater integration of the relevant mathematics with the engineering subjects, in other words, transferring a substantial part of the mathematics taught from separate service modules into the engineering modules themselves is another suggestion [15]. For example, an undergraduate chemical engineering course concerned with rates of reaction might teach coupled ordinary differential equations in addition to the relevant chemistry. In this way, students immediately see the relevance of what they have been taught, and the examples they are given will be more appropriate than the more general or possibly even contrived examples given to them in a general service mathematics course.

Some educators have argued that engineering mathematics should be taught by engineers and not by mathematicians. In a report entitled "*Engineering Mathematics Should be Taught by Engineers*" [16], the authors make the case that engineers would make better teachers of engineering mathematics because they are "unlikely to become entangled with providing unnecessary proofs", and will not provide "unrealistic examples". The authors of the same report also suggest that engineering lecturers can "better relate" to their students. I will consider the issue of "unnecessary proofs" later. Indeed, improving the relevancy of the mathematics taught to students is a key issue in the teaching of mathematics to engineers and scientists, of which some observations and suggested courses have action have been discussed above. The third point made by the authors about having empathy with the students is also important. It is natural to assume that an experienced mechanical engineer, for example, would usually be more attuned to the needs of mechanical engineering students than a person who exclusively thought of themselves as a professional mathematician. There are, however, also those strange animals who might think of themselves as being exclusively engineering mathematicians. And if he or she is an effective engineering mathematician then they will make it a priority to empathize with the needs of their students, teaching what is needed and emphasizing what is especially important rather than being academically self-indulgent. There is surely a strong case for saying, as with all academic disciplines that the most effective teaching is delivered by specialists who are cognizant of the needs of the students and who are broadly familiar with the courses they are pursuing. Poor mathematics teaching in schools, for example, is often said to occur because it is not carried out by specialists, or even by people who especially like or enjoy the subject. This is often cited as a major reason why students remain weak in mathematics and in turn view the subject with some misgivings. With regard to university-level teaching of the subject, specifically in the context of engineering mathematics taught as a service course, the syllabus of a particular course should have been developed in cooperation with the various engineering schools and should reflect their needs rather than just representing the way things have always been done in the past or the particular interests of the engineering mathematics lecturers.

In assessing the best ways to help students through teaching, it is also relevant to think about the best way students can be helped or motivated to teach themselves. I can still recall as a high school pupil hearing one of my teachers saying that mathematics was different from other subjects in that you only really learned it by actually doing it yourself rather than being told about it. Arguably, this is true of all technical subjects to some extent, but it seems to be especially true of mathematics, where understanding and the development of objective problem-solving skills clearly trumps memorization and subjectivity. Indeed, the most effective way to achieve a deeper understanding of a mathematical technique or concept is to have to judge that it is appropriate for the problem at hand, possibly selecting it from a number of possible alternatives and perhaps using it in conjunction with some other techniques. Then it may have to be adapted into a suitable form and used effectively to find the solution. When providing assistance to students solving problems, it is generally accepted that tutors should only give guidance when it is felt that the student has genuinely made some effort to break into the problem. Many students when presented with a new problem will first look for a direct correspondence between the current one and the solution of a previous example given to them by their lecturer. If one is not immediately forthcoming, disillusionment may set in only to be followed by the familiar plea for help: “How do you do this problem, sir?” Some analogy with spoon feeding is very appropriate in this situation. Students that overly rely on immediate assistance fail to develop the necessary confidence and elan that should be part of their educative process. Moreover, they will fail to develop the necessary tenacity needed later to solve problems involving the application of mathematics to real life situations. Ideally, the students will have already attempted and had time to think about the more demanding problems before attending a scheduled tutorial session. Giving students the time to think, by not making excessive and unrealistic demands on their time and capabilities is also important.

Sometimes, when teaching engineering mathematics, it is really necessary to state the obvious. What may seem obvious or self-evident to someone who has been teaching or practicing the same subject for years will not always be so for a student meeting it for the first time. Students occasionally get held back in their understanding of something just for want of a simple, even trivial, additional explanation or statement. The same thing also applies to textbooks where students are often frustrated when a key step is omitted from a proof, derivation or solution, maybe because it seems to be obvious or transparent to the author or authors of the book and hence not worthy of inclusion. For example, looking at the customer reviews of a revised edition of one famous engineering mathematics textbook (on the commercial Amazon booksellers site on the worldwide web) I came across the following comment: …“*I think they've packed too much in sometimes, without covering it properly, or just by omitting things because they are supposedly ‘obvious’; well not to everyone*”…[17]. It should be also stated, however, that the reviewer does go on to indicate that the book in question is a good mathematics textbook, and that the complaint is motivated by how things have been omitted in comparison to an older edition of the same book.

At Asian University, small class-sizes have permitted us to have an effective tutorial system where students are able to receive a relatively large amount of individual attention from tutors. Homework assignments can be marked and returned quickly to the students. Rapid feedback allows problems or misconceptions to be quickly identified and hopefully remedied. Another advantage is the greater degree of class participation that is possible in smaller groups. Group discussions about problem-solving are a very effective and efficient way to learn. Students often have common difficulties or share the same misconceptions. They also learn well from each other’s questions and difficulties. In fact, it has been claimed by some educationalists that seventy per-cent of what students learn is from discussions with their peers, the remainder being supplied by their lecturers. Moreover, students may also at times be able to assist lecturers or tutors, especially in the teaching of non-native English speakers (or whatever is the language of instruction) and can provide a translation or a synonym for an unfamiliar word or technical term.

At Asian University, we run a short course called the Intensive Academic Programme (IAP) prior to the start of each academic year where new engineering students are given courses in English communication skills, basic mathematics and physics. Apart from acting as a brief revision course for basic subjects and also introducing students to their teachers and university life in general, another important aim of the IAP course is to allow non-native speakers of English to become acquainted with some of the terminology they will encounter later on in their engineering degree courses. Although students may be familiar already with a particular concept, they may get confused when hearing it referred to in a variety of ways. One example is the “gradient”, for which the terms: slope, derivative, or rate of change could also be used, yet a non-native speaker may only be familiar with one of these.

4. The role of informal methods in teaching engineering mathematics

Most of us will already be familiar with informal, and sometimes irreverent, self-teaching guides for various subjects, typified by the so-called Dummy series [e.g. 18]. As someone who has used some of these books to bridge gaps in my computing knowledge, I can vouch for their effectiveness. These books are appealing and refreshing in their open and honest approach to instruction. They also entertaining and, dare I say it, fun to read. Another famous example (though by no means one for dummies!) where an informal and engaging style is used is the Feynmann Lectures on Physics series [19]. Reading Feynmann's Lectures on Physics, with its mix of informality, acute perception and somewhat folksy language is a bit like having the essence of a difficult lecture explained by an intelligent and knowledgeable friend. I believe that the use of informal, and even entertaining or humorous, language can be very effective tool in teaching mathematics to engineers, as well as in making the subject generally less inhibiting.

I know from personal experience that the injection of some informality, including humour, into the teaching of engineering mathematics can have a very positive impact on students. Moreover, some self-deprecation on the part of the lecturer can also help boost students' confidence. For example, if a lecturer owns up to the occasional error in a calculation or derivation, or even confesses to having some difficulty with some aspects of an advanced concept, then this can reassure students that it is normal to experience such things when doing mathematics. Students may then appreciate that they should not become unnecessarily disheartened when things are difficult or when they go wrong. Moreover, since mathematics is sometimes seen as dry and unexciting subject, albeit an occasionally useful one by those not studying it as their main course of study, anything that makes it more accessible is surely a good thing.

When discussing the formative years of James Clerk Maxwell [20], Ian Hutchinson (Head of the Department of Nuclear Science at M.I.T.) states that: "Undoubtedly his father's patient informal tutoring was an abiding formative influence". In other words, patient, informal tutoring was an "abiding formative influence" of arguably the greatest engineering mathematician of modern times. This is surely a point worth noting, especially with the knowledge Maxwell was no child prodigy, but rather someone who gradually developed his formidable mathematical skills and insights over time. It is also noted [Ibid] that he was someone who had not responded well to a stern and rigid tutor originally hired by his father (see Fig. 4).

Fig. 4: Maxwell escaping his tutor in a washing tub.

It is also important to develop the habit of exploratory or heuristic thinking when solving problems in mathematics. An immediate example that comes to mind is in the simple harmonic motion equation: $x'' + \omega^2 x = 0$. Usually, when I tell first-year students that we can see, or "guess", that the solution is: $x = A\cos\omega t + B\sin\omega t$, many of them are initially unimpressed or unconvinced by this statement. Even after the "guess" can be shown to be correct by direct substitution into the equation, some doubts about how the solution was obtained in the first place still linger. It is somehow felt that this is not a valid or proper way of doing mathematics, and that perhaps it is not formal enough. Maybe it is even something of a cheat, and that some prior step should have been carried out to obtain the solution rather than apparently just pulling it out of thin air.

In considering the issues surrounding formality and informality in engineering-mathematics teaching, the thorny issue of providing proofs will inevitably arise. It is clearly not possible to prove everything rigorously, but on the other hand merely just stating results and telling students to use them is clearly also not desirable. The former approach would be overwhelming and impractical for both students and staff, whilst the latter might encourage a mindset of rote memorization and would not be beneficial to the development of essential logical and mathematical skills. Nevertheless, having said this, I do not think that teachers of engineering mathematics should be afraid of using less formal or heuristic proofs to justify results, or at least to hint that the results might be true. Indeed, such approaches are often more natural, and correspond to how certain results were actually discovered in the first place. A more rigorous or formal justification of the result often follows afterwards. After all, intuition and insight has often been a strong force for development in mathematics, physics, chemistry and a host of other fields throughout human history.

With regard to the issue of providing a convincing mathematical proof for a given result, I can recall reading the following quote (but sadly I do not have a reference for it) said of a professor to his graduate student when presented with a mathematical proof: "Don't just prove it to me, convince me". The sentiment of this statement is very pertinent in the case of proving the validity of mathematical results to many engineering and science students. Sometimes, an informal proof is the most effective way to proceed before (if necessary) finally providing a formal one. The informal explanation often allows students to appreciate the validity of a result in a way that a formal proof might obscure. An example I recently experienced of this was when a student asked me to show (or prove) that the divergence operator in polar coordinates is given by:

$$\nabla \cdot \vec{V} = \frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z}$$

The student had been trying to transform a given result from Cartesian coordinates into polar coordinates, but for some reason had been unsuccessful. I suggested that an alternative and perhaps more natural approach was to try and have some picture, or analogue, of what divergence actually meant in a physical sense and to combine this with a polar coordinate view of the world. To understand the meaning of divergence, I suggested the idea of a small volume in space with a "flow" passing through it, and drew the sketch shown below in Fig. 5:

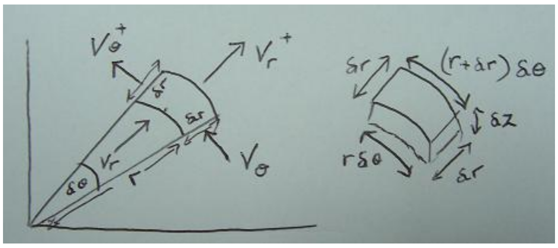


Fig. 5: Informal sketch illustrating the concept of divergence: $\Delta \cdot \vec{V}$.

The formal definition of divergence is given by $\lim_{\delta v \to 0} \frac{\iint_s \vec{V} \cdot d\vec{A}}{\delta v}$, where $\vec{V}$ is some vector property of the "flow", dA is a infinitesimally small element of area and δv an infinitesimally small element volume that is made to tend to zero (making the divergence a property of a point in a space). The flux integral in the numerator represents a summation over all the surfaces of how much of the "flow" property remains within the small element of volume. In other words, this is an "input" minus "output" calculation taken over all the corresponding faces of the small element of volume δv enclosing a particular point in space. The orientation and magnitude of the faces on the volume element, as expressed by the area vector in the vector scalar product $\vec{V} \cdot d\vec{A}$, performs the "in" minus "out" calculation as required. Moreover, the variation of the "flow" property within the volume in each of the coordinate directions is expressed by first-order Taylor expansions (e.g. $V_r^+ = V_r + \frac{\partial V_r}{\partial r}\delta r$). From this picture, the desired result was eventually obtained in a few more relatively simple steps. The beauty an informal approach like this is that it uses a very familiar and easily-understandable concept. In this particular example, it could be reduced to the statement: "what remains inside is equal to what goes out minus what comes in". Using a conceptual basis like this prevents the student from becoming lost in a morass of symbols and operators that may seem to be far removed from their physical significance. This particular example also happens to bring out what is surely an extremely vital skill that needs to be taught early and constantly re-emphasized throughout any engineering mathematics course, namely the ability to linearize expressions, or more generally the ability to use Taylor series expansions in a variety of guises. This will be discussed in more detail later.

Some other issues to consider here are the reason for giving proofs in engineering and service mathematics courses and the extent to which they are needed. As pointed out in [21], the rôle of proof in standard scientific research and the rôle of proof in the classroom is quite different. In research, its purpose is for the assurance of truth, whilst in the context of teaching the primary purpose of proof is to *explain.* I remember as a first-year undergraduate, studiously working through and compelling myself to understand the large number of proofs given in lectures. I knew very well, for example, how to prove that a determinant was an "alternating multi-linear function", but recall feeling cheated when the mathematics lecturer announced just before the Christmas examinations that the most important thing was to "know the results" and "not to worry too much about the proofs". In other words, in terms of passing the examination I was better off being able to calculate a determinant than in actually having any insight into what one actually was. In hindsight, of course, I came to realise that understanding the proofs had also allowed me to gain a better understand of the mathematics. That being said, however, the final emphasis in engineering mathematics is always going to be on the application of results. For example, being able to demonstrate use of the convolution theorem for integrals in an examination to determine the response of a system should have some priority over being able to prove it. The extent to which mathematical proofs are needed (and their level of formality) is ultimately a matter of judgment taking into account their rôle as a tool for explanation and what can be assumed to be axiomatic.

5. Teaching resources

In the (mainly long) past, many of the available engineering-mathematics textbooks were undoubtedly dry, uninspiring and even daunting tomes. Some even appeared to have been written specifically for people who already knew the subject rather than actually for genuine learners. But nowadays there are many excellent engineering-mathematics textbooks available that are both comprehensive in scope and appealing to students. Some have gained universal acceptance as standard texts by lecturers [e.g 22]. Such books are genuinely useful to students as a supplement to their lecture notes, as a source of useful exercises and also for self-study. As well as standard format text books, with explanations, examples and exercises, programmed-learning engineering mathematics textbooks can also be extremely effective, especially for students who might otherwise struggle with particular concepts. Students are encouraged to work through the steps in a problem and, depending on their answers at various stages, are directed onwards to the next stage or backwards to review earlier material. One outstanding example of programmed-learning in engineering mathematics is the universally praised: "Engineering Mathematics" by K.A. Stroud [23]. I have used earlier editions of this book in the past, but was interested to read what current engineering mathematics students think of this book. One of the customer reviews of the latest version of the book on the Amazon book site on the web typifies the praise that has been heaped on the various editions of this book: "*My God! What a revelation, for the first time here was a maths text book that helped me to understand exactly what was going on and took me step by step through increasingly difficult problems.*" [24]. ("Advanced Engineering Mathematics" by the same author uses a similar approach.)

Mathematics books of whatever form are enhanced in their effectiveness by accompanying material on CD ROMs or by providing access to a companion website for both staff and students. Most modern engineering mathematics textbooks provide at least one of these two options. There is also an increasing number of websites and forums devoted to all aspects of mathematics and physics. Students, researchers or the general public can post questions or problems on these forums, and provided the question or problem is sufficiently well-posed, a solution to the problem, or at least a helpful comment, is usually forthcoming. In the past, a student who was unable to get enough help from a lecturer or tutor for an assignment might turn to a fellow classmate for help. This is, of course, not the only way in which the internet can be used as an educational tool. There are many websites and resources available on the internet devoted to every aspect of mathematics and at all levels. As with so many other things, the internet will, or already has, revolutionized the teaching of engineering mathematics.

The availability of computer-algebra software packages is another example of a revolutionary development in the teaching of engineering mathematics. It offers new and far-reaching possibilities for the teaching of the subject. It is understandable that there might be some slight reservations about such a tool, much in the same way there were about the use of electronic calculators in school mathematics teaching, specifically in tests or exams, when they first appeared around the mid 1970s. The feeling that it is detrimental for students to be helped with numerical calculations, and now also with algebraic manipulations and operations in calculus, is clearly motivated by a fear that students relying on such tools will not develop essential skills or gain necessary insights into the subject. In the case of computer-algebra packages, there might also be the feeling that students will be able to obtain answers by shortcutting the usual steps needed in order to obtain solutions to problems. In other words, students may be able to obtain correct answers without necessarily fully understanding how they arrived at them. A prime example of this is in calculus where functions can be symbolically differentiated, or integrated, merely by highlighting a particular variable in a term in a spreadsheet and selecting a menu option in the software. Some people may ask: why fret over the level of proficiency a student (and later on perhaps, a practicing engineer) has in applying, for example, the quotient rule or the chain rule to a function when a piece of software can shoulder the burden? Moreover, as long as the general concept of a derivative is understood, surely that is good enough. In response to this, many would say that within reasonable limits this sentiment is correct. But concepts still have to be properly understood; familiarity and proficiency with the basic results is what really assists in the development of this understanding. Lessons can still be structured so that during problem solving, computer-algebra software is used to assist with the more mundane tasks as well in checking the validity of final results. Furthermore, as a result of computer-algebra software, there is now greater scope for giving realistic problems, allowing a more effective use of the time available for learning. Most computer-algebra software packages contain programming features that are relatively easy to master, with clear and almost self-evident syntax, dispensing of the need to spend (i.e. waste) fruitless hours "debugging" as well as learning the syntax and idiosyncrasies of a particular language, such as was the case with the old technical programming languages: activities, which I know from experience were the bane of engineering mathematicians of whatever station in the past. The new opportunities offered by this technical revolution should, in principle, lead to students gaining a deeper understanding of the mathematics much more efficiently. As an example, students will be able to quickly graph and then interpret or check results. They will also be able to explore with ease the effect of changing parameter values.

With sufficient imagination, the possibilities for the effective use of computer algebra software in engineering-mathematics teaching are boundless. What is more, this is the way engineering problems are increasingly now being solved in real life and will continue to be so in the future. Tools such as this are now seen as indispensable to the engineer as slide rules once were half-a-century or so ago. Common sense and judgment still, however, needs to be used in applying new technology to mathematics teaching, especially with regard to the software used. For example, I had some experience several years ago of teaching in an institution where new technology was not really being used as an effective teaching aid for mathematics, rather it was being used in an almost fetishistic way. In this particular case, the actual use of new technology was almost an end in itself. Students were forced to spend lessons keying in letters in response to multi-choice questions. To make matters worse, some of the multi-choice problems were ambiguous, ill-posed or just plane unimaginative. And if that wasn't bad enough, the supposed answers were not always correct. I should also add that the software frequently crashed, wiping out in the process students' scores for that session, meaning that the whole sorry exercise had to be repeated again in the future. It all looked impressive for public relations purposes, even though the students were using computers in a way that was actually less effective than traditional ways of teaching.

Another issue in the effective application of computer-algebra software for the teaching of engineering mathematics, especially with first-year students, is that of being able to actually convince them to make use of the software in the first place. Many students are initially skeptical of the benefits, and see it as just one more thing they have to learn rather than something that might actually make their lives easier. In [25], the author points out that: "familiarity with the software is initially a barrier to student success in solving first-year problems, and that very few students have any experience in writing software input commands in any language or for any application". In my mathematics and other engineering classes, I have tried to encourage students to make greater use of packages such as *Mathcad*. I have especially emphasized its use in checking their results, or in assisting with some of the more tedious and involved algebraic manipulations incidental to the problem at hand. In more advanced courses where students have to solve, for example, partial differential equations numerically, a well-documented software worksheet may constitute part of the work they need to submit as part of their assignment. I have also started to include the process of writing, or completing, computer-algebra software worksheets as part of examination questions (see Fig. 6). External examiners have accepted such questions, but naturally raise issues about whether familiarity with a particular software package has been stipulated as part of the syllabus for the subject in question and whether this might provide some obstacles in answering the question correctly.

```
for i ∈ 0..n
  u[i,0] ← f[i]
for j ∈ 1..m
  | u[0,j] ← 0
  | u[n,j] ← 0
for i ∈ 1..n − 1
  u[i,1] ← 0.5·(u[i−1,0] + u[i+1,0]) + k·g[i]
for j ∈ 1..m − 1
  for i ∈ 1..n − 1
    u[i,j+1] ← u[i−1,j] + u[i+1,j] − u[i,j−1]
u ____________________
```

Fig. 6: Fragment of *Mathcad* worksheets can be used in an advanced engineering mathematics exam question.

6. What are the essential skills, concepts and techniques that need to be taught and encouraged?

The development of a general fluency in applying mathematical-reasoning in conjunction with engineering insight is a key aim of engineering-mathematics courses. Another important skill that needs to be taught is the ability to make reasoned assumptions and approximations when building mathematical models. Developing these general skills is important given the transient nature of techniques and the fact that applications change. An analogy would be in computer science where the development of the necessary logical skills that allow a computer programme, for example, to be written should take priority over knowing in detail the specifics of a given computer language, which may anyway eventually change or become obsolete.

There are certain concepts that reoccur repeatedly in many different areas of engineering mathematics. For this reason, they are important, and need to be taught as early as possible, and re-emphasized as often as possible. When writing this paper, I thought about the different techniques and results from mathematics that I have actually used in research and in practical applications to industrial problems. I decided that the most useful thing (in the sense that I had needed to use it the most often) was the ability to expand functions using one, or multi-dimensional, Taylor series expansions, and to use them to make (usually linear) approximations. The linearisation of non-linear functions was an activity that seemed to be ever-present. Indeed, throughout engineering and physics, non-linear systems are linearised in order to obtain expressions valid for small perturbations about some nominal point. This is even done in high-school physics when analyzing the motion of a simple pendulum for small angular displacements about the vertical. The ubiquitousness of linearisation (i.e. using the first-order terms in a Taylor series expansion of a given function) coupled with the assumption that it represents a valid approximation or assumption is the reason why it is often jokingly said that the first law of engineering is: "*All systems are linear!*". General familiarity with series expansions is essential in all branches of engineering (dynamics, control theory, error analysis etc. etc.), as well as in the theoretical aspects of engineering mathematics (e.g. optimization, numerical integration and differential calculus).

I think it is also very important that students understand how to define a system and its boundaries early on in their studies of engineering mathematics. Gaining an understanding of some of the more difficult or potentially confusing results in mechanics, fluid mechanics and thermodynamics, for example, is made easier if the idea of how a system is defined and how it interacts with outside influences through its boundaries has already been internalized by the student. Isolating some part of a structure or machine that is of interest, or some control volume in a fluid flow, and defining this to be the system is often a crucial first-step in analysis. In this context, a relatively simple example that comes to mind (but one that is absolutely crucial in mechanical engineering) is the idea of a free-body diagram showing the forces and moments acting on a body. In drawing a free-body diagram, we are in essence defining a system with its boundaries and are indicating the external influences (forces and moments) acting on it. This process often involves making an imagined cut or slice through a physical object, such as in the case of analyzing a loaded beam (see Fig. 7).

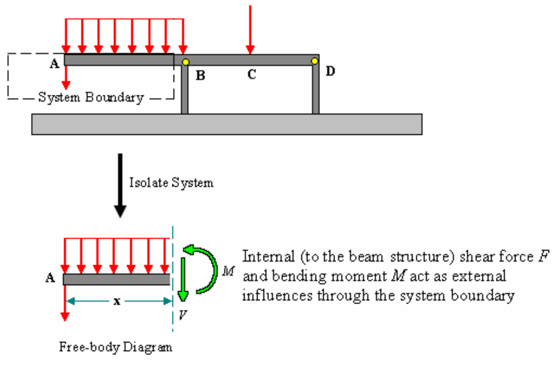


Fig. 7. Defining a system in a loaded beam.

Students may initially have difficulty in conceptualizing an imaginary boundary. They often also need convincing that only the forces (or moments) acting immediately on our isolated system through its boundary will have any effect on it or that the forces or moments exerted by the system on any other body do not need to be taken into account in the analysis. It is frequently, and mistakenly, believed that the forces or moments existing in the wider picture are somehow not being taken into account. But in actuality they are, with their effects being implicit in the forces and moments acting at the system boundaries and as such are already represented on the free-body-diagram. In my teaching, When they arise, I try to counter such misapprehensions by treating the system as if it was something really capable of feelings; I tell the students that the system (in whatever context) "doesn't care what is happening elsewhere", but is only concerned with what it "feels" or experiences directly at its boundaries. The example of being pulled by holding hands with two other people who are themselves holding hands with others and in turn being pulled gives the students a clearer picture of this point.

One more concept involving systems that I believe is crucial in applying mathematics successfully to engineering is system identification (together with the associated concept of parameter estimation). Traditionally, system identification involves finding both an adequate structure for a mathematical model (in other words, the general form of the equations to be used) as well as estimating the specific values of the coefficients or parameters within the model (i.e. parameter estimation). For example, some initial analysis may indicate that a particular system can be represented adequately by a set of coupled linear first-order ordinary differential equations (obtained perhaps following the linearisation of an original non-linear representation, hence being valid for only small-scale perturbations about a nominal point). The model may then be written in linear state-space form, and includes parameters as elements of both the state matrix A and the measured input matrix B:

$$x' = Ax + Bu$$

Given the model structure indicated above, the problem is then to find optimal estimates of those unknown elements of A and B. Optimization and statistics (typically involving 'least-squares" methods) as well as engineering insight, are key components of this task. In general, the mathematical modeler, or "system identifier", thus first has to establish in a given situation the structure of an adequate model, and then has to determine accurate values of the parameters within that particular model. In first establishing what actually constitutes an adequate model, it is important to appreciate that in all practical cases there is no such thing as a "true" model. The desired outcome in applying mathematics to an engineering situation is to obtain a model that serves the required purpose. It is important for students to appreciate that all practical mathematical models are based on assumptions that determine the limits of their validity. The myth of the "true" model is still, however, very strong even amongst practicing engineers. For example, [26] cites the following comment: "*A favourite form of lunacy among aeronautical engineers produces countless attempts to decide what differential equation governs the motion of some physical object, such as a helicopter rotor… But arguments about which differential equation represents truth, together with their fitting calculations, are wasted time.*" A familiar example from physics can perhaps further clarify this idea.: Newtonian mechanics is perfectly adequate for making all the calculations necessary for sending a man to the moon, but in light of Einstein's special theory of relativity it is inappropriate to claim without reservation that such a mathematical model based entirely on Newtonian mechanics is the "correct" model; it is certainly adequate, however, for the purpose of calculating the flight path to the moon. Incorporating special relativity into the model would introduce unnecessary complications without giving any significant improvements in its predictive qualities. This highlights an important principle in mathematical modeling known as the "principle of parsimony", which states that any mathematical model should be as simple as possible; this is really just another way of expressing a guiding principle in philosophy, first suggested by the 14th-century philosopher, William of Ockham, known as Ockhams' razor [27], which states that the simplest explanation of any phenomenon, making the least assumptions, is always the best one. As a general rule, simple mathematical models are preferred to over-wrought or over-parameterized models that will have poorer predictive qualities.

The practical use of mathematical models to predict results will inevitably require some analysis of the errors or uncertainties. In some cases, such as with Kalman filters, this will incorporate uncertainty in both the mathematical model and in any measured inputs used to "drive" the model. Moreover, a common (but not always justifiable) modeling assumption is that all uncertainties have Gaussian or normal distributions. This assumption is valid sometimes because of the central-limit theorem of statistics, but just like the assumption of linearity discussed earlier it is also often made because it is just mathematically convenient to do so. Modeling uncertainties that do not have a normal distribution is inherently more difficult. Understanding and appreciating uncertainty in data is essential in engineering. And although it is often valid to do so, prospective engineers should also be taught to be wary of assuming that all uncertainties have a Gaussian distribution.

Compiling a complete list of all the other essential skills that might be considered necessary for students studying engineering mathematics would probably just end up look like a copy of most engineering-mathematics syllabi. Therefore, I will avoid doing this and just mention five more things I feel really are the *sine qua nons* of an engineering-mathematics education; these are: a complete understanding of the nature of a function; total competence in algebraic manipulation; familiarity with the general concept of using transforms (e.g. Laplace, Fourier, or Z-transforms) to convert a problem involving derivatives into one having a more convenient algebraic form; confidence in the use of dimensional analysis to validate expressions and to predict the forms of relationships; and a firm grounding in probability and statistics, especially probabilistic distributions with a view to their use, for example, in reliability analysis, error analysis, control-system design or optimal estimation.

7. Effective testing and grading of engineering mathematics

Examinations are like democratic systems of government: neither is perfect, but the alternatives are not without their faults. And so given that examinations and tests will be how students' understanding of the subject is assessed for the most part, this then begs the question: what constitutes a good examination question in engineering mathematics? This is surely a topic worthy of a study in its own right, but at the very least I would suggest that a good engineering-mathematics examination question is one that allows an average student to make some initial headway, and to demonstrate some understanding of the subject matter, without being thwarted or stumped at the very beginning. An ideal examination question would become progressively more demanding (i.e. be "wedge-shaped") allowing it to differentiate between an excellent student and an average one.

It seems to be common practice to mark engineering examination questions positively, that is not overly penalize students for mistakes made at earlier stages in their solutions, especially numerical errors, if they are able to demonstrate correct and logical progression of the problem at the later stages.

The use of examinations as a tool for assessment, however, is clearly more problematic in some areas of engineering mathematics than in others. For example, much of the important numerical analysis in an engineering-mathematics course does not lend itself easily to the time constraints of an examination. As a result, this may distort the teaching process in favour of more examinable but not necessarily more important aspects of the course. For example, lengthy and time-consuming numerical problems, such as the numerical solutions of partial differential equations, cannot be easily incorporated fully into standard examinations. Another alternative (particularly relevant to the type of numerical problem just mentioned) might be to allow for some weighting in the final score for individual mini-projects carried out by students (this would be in addition to marks awarded for the usual shorter problem-based assignments). Such projects could also be a vehicle for students to become familiar with, and to use, computer -algebra or simulation software packages.

Mathematical modeling skills, as discussed earlier, would also be further developed and tested satisfactorily by mini-projects more so than by the traditional approach primarily involving examinations. This type of activity would give students time to assess a problem thoroughly, and to appreciate more deeply how mathematics is really applied in the world of engineering.

In summary, a compromise has to be struck between the use of formal written examinations and various forms of continuous assessment, including projects. This should be based on what is considered the best option for testing the particular subject matter. It also has to be acknowledged, however, that continuous-assessment assignments and projects have their weaknesses as a tool for assessing students where individual work might not be quite as individual as the lecturer intended it to be!

8. Conclusions

Reading through recent literature on engineering-mathematics education reveals that there are many challenges ahead for the teaching of the subject. The increasing numbers of people who now have access to higher education, and the consequent wide range of abilities, backgrounds and motivations of those taking service courses in mathematics, presents some unique problems. There are no quick fixes or easy solutions to these problems. It is clear, however, that mathematics courses need to be sensitive to the needs of the engineering courses they are serving. Certain key skills and concepts need to be taught as early as possible and emphasized as often as possible. There is also a need to anticipate, whenever possible, which areas or topics need to be taught before others.

The continuous development of transferable and general mathematical modeling skills is an extremely important aspect of engineering mathematics courses, and should take precedence over the learning of specific techniques that may become less relevant with time.

With regard to the range of abilities and levels of interest students are likely to bring with them when they first start service mathematics courses, encouraging the use of more programmed self-study material, together with a more imaginative and sometimes less formal approach on the part of teachers, will be beneficial.

It is evident that computer-algebra and other types of software, such as simulation software, will play an increasingly important role in the mode of teaching engineering mathematics. Online resources will develop and continue to offer great possibilities for both teaching and self-study. Indeed, it is safe to say that we are now on the cusp of a paradigm shift in terms of how engineering mathematics is taught and how students will learn it as a result of the digital revolution.

Finally, through good and effective teaching of the subject, mathematics should hopefully become to be seen by students as something that is directly relevant, useful and indeed essential to engineering rather than a complicating hindrance that should, whenever possible, be disassociated from it. If this can be achieved, even just partially, then mathematics and engineering will continue to have a productive and symbiotic relationship. The inspirational examples set by people such as Galileo, James Clerk Maxwell and all the other countless great thinkers who have managed to successfully combine engineering acumen, technical insight and mathematical analysis throughout the ages, all give testimony to this fact.

References:

1. Taylor, Janet A and Mander, David. "*Managing diversity in first year service mathematics course: integration of effective numeracy teaching principles and management theory*". First Year in Higher Education, 2002 Conference, The University of Canterbury in conjunction with Queensland University of Technology. Christchurch, New Zealand. 8-10 July 2002.
2. London Mathematical Society. "*Tackling the Mathematics Problem".* London, London Mathematical Society, Institute of Mathematics and its Applications, Royal Statistical Society, 1995.
3. Croft, Tony. "*Mathematics for all - supporting your students with mathematics and statistics at the transition to university*". Dyscalculia and Dyslexia in Higher Education**:** "Forging the Future"**.** Loughborough University, April 2006.
4. BBC, "*A-levels show rise in top grades*". http://news.bbc.co.uk/2/hi/uk_news/education/3577342.stm, August, 2004.
5. Smith, Alexandra. "*Maths teacher shortage 'threat' to UK competition*". Guardian Unlimited, UK, March 8th, 2006.
6. http://en.wikipedia.org/wiki/Human .
7. Lieberman, Philip. "*The Evolution of Human Speech: Its Anatomical and Neural Bases*". Current Anthropology, Volume 48, No. 1, February, 2007.
8. http://en.wikipedia.org/wiki/Prehistoric_music
9. http://en.wikipedia.org/wiki/Pre-historic_art
10. http://en.wikipedia.org/wiki/Mathematics
11. http://en.wikipedia.org/wiki/Stonehenge
12. Tarnas, Richard. "*The Passion of the Western Mind*", page 263, Pimlico, Random House. 1991.
13. Drake, S., "*Galileo Gleanings XXII: Galileo's Experimental Confirmation of Horizontal Inertia: Unpublished Manuscripts*", *Isis*, Vol 64, 291-305. 1973
14. Carpenter, Jenna P. et al., "*Mathematics and Chemical Engineering Education*". Joint Session on Mathematics Requirements in the Chemical Engineering Curriculum. 2001 ASEE Annual Meeting, Albuquerque, New Mexico, June, 2001.
15. Niklasson, Claes. "*Integration of Mathematics/Numerical Analysis with Chemistry/Chemical Engineering*". (Alternatively titled: "*How Can Learning in Chemistry and Chemical Engineering Be*

Improved by Revised Mathematics?”). ASEE 2002 Conference: Global Changes in Engineering Education. Berlin, Germany, October, 2002.
16. Ahmad, J et al. “*Engineering Mathematics Should be Taught by Engineers!*”. Undergraduate Mathematics Teaching Conference. Proceedings of the 27th Conference. University of Birmingham, UK. September, 2001.
17. ‘Rumplestiltskin’. “*Disappointing*”, http://www.amazon.co.uk/Engineering-Mathematics-5th-Programmes-Problems. September, 2006.
18. Ryan, Mark. “*Calculus For Dummies*”. Wiley Publishing, Inc, ISBN: 978-0-7645-2498-1, 2003.
19. Feynman, Richard et al. “*The Feynman Lectures on Physics: Mainly Mechanics, Radiation, and Heat*”. Addison-Wesley Publishing Company, Sixth Printing, ISBN: 0-201-02010-6-H, 1977.
20. Hutchinson, I. “*James Clerk Maxwell and the Christian Proposition*”. The Faith of Great Scientists. MIT Independent Activities Period, Seminar and Discussion Series, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.January, 2006.
21. Rowland, T. “*Proof and generic examples in number theory*”. Undergraduate Teaching Conference 2000, Proceedings of the 26th Conference, Homerton College, University of Cambridge.
22. Kreyszig, E. “*Advanced Engineering Mathematics*”. John Wiley and Sons, Inc. Eighth Edition, 1999.
23. Stroud, K. A. “*Engineering Mathematics*”. Palgrave Macmillan, 5th Rev Ed edition (23 Feb 2001).
24. ‘Mark’. “*This book saved my life!*”. http://www.amazon.co.uk/Engineering-Mathematics-5th-Programmes-Problems. September, 2006.
25. Rielly, Chris. “*The Application of Computer Algebra Software in the Teaching of Engineering Mathematics*”. Engineering Subject Centre Resource Database: Professor Chris Rielly, Department of Chemical Engineering, Loughborough University, U.K. (http://www.engsc.ac.uk) (http://www.engsc.ac.uk/downloads/miniproject/maple.pdf).
26. Maine, R. E. , Iliff, K. W. “*The theory and Practice of Estimating the Accuracy of Dynamic Flight Coefficients.*” NASA Reference Publication 1138, February, 1985.
27. http://en.wikipedia.org/wiki/Occam's_Razor

Future of power engineering education in developing countries

Mukhtar Ahmad

Department of Electrical Engineering
Aligarh Muslim University, Aligarh 202002

Abstract

Power engineering is perhaps the oldest branch of electrical engineering. However, during the decades of 1970 and 1980 there was a decline of interest among the students of electrical engineering to opt for power engineering in most of the developed countries. The trend is catching up with developing countries also. Now the big question is what new courses may be introduced to attract the students to power engineering. If many new courses are introduced then some old courses will have to be dropped. In this paper an attempt has been made to devise a balanced curriculum taking care of all important courses basic to power engineering keeping intact and adding few new courses.

Keywords: power engineering; new courses; electrical engineering

1. Introduction

A large number of power engineers are retiring every year. However, not many bright students are enthusiastic about specializing in power and energy engineering [1-5]. The enrollment in power engineering course is not uniform in all the countries. It is a function of population, job opportunities, hiring rates and of course the interest of students. In developing countries like India the enrollment in power courses is not that low compared to some of the developed countries. However, the first choice of students is still towards the IT or communication courses. If the trend has to change now the power engineering curriculum must offer something new to attract students

With the advent of deregulation of the electric power industry and lure of high paying jobs in information technology sector, the revision in power engineering curriculum is necessary to attract bright and best students to power and energy engineering. The power engineering education committee of IEEE is taking active steps in modifying the curriculum so that new attractive courses may be introduced at the under graduate level. The main problem in introducing new courses at UG level is the slots to accommodate these courses. The basic courses that are the part of the curriculum can not be ignored, otherwise it may result in production of engineers lacking basic knowledge of present day power systems.

In this paper an attempt has been made to introduce few new courses only, while putting some of the new material in the old courses.

2. Course structure

In all Indian universities, the academic year is divided into two semesters. The bachelor's degree of engineering requires four years of education, after twelve years of schooling. In first year of the Electrical engineering a student is required to study one course on basic electrical science, and rest of the courses are from basic sciences and other courses like computer programming, engineering drawing etc. In second year, the courses generally offered at various colleges include one course each on electrical machines, power system, instrumentation, circuits and fields, Digital and analog electronics etc. These courses are basic for any electrical engineer and can not be replaced. Thus we are left with third and fourth year where few new courses can be introduced. At third year generally second course of machines, second and third course in power system, microprocessor system power electronics, and few electives are available.

The electives generally include two open electives, and one elective from the department. In fourth year apart from core courses such as control system, electric drives, electric power utilization, switchgear and protection, there are two to four departmental electives, and two open electives. A student also undertakes one project at the final year.

It is very important to provide projects involving computer simulation as well as fabrication circuits. In many universities a student is advised to take a short project and a detailed project. Thus the new courses can be accommodated among the departmental electives. However, if the student takes the electives from the courses that are now introduced (list follows), then he has to sacrifice some of the courses considered important from the point of view of energy industry. In this paper therefore, it is suggested that instead of introducing all the new courses at the undergraduate level, their introduction may be provided in the courses already taught. Few courses may be introduced as optional courses at fourth year level, and some courses may be taught at graduate level.

3. Proposed curriculum

Due to the changes happening in the power industry as a result of deregulation, and quest for effective and environmental friendly solutions to energy problems, the power engineering education needs introduction of few new courses not taught earlier. Also the syllabi of the courses taught earlier have to be revamped to make it computer oriented. The importance of following courses can not be over emphasized. However it will be very difficult to include all these courses without dropping some of the conventional ones. The courses that have become important now are listed below:

1. Deregulation in Energy sector
2. Alternative Energy Sources
3. Applications of IT in power industry
4. Application of Power Electronics in Power Transmission and Distribution
5. Power Quality
6. Asset management
7. Distribution system management

The distribution system is almost as important as generation and transmission, but many universities do not teach it at all. It is getting increasingly clear that electric distribution systems are undergoing rapid changes due to deregulation, the penetration of distributed generation and power electronics technologies, and the adoption of efficient computation, communications, and control mechanisms [6]. It is therefore proposed that this course may be introduced if it is not available. The course material of the above six course (1-6) may be introduced in the courses already taught as suggested below.

The introduction to alternative energy sources may be taught along with the conventional power generation course. Power quality may be introduced in utilization of electrical energy. The course on asset management may be offered as open elective. The idea of deregulation may be given in power system operation and control or power system protection. Applications of IT in power industry may be offered as open elective. The courses on power electronics may have to be increased in order to include many applications in power industry.

However, the courses as listed above may be offered as electives and may be grouped together so that a student may take only two courses out of six as listed above. Another option is to introduce integrated five year program in which bachelor and master degree both are given at the end of five years. Such a course is being offered in few Indian Institute of Technologies in India. The student then can select optional subjects to graduate in Power System Engineering with specialization mentioned. It is also important that the universities start interacting with power industries and the projects shall be more and more industry oriented [7]. The industries may sponsor some of these projects and the students are given fellowship at the end of four years. It must be emphasized here that unless the power industry is serious about their requirements of power engineers by providing funding to the universities and giving proper salaries to the engineers comparable to IT industry, the universities shall not go for more enrolments.

4. Conclusions

The deregulation and restructuring of power industry along with computer applications and use of power electronics in power industry requires major changes in power curriculum however it is not possible to do away with basic courses so important for understanding the basic electrical engineering. It is therefore, proposed to have new structure introducing recent trends in the curriculum along with more optional courses. The main emphasis should be on projects that must be supported by the industries. Tutorial course may also be given by people from industries and senior teachers to introduce the trends in power industry.
The possibility of giving the bachelor and masters degree at the end of five years is an option that can also be considered. The solutions may be different for different countries depending on the local conditions.

References

1. AG Phadke, “CAP Forum: Power engineering education: crisis revisited, IEEE Computer application in Power Magazine Vol.15, No.4 Oct.2002.
2. B.H .Chaudhry, “Power education at crossroads” IEEE SpectrumVol.37 No.10 October 2000.
3. G T Heydt and V Vittal,” Feeding our Profession” IEEE Power and Energy Vol.1. No.1 Jan/Feb 2003
4. G.G karady and GT Heydt, “Increasing student interest and comprehension in power engineering at graduate and undergraduate level” IEEE Transactions on Power Systems Vol.15 No.1 Feb.2000
5. P. W. Sauer, G.T.Heydt, and V. Vittal, “The State of Electric Power Engineering Education” IEEE Trans. On Power Systems, Volume 19, issue 1, pp 5-8, Feb 2004

6. S. S. Venkata, A .Pahwa,R.E. Brown, and R.D. Chirstie, " What Future Distribution Engineers need to Learn" IEEE Transactions on Power Systems , Vol19 issue 1 pp. 17-23, Feb 20047.

7. J Bohman, B.A. Mork, and D.O. Witanen, "Power engineering design projects: capston team projects versus topical design courses" IEEE Transactions on Power Systems, Vol. 19 Issue1, pp152-156, Feb. 2004.

Innovative Design Component in Engineering Curriculum

Othman O. Khalifa [a], Meftah Hrairi

Faculty of Engineering
International Islamic University Malaysia (IIUM)
Gombak, Malaysia
[a] khalifa@iiu.edu.my, Fax: +603 61964488

Abstract

The Design component is probably the most important requirement for engineering graduates since it integrates a plurality of courses and trains them on practical and real problem solving. It is a creative way to add new learning experiences to the lives of students. The term “design” has developed progressive association with engineering and technology. There has been a realization that design experience needs to be presented, not only in capstone design projects, but also throughout the curriculum, as they are very important in the first years of a technical education. In a standard design project, the student, under the continuous guidance of the supervisor, is expected to start with a problem definition or description, propose some solutions and iterate, if necessary, to arrive at an optimum solution complying with the specified design considerations or criteria. The benefit and impact of the design component on the student will be increased if the design were to be executed in the workshops and experiments were carried out to test the product in real life situations. In this paper, an overview of Design Component in engineering education is introduced. The objective of engineering design and fundamental limitations is presented. Engineering design process and managing is listed. Some of the impact that implied in engineering is highlighted.

Keywords: engineering education; design projects; innovation; course development

1. Introduction

Design is widely considered to be the central or distinguishing activity of engineering [1]. It has also long been said that engineering programmes should graduate engineers who can design effective solutions to meet social needs [2]. Despite these facts, the role of design in engineering education remains largely. Historically, elements of hands-on practice and scientific fundamentals have been well balanced in academic engineering curricula. Most instructors were practicing engineers themselves who focused on solving concrete problems, and their students learned to conceptualise and design products and systems. However, after the Second World War, the rapid expansion of scientific and technical knowledge saw engineering education evolve into the teaching of engineering science with less emphasis on actual engineering practice. This has led to an increased concern that graduating students, while technically adept, lack many skills needed in real world situations. Industry and related organisations have reacted by creating various requirements of attributes vital for graduating engineers, and in the late 1990s a group of university engineering educators started to examine the issue. Their work has resulted in a new approach to engineering education, the aim of which can be stated: Graduating engineers should be able to **C**onceive, **D**esign, **I**mplement and **O**perate complex value-added engineering systems in a modern, team-based environment. The acronym CDIO has become the trademark of this approach and a detailed educational syllabus has been developed [3].

This new educational model is integrated, with mutually supporting disciplines and a multitude of projects. Students learn through experience about system building through team-based design-build-operate projects and develop a deep working knowledge of the fundamentals. This is particularly important nowadays when we can no longer expect the students arriving at the universities to have the hands-on experience of earlier generations that provided the necessary foundation upon which to base the forming and testing of hypotheses in an abstract fashion. However, first-year introductory courses are common in engineering education programmes, and the objective of each of them is to attract students to engineering and to motivate them for engineering studies. A sample of such courses includes introduction to mechanical engineering (McMaster University), introduction to automotive engineering (International Islamic University Malaysia), introduction to electrical engineering (University of South Australia), introduction to aerospace engineering and design (MIT), etc. The infusion of design components into these first year courses, known in some institutions as Mini project, will alleviate the curricular disconnect with first-year students who often did not see any engineering faculty for most of their first two years of study [4, 5]. During this period, first-year project and design courses emerged as a means for students to be exposed to some flavor of what engineers actually do [6, 8] while enjoying an experience where they could learn the basic elements of the design process by doing real design projects [9, 10].

The intention of providing such courses is to introduce the students to the disciplines that will follow, giving them some confidence in handling basic tools and theories and providing a motivational link for their chosen academic careers. The mission of engineering design is to teach students engineering design with an innovative and practical approach to ensure that the students are adequately equipped to apply their knowledge and skills in industry. Project courses in which students design, build and test a device on their own are increasingly being used in engineering education. The reasons include that such projects not only train design skills but can also be exploited in order to increase student motivation, to give an improved understanding of engineering science knowledge and to practice non-technical skills such as teamwork and communication skills. However, design-build-test (DBT) experiences may also be costly, time-consuming, require new learning environments and different specialized faculty competence [11].

Historically, engineering curricula have been based largely on an engineering science model, in which engineering is taught only after a solid basis in science and mathematics. This model is sometimes unfairly characterized as the “Grinter model,” an attribution that ignores many other recommendations in the Grinter report [12], some of which are being independently revived today. The first two years of the curriculum which in many respects have changed little since the late 1950s [13] are devoted primarily to the basic sciences, which served as the foundation for two years of engineering sciences or analysis where students apply scientific principles to technological problems. The resulting engineering graduates were perceived by industry and academia as being unable to practice in industry because of the change of focus from the practical to the theoretical [14]. What is now routinely identified as the capstone (design) course eventually became the standard academic response, with the strong encouragement of the ABET engineering accreditation criteria [7]. The capstone course has evolved over the years from made up projects devised by faculty to industry-sponsored projects where companies provide “real” problems, along with expertise and financial support [7, 8].

Though the presence, role, and perception of design in the engineering curriculum have improved markedly in recent years, both design faculty and design practitioners would argue that further improvements are necessary [4, 17]. There have even been formal proposals for curricular goals and assessment measures for design-based curricula

2. Objectives of Engineering Design

As quoted from ABET, engineering design is the process of devising a system, component, or process to meet specific needs. It is a decision-making process (often iterative), in which the basic sciences, mathematics, and engineering sciences are applied to convert resources optimally to meet a stated objective. Among the fundamental elements of the design process are the establishment of objectives and criteria, synthesis, analysis, construction, testing, and evaluation. The engineering design component of a curriculum must include some of the following features: development of student innovativeness, use of open-ended problems, development and use of modern design theory and methodology, formulation of design problem statements and specifications, consideration of alternative solutions, feasibility considerations and detailed system description. Further, it is essential to include a variety of realistic constraints such as economic factors, safety, reliability, aesthetics, ethics, and social impact. Courses that contain engineering design normally are taught at the upper division level of the engineering programme. Some portion of this requirement must be satisfied by at least one course, which is primarily design, preferably at the senior level, draws upon coursework in the relevant disciplines.

Therefore, the main objective of the design component is to provide a real-world design experience to students by applying and integrating the knowledge gained from their undergraduate programme. In addition to assessing the student's performance a few of the other objectives of the engineering design component in engineering curriculum are [20]:

1. To give students experience of experimental techniques, analysis of experimental results, practical constraints, comparing theoretical knowledge with practical situations, problem solving techniques, preparing detailed reports of their own work, writing briefly, explicitly, and with authority, and presenting verbal reports to an audience and replying to a discussion.
2. To improve the students ability and/or motivation to identify the critical features of a problem, to solve engineering problems by sub-dividing the overall problem into smaller, more easily solvable problems, to organize their work to meet a specification and a deadline and adapt to the implications of changes as work proceeds, to stick to an approach and know when it is dead, to survey progress and problems ahead and to self-learn.
3. To increase a student's self-confidence and demonstrate ethical responsibility and practice by being a significant contributor to the design group effort (i.e., not loafing).

Although many of these objectives can be satisfied in other ways, engineering design components are of value because they give the student a problem that is extended, open-ended and often realistic. A design project is an attempt to solve a practical problem and to assess the effectiveness of a solution to it within the limitations of time, material resources, and experience available.

3. Impact of Engineering Design Components

Project courses in which students design, build and test a device on their own are increasingly being used in engineering education. The reasons include that such projects do not only train design skills but can also be exploited in order to increase student motivation, to give an improved understanding of engineering science knowledge and to practice non-technical skills such as teamwork and communication. Engineering Design is the systematic, intelligent generation and evaluation of specifications for things/devices whose form and function achieve stated objectives while satisfying given constraints. More recently the term design component has been applied in engineering education. It can be more or less specific, but can never be completely specified. The impact of engineering designs can only be measured in terms of particular implementations, and these can vary widely depending on the participant's needs, interests, abilities, interpretations, interactions, and goals.

Hence in evaluating any design, it is important to keep in mind the limitations of the evaluation. The effectiveness of a design in one setting is no guarantee of its effectiveness in other settings. This is a fundamental problem pervading all education research, which we return to later when we discuss the limitations of design in engineering for carrying out education process.

4. Engineering Design process

Engineering design was developed as a way to carry out formative research to test and refine educational designs based on theoretical principles derived from prior research. In a standard design project, the student, under the continuous guidance of the supervisor, is expected to start by a problem definition or description, propose some solutions and iterate if necessary to arrive at an optimum solution complying with the specified design considerations or criteria. This approach of progressive refinement in design involves putting a first version of a design into the world to see how it works. Then, the design is constantly revised based on experience, until all the bugs are worked out. However, progressive refinement was introduce in the car industry by the Japanese, who unlike American car manufacturers, would update their designs frequently, rather than waiting years for a model changeover to improve upon past designs. Engineering design is not aimed simply at refining practice. It should also address theoretical questions and issues if it is to be effective. There are many models describing the steps which are followed in a typical process of engineering design. This simplistic representation can be made much more elaborate by refining each of the design stages into many sub-processes. It is also implicit to this model that much iteration may occur among the sub-processes [20]

4.1. Problem Identification

Design process typically starts with identifying a problem or a need which has necessitated the design activity. This step is of tremendous importance since it establishes the foundation and the direction for all further effort. Most engineering problems are ambiguous at the onset. They usually reflect an abstract desire or a vague need expressed by a client (customer, supervisor, market, etc.). It is the responsibility of the design team to clearly identify the project goals before setting out to create a design solution. Ill-defined goals are a major contributor to wasted design effort and failed projects.

As a part of problem identification, the design team must determine the answer to questions such as the following:

1. What does the client want?
2. What does the client "really" want?
3. What are the real causes of what the client presents as the problem?
4. Having thoroughly clarified the client's needs, what are the specific goals that the design solution must meet in order to completely solve the client's problem?
5. What are the constraints (budget, material and manufacturing resources, time, labor skills, etc.) within which the design solution must be accomplished? Only after a reasonable sense of clarity about the problem is established should the design team engage in the actual process of designing a solution. There are several methods for clarifying design goals and constraints: Objective or Decision Tree, Quality Function Deployment (QFD), House of Quality, etc.
6. END RESULT: a clear agreement on what the true design goals and constraints are.

4.2. Ideation

Once the design goals and constraints are clearly specified, the design team can turn its attention to generating conceptual (preliminary) solutions. Conceptual ideas may be completely new and innovative, or they may be clever ways of transforming an existing design to meet the current design challenges.

During the ideation stage, it is important to generate an atmosphere which induces creativity and freethinking. There are numerous methods which enhance the ability to create design ideas: Brainstorming, Checklisting, patent and literature search, etc. The following points can help improve the ideation stage:

1. Almost all design problems can be broken down into a collection of smaller subproblems. Conceptual ideas can then be generated for each of the sub-problems separately.
2. While in the process of ideation, one must be careful not to let feasibility concerns get in the way of creativity. These concerns usually have to do with not knowing if the idea will work, or not knowing if the design can be built. Feasibility issues should be addressed in the next stage. a few conceptual designs which may have the potential to satisfy the design goals and constraints.

4.3. Feasibility

Once a few conceptual designs have been generated, they need to be evaluated against the goals and constraints of the project. At this point, there may be a need for re-defining (re-clarifying) the design goals and constraints. Also, rough (first-order) calculations may be needed to establish feasibility of the design. One refined conceptual design which has a reasonable chance of success.

4.4. Detailed Design & Analysis

Following the selection of a viable conceptual design, the design team must use its analytical and scientific skills to transform the concept into reality. During this stage, all the components and sub-systems of the proposed design have to be given specific sizes, dimensions, power ratings, capacities, etc. Various analysis tools may be used during this stage. These include: hand calculations, computer simulations, computer-aided design, and graphic representations. As computations progress, the need for design refinement may arise several times, and the team may have to re-visit all of the previous design stages to improve its design. justification and documentation of the proposed design in such a way that it can be manufactured or re-traced by another team.

4.5. Implementation

Although it could be argued that implementing a design is not a part of the design process, the culmination of the design process is its implementation. As a matter of fact, designs which ignore this stage often run into long lead-times and time consuming revisions because some of the implementation constraints were overlooked.

Upon completion of the detailed design stage, the results must be presented in a "workable" form. This form depends on the nature of the final product. For example, if a prototype is to be fabricated, implementation would mean to produce working drawings, specify manufacturing tooling and processes, and finally fabricate. On the other hand, the implementation stage may require documentation (perhaps for a patent) and/or development of a marketing strategy. In most cases, the implementation stage will require many elements including: working drawings, documentation, tooling, prototyping, marketing, working drawings, documentation, plans for manufacturing and/or marketing, and functional prototypes.

5. Managing Engineering Design

In recent decades designers have helped develop an increasingly complex, human-built world that includes ambitious large-scale engineering projects. At the same time, designers are making engineered products and systems increasingly complex as they work to improve robustness by increasing the number of components and their interdependencies. Further, designers are now required to expand the boundaries of the design to include such factors as environmental and social impacts in their designed systems. These trends suggest that engineering designer's need skills that help them cope with complexity. In response, many universities have created specialized programs for system design, systems engineering, and closely related areas. Despite its creative mood, product design needs to be managed. Students have to be trained and be ready for the job market since most of companies employ top-down decision-making to create environmental stability for those who do product design. Such an approach requires that deadlines for producing designs be set in advance and that those deadlines be taken very seriously. This emphasis on time-based deadlines creates a natural pressure for design engineers to attempt to modify existing designs without the kind of thorough analysis and testing that would normally be desirable.

The important point here, from the perspective of managing the engineering design interface, is that in some working environments, design engineers have become accustomed to meeting what they perceive as being serious though not critical project deadlines. The deadlines are serious enough that one does not wish to fail to have a product design ready at the pre-established time. Apparently, organizationally imposed criteria-the "timely hand-over" of a design-can dominate the professional norm of producing a high-quality design. What makes this compromise possible for the design engineer are two other elements of the work culture: it is acceptable, and even expected, that change orders be issued to "fine tune" the design after its release; and that it is usually someone else's responsibility at that point.

6. Design engineering fundamental limitations

When the design is carried out in the messy situations of actual learning environments, such as classrooms or after school settings, there are many variables that affect the success of the design, and many of these variables cannot be controlled**.** Also, Designers usually end up collecting large amounts of data, such as video records of the intervention and outputs of the students' work, in order to understand what is happening in detail. Hence, they usually are swamped with data, and given the data reduction problems, there is usually not enough time or resources to analyze much of the data collected.

7. Conclusions and recommendations

In this paper, engineering design components in engineering education has been discussed. The need for engineers that are creative and can solve real problems has always been apparent. However, the need for engineers that integrate their engineering knowledge, specific design objectives, and decision making in a systems context has recently become a concern. Therefore, the challenge before engineering educators is to provide an educational experience so that integrated design problems can be addressed from a larger systems perspective. Engineering students must graduate with command of a vast body of technical knowledge. They must possess personal, interpersonal and system building skills to function in teams, and be prepared to produce products and systems. Their education must have been structured under a curriculum blending ability to combine technical expertise with ethical, innovative, philosophical and humanistic acumen. Educators still try to find the best approaches to introduce design in engineering education. In order to develop the best curriculum for its students, each programme needs to consider its objectives, faculty, student quality, facilities, and the need of its constituencies. There is no universal solution and the development of curriculum within given limitations is also a design problem itself.

Engineering programmes and their laboratories in which design can be conducted, while paying attention to the admonition that such outcome must be conducted in an ethical fashion

References

1. H.A. Simon, The Sciences of the Artificial, 3rd ed., Mass.s: MIT Press, Cambridge, 1996.
2. S.D. Sheppard, A description of engineering: an essential backdrop for interpreting engineering education, Proceedings (CD), Mudd Design Workshop IV, Claremont, Cal.: Harvey Mudd College, 2003.
3. E. Crawley, The CDIO syllabus: a statement of goals for undergraduate engineering education, MIT CDIO Report #1, Massachusetts Institute of Technology, Cambridge, MA, USA, 2001.
4. A.M. Agogino, S.D. Sheppard, A. Oladipupo, Making connections to engineering during the first two years, Proceedings of the 1992 Frontiers in Education Conference, Institute of Electrical and Electronic Engineers, 1992.
5. M.J. Pavelich, B.M. Olds, R.L. Miller, Real-world problem solving in freshman/sophomore engineering, in Gainen, J. and Willemsen, E., eds., Fostering Student Success in Quantitative Gateway Courses, San Francisco, Cal.: Jossey-Bass, 1995.
6. J.W. Dally, G.M. Zhang, A Freshman engineering design course, Journal of Engineering Education, 83 (2) (1994) 83–91.
7. R.G. Quinn, Drexel's E4 program: a different professional experience for Engineering Students and Faculty, Journal of Engineering Education, 82 (4) (1993) 196–202.
8. J.E. Froyd, M.W. Ohland, Integrated engineering curricula, Journal of Engineering Education, 94 (1) (2005) 147–164.
9. C.L. Dym, Teaching design to freshmen: style and content, Journal of Engineering Education, 83 (4) (1994) 303–310.
10. C.L. Dym, L. Little, Engineering Design: A Project-Based Introduction, 2nd ed., John Wiley, New York, N.Y, 2003.
11 J.H. McMasters, Influencing engineering education: one (Aerospace) industry perspective, International Journal of Engineering Education, 20 (3) (2004) 353–371.
12. A.L. Brown, Design experiments: theoretical and methodological challenges in creating complex interventions. Journal of the Learning Sciences, 2 (2) (1992) 141-178.
13. A. Collins, Toward a design science of education, In E. Scanlon & T. O'Shea (Eds.), New directions in educational technology, New York: Springer-Verlag, 1992.
14. Ertas, A., Jones, J. C., *The Engineering Design Process*, John Wiley and Sons, New York, 1996.
15. J. Malmqvist, P.W. Young, S. Hallström, J. Kuttenkeuler, T. Svensson, Lessons learned from design-build-test-based Project courses, Proceedings of the International Design Conference, Dubrovnik, 2004.
16. L.E. Grinter, Report on the evaluation of engineering education, Engineering Education, 46 (1956) 25–63.
17. C.L. Dym, Design, systems, and engineering education, International Journal of Engineering Education, 20 (3) (2004) 305–312.
18. A.J. Dutson, R.H. Todd, S.P. Magleby, C.D. Sorensen, A Review of Literature on Teaching Design Through Project-Oriented Capstone Courses, Journal of Engineering Education, 76 (1) (1997) 17–28.
19. A. Bright, Teaching and learning in the engineering clinic program at Harvey Mudd

College, Journal of Engineering Education, 83 (1) (1994) 113–116.
20. C.L. Dym, Learning engineering: design, languages, and experiences, Journal of Engineering Education, 88 (2) (1999) 145–148.

Revisiting the Engineering Mathematics Curriculum Offered to Engineering Undergraduates

Maz Jamilah Masnan, Abdull Halim Abdul, Aishah Mohd Noor

Institut Matematik Kejuruteraan, UniMAP

Abstract

Univerisiti Malaysia Perlis (UniMAP) is an engineering university. Among the courses that undergraduates are required to take are the Engineering Mathematics 1, Engineering Mathematics 2 and Engineering Mathematics 3. *Institut Matematik Kejuruteraan* acts as a service centre to offer the courses to students from various engineering fields. The engineering mathematics curriculum is established to provide students with appropriate mathematical theories, principles and applications to be applied in their engineering branches. The engineering mathematics syllabus is taken by electrical and electronics based engineering, mechanical based engineering, bio-process engineering and bio-medical engineering students are the same. It is the primary goal of this research paper to identify whether the engineering mathematics curriculum has successfully bestowed the mathematics requirements to the undergraduates. Various issues including the relevance of the particular topics covered in the syllabus to meet the engineering needs, the suitability of common or standard engineering mathematics curriculum to different engineering fields, the appropriate applications topics for in class discussions and some other posted problems will be discussed.

Keywords: engineering mathematics; curriculum

1. Introduction

The process of rendering mathematical models more precise is unending, since we cannot limit the development of knowledge. As a result, we come to the conclusion that the accumulated mathematical knowledge necessary for the study of the quantitative laws of engineering must inevitably change with time, expanding with the advances in technology and industry [1]. As a centre accountable to plan and monitor the engineering mathematics curriculum offered to various disciplines of engineering undergraduates, it is IMK's concern to ensure that the established curriculum has successfully bestow the appropriate mathematical skills, theories, principles and applications to be applied in different engineering branches.

At the moment, engineering undergraduates comes from sixteen different engineering programs offered by eight engineering centre namely Microelectronics, Computer and Communication, Mechatronics, Electrical System, Manufacturing, Materials, Bioprocess, and Environmental. For mathematics to be effective in different engineering background, we believe that the delivery of the mathematics must be at the right time, in the right dose, and above all correspond to the needs. Hence, the objective of this research is to revisit the engineering mathematics curriculum offered to the engineering undergraduates, which associated with the content, relevance of topics taught, suitability of custom curriculum, teaching and assessment, and perception towards students' competence in mathematics. We perceive this effort as an initial step to recognize adjustments that may be needed in attaining the curriculum's goals, and to comprehend with the needs of the engineering programs specifically from the engineering lecturers' point of view. Later, suggestions will be proposed in accordance to what have been pointed out through the study.

The research adopted in this work is based on surveys conducted on lecturers and program coordinators from all the engineering departments. About 120 questionnaires were distributed throughout the departments and we successfully received 35 feedback. However, only 34 questionnaires were analyzed while the rest was incomplete. The questionnaire covered four parts as follows: Part A Background, Part B Present engineering mathematics courses for review (includes three mathematics courses), Part C Feedback concerning engineering mathematics courses, (i) Engineering mathematics curriculum: relevance of topics covered, (ii) Suitability of common engineering mathematics curriculum to different engineering fields, (iii) Teaching and assessment of engineering mathematics curriculum, (iv) Lecturer's perception towards students' competence in engineering mathematics, and finally Part D Suggestion and comment.

For each statement in part B there were five choices reflecting the related contents of engineering mathematics taught namely, not related, quite related, fairly related, related and strongly related. While for part C, feedbacks were drawn from five choices ranging from strongly agree, agree, neither agree nor disagree, disagree and strongly disagree.

2. Present mathematics curriculum

Present mathematics courses are spread over three consecutive semesters and cover all topics that are relevant to the engineering. In order to facilitate the basic engineering mathematics needs, engineering mathematics I (EQT 101) that comprises four main topics; complex numbers, matrices, vectors, and probability and statistics is taught to the first semester undergraduates. Besides able to define principles, identify, analyze and solve engineering mathematics problems, it is hoped that at this initial stage, students will able to develop skills in analytical, logical and critical thinking in problem solving. The ability is further developed in the second semester through topics in engineering mathematics II (EQT 102); differentiation and integration, differential equations of first order and second order, Laplace transforms, and Fourier series. The final engineering mathematics paper is engineering mathematics III (EQT 203). In this course, students are taught partial differential, vector calculus, and numerical methods. Our primary concern through the final paper is that we have taught all the required mathematical knowledge that they will need for their engineering study and future endeavors.

After five years serving the custom curriculum, there is a feeling within the institute that some of these courses are of little relevance to some of the engineering programs. This is due to feedbacks from the engineering lecturers and IMK's lecturer generated through educational discourses, meetings, and colloquiums concerning the mathematics curriculum. There were also suggestions from few engineering lecturers to drop certain topics and be replaced by advanced topics. However, being a service institute, it is a hard decision to make. These pose questions as to whether the curriculum should be revised, and changes should be made according to the engineering needs. The main aim here is then to see whether the current mathematics curriculum is relevant to the engineering program needs. This can be achieved by giving the engineering lecturers a room to observe and evaluate the engineering mathematics syllabi that are taught to their students.

Table 1 depicts the percentage of engineering lecturers' response that the contents of EQT 101 are related to their engineering needs. We can see that majority do agree that the contents of this course are related except for the complex numbers which shows only half of them feel that it is related. There are few suggestion posted such as to eliminate unnecessary subtopic in matrices and statistics. The need to enhance each topic in this course must be done delicately since this is the first engineering mathematics that students will register. As an introduction course, the content should be organized and taught in such a way that student should able to relate knowledge he acquire to the engineering he specialized in.

Table 1. The percentage of response that the contents of EQT 101 are related to engineering needs

Course Content	Percentage
Complex Number	53%
Matrices	62%
Vectors	68%
Statistics	62%

Table 2 shows the engineering lecturers' feedback towards the contents of EQT 102. Unlike EQT 101, some of the percentage of response towards this course is higher. As a continuation course, at this level, students are exposed to higher mathematical skills that serve the principal of learning the engineering mathematically. The use of tables should not be restricted during learning differentiation and integration. It is because today's mathematics is not about how to remember more formulas, rather interprets what appeared in the results. At this point also, teaching the course from the application perspective is very crucial. Only through learning mathematics with an application, students can build confidence and understanding towards their engineering branches. Furthermore, application must be taken from both mathematics and engineering text books.

Table 2. The percentage of response that the contents of EQT 102 are related to engineering needs

Course Content	Percentage
Differentiation and Integration	85%
First Order Differential Equations	82%
Second Order Differential Equations	65%
Laplace Transforms	56%
Fourier Series	53%

Table 3. The percentage of response that the contents of EQT 203 are related to engineering needs

Course Content	Percentage
Partial Differential Equations	47%
Vector Calculus	59%
Numerical Methods	62%

3. Feedback concerning engineering mathematics courses

Besides feedback on the content of the curriculum, we are expected to find guides towards our current services. Apart from the curriculum itself, we believe that there are additional points should be highlighted with regard to the issue.

3.1. Engineering mathematics curriculum: relevance of topics covered

Mathematics is a mean to study engineering subjects. Thus, curriculum should become a keyword to educator to teach what is necessary for the students, and they must present the content in such a way as to show students why the subject is being studied [1]. The mathematics taught must be seen by the engineering lecturer to be relevant to their engineering branch. Table 4 shows more than half of the lecturers think the topics covered in each courses are relevant. With regard to revised the content, only 44% of the respondent felt that revision is required. The figure however does not indicate a strong evidence to do revision, but it gives us clue that further exploration is needed.

Table 4. The present three engineering mathematics courses meet the engineering requirement.

Feedback	Frequency
Strongly Agree	4
Agree	20
Neither Agree nor Disagree	9
Disagree	0
Strongly Disagree	1

Another concern relates to the issue of relevance of topics covered is to identify specifically when do the mathematical skills that are required in the engineering are being taught. This is very important so that student can apply what they have learnt in the mathematics courses for their engineering studies. Besides, this can avoid students from losing the connection between the mathematical theory and its relevance to the engineering application. Generally, only 44% disagree that the timing of teaching the engineering mathematics is inappropriate as depicted in Table 4.

Table 5. The timing of teaching engineering mathematics courses is inappropriate.

Feedback	Frequency
Strongly Agree	2
Agree	4
Neither Agree nor Disagree	13
Disagree	14
Strongly Disagree	1

An important finding that we discover is that different engineering programs should take the same engineering mathematics courses. About 50% of the respondents disagree when they were asked different engineering mathematics courses should take different mathematics courses. All the above findings support that the present topics covered in the three mathematics courses are relevance, subject to suggestion in previous section.

3.2. Suitability of custom engineering mathematics curriculum to different engineering fields

In light of this concern, we raised three issues to be evaluated by the engineering lecturer. The first issue to be highlighted is whether the engineering mathematics curriculum suits the engineering needs. About 76.5% of the respondents perceive that the present engineering mathematics courses suit the engineering programs needs.

Table 6. The present three engineering mathematics courses suit the enineering program needs.

Feedback	Frequency
Strongly Agree	5
Agree	21
Neither Agree nor Disagree	6
Disagree	2
Strongly Disagree	0

Table 6 shows the result in detail. When they were asked upon custom engineering mathematics applications taught to different engineering courses, 62% view it as acceptable. However, 71% agreed that some of the specific topics in the engineering mathematics courses should directly discussed the core engineering area.

3.3. Teaching and assessment of engineering mathematics curriculum

An aspect of teaching discussed is directly connected to the particular content of the engineering mathematics courses. Through our experience, some topics in the courses may be best taught by the engineering lecturer who already masters the area. From the feedback concerning the matter, basically about 41% respondents agree, 35% respondents disagree, while 24% respondents unsure that certain engineering mathematics topics should be taught by the engineering department. Please refer Table 7 for further detail.

Table 7. Certain engineering mathematics topics should be taught by the engineering department.

Feedback	Percentage
Strongly Agree	9%
Agree	32%
Neither Agree nor Disagree	24%
Disagree	32%
Strongly Disagree	3%

Good communication between the mathematics and engineering department appear to be beneficial to both departments in many aspects. Such aspect includes the development of teaching and learning of the engineering students with respect to the mathematical skills required. Being a service department, IMK definitely cannot design the engineering mathematics curriculum from their own viewpoint. Thus, apart from IMK, the engineering department should also responsible in the development of the engineering mathematics curriculum. Fig. 1 shows that 79% of the respondents agreed upon the point.

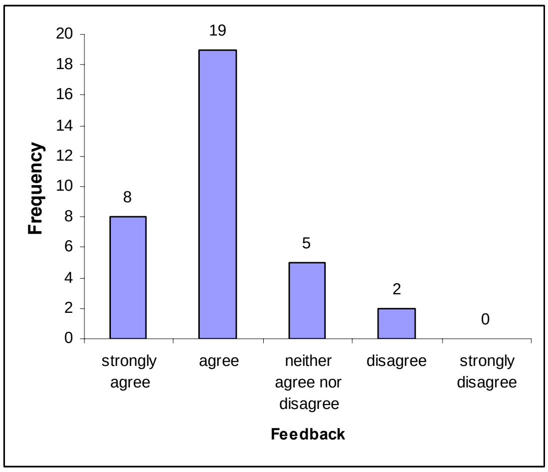


Fig. 1. Engineering centre should involve in the development of mathematics curriculum.

3.4. Lecturer's perceptions towards students' competence in engineering mathematics

One way to find out student's competence is through their ability to integrate what they have learnt in the mathematics courses to their engineering education. And this can be observed by the engineering lecturers who directly involve in such particular process. In one way or another, it also portrays the effectiveness of the engineering mathematics curriculum. With regard to students ability to use mathematical knowledge whenever needed, only 50% respondents agree upon the point.

Table 8. Students lack of the competence in using mathematics to solve engineering problems.

Feedback	Percentage
Strongly Agree	12%
Agree	44%
Neither Agree nor Disagree	29%
Disagree	15%
Strongly Disagree	0%

On the other hand, when they were asked whether their students lack the competence in using mathematics to solve engineering problems, the frequency is even greater. Look at Table 8 for further detail. The final issue to be discussed is the concern on technology in learning. Feedback in Table 9 serves the finding on the issue. Majority agree that one of the engineering mathematics courses should be taught with an application of appropriate mathematics software.

Table 9. Teaching one of the engineering mathematics courses should be with an application of appropriate software.

Feedback	Frequency
Strongly Agree	8
Agree	14
Neither Agree nor Disagree	11
Disagree	0
Strongly Disagree	1

4. Conclusions

This paper has looked at the effort to revisit the engineering mathematics curriculum offered to the engineering undergraduates, which associated with the content, relevance of topics taught, suitability of custom curriculum, teaching and assessment, and perception towards students' competence in mathematics. Based on the findings, the following conclusions can be drawn. The content of three engineering mathematics courses is assumed to be related to the engineering branches, however subject to some suggestions for adjustment. The contents of each course are also relevance, but further discussion on application which relates to specific engineering area is expected. Again, findings that relates to the custom engineering mathematics curriculum require specific attention to the core engineering branches. With regards to teaching and assessment, basically both centers are responsible in designing the engineering mathematics curriculum. However, in terms of student's competence, it seems that the teaching and learning aspect in the curriculum need to be scrutinized, especially on the approach and technological aspect. It is hoped that these finding would enlighten the effort of enhancing the engineering mathematics curriculum as a whole for the benefit of the future engineers.

References

1. B.V. Gnedenko, Z. Khalil, The mathematical education of engineers, Educational Studies in Mathematics 10 (1979) 71-83.
2. S. Williamson, C. Hirst, P. Bishop, T. Croft, Supporting mathematics education in UK engineering department, Int. Conference on Engineering Education, 2003.
3. K. Willcox, G. Bounova, Mathematics in engineering: identifying, enhancing and linking the implicit mathematics curriculum, Proceeding of the 2004 American society for engineering education annual conference & exposition, 2004.
4. F. G. Hayati, A. N. Ali, Mathematics for electrical engineering programmes: an engineering perspectives, METSMAC

Mathematical thinking abilities of first year engineering students of different pre-university mathematical backgrounds

Khamisah Jafar [a], Firdaus Mohamad Hamzah [a], Abdul Halim Ismail [b], Azami Zaharim [a], Nik Rusdi Yaacob[b], Haliza Othman [b], Farah Liza Mohd Isa [b]

[a] Engineering Education Research Group,
The Faculty of Engineering, UKM
[b] Unit Pengajian Asas Kejuruteraan,
Fakulti Kejuruteraan, UKM
E-mail : khamisah@vlsi.eng.ukm.my , fmh@vlsi.eng.ukm.my , abdhalim@vlsi.eng.ukm.my ,
azami@vlsi.eng.ukm.my, rusdi@vlsi.eng.ukm.my , haliza@vlsi.eng.ukm.my , f_liza@vlsi.eng.ukm.my
Tel: 03-89216681 Fax: 03-89216960

Abstract

At the workplace, an engineer usually faces a multitude of problems. To adapt engineering students to the working world, it would be prudent to prepare them with solid analytical abilities. The role of mathematics courses in the engineering curriculum is not just to provide technical competencies in problem solving, but more importantly, it is to inculcate and enhance the thinking abilities in students. Engineering mathematics curriculum at universities generally assumes that certain basic mathematical thinking skills have become ingrained in a student prior to entering university through years of problem solving practice at school and deeper conceptual understanding during pre-university mathematics experience. However, the general consensus among mathematics instructors is that students today exhibit serious deficiencies in these skills; in addition they also have serious misconceptions towards mathematics, observed from their answers to assignments and exam questions. The purpose of this study is to investigate the mathematical thinking ability of first year engineering students in the Universiti Kebangsaan Malaysia. For this study, mathematical thinking abilities measured are, 1) fault finding and fixing skill, 2) plausible estimation skill, 3) creating measures skill, and 4) convincing and proving skill. The instrument used was a set of five general mathematical questions designed to assess these skills. Students are asked to choose the best answer from a set of given answers to each question, and they are also required to justify their choice of answer. Entrants to the faculty are considerably heterogeneous in terms of mathematical adequacy, with intakes coming from three main pre-university backgrounds, namely STPM, Matriculation and Diploma students from various institutions. Results indicate some worrying deficiencies in fault finding and convincing and proving skills across all groups of pre-university background, and also in categories of mathematics achievement.

Keywords: mathematical thinking skills; fault finding and fixing skill; plausible estimation skill; creating measures skill; convincing and proving skill

1. Introduction

The engineering profession is a bridge between science and mathematics and the technological needs of all people (Henderson, 2003). An engineer's job usually involves a host of problems or variations on known problems with multiple solutions subject to constraints on various factors. According to Henderson, in the most general interpretation, every problem-solving activity is an application of mathematical reasoning. Here mathematical reasoning involves applying mathematical techniques, concepts, and processes, either explicitly or implicitly, in other words, mathematical modes of thoughts that help us solve problems in any domain. Basically, the role of mathematics courses in the engineering curriculum is not just to provide technical competencies in problem solving, but more important is to inculcate and enhance the thinking abilities of students.

Mathematical thinking can be defined as 'the development of a mathematical point of view – valuing the process of mathematization and abstraction and having the predilection to apply them; and the development of competence with the tools of the trade, and using those tools in the service of the goal of understanding structure. (Schoenfeld, 1992). Engineering mathematics curriculum at universities generally assumes that certain basic mathematical thinking skills have become ingrained in a student prior to entering university through years of problem solving practice at school and deeper conceptual understanding during pre-university mathematics. However, the general consensus among mathematics instructors is that students have serious deficiencies in these skills; It was also observed that they have major misconceptions towards mathematics, observed from their answers to assignments and exam questions. What is more worrying is that in the practice of teaching and assessment of mathematics courses, some important forms of mathematics such as theorems and proofs are disappearing. In the past, mathematics education for engineers has typically been about techniques which were needed to perform practical calculations. Now even the techniques have disappeared, to be replaced by computer software (Kent P. et. al. 2002)

2. Background

The present study extends a previous similar study on engineering thinking ability of first year engineering students at the same university. Observations on a few years of students' mathematical practice and performance suggest factors critical in affecting the academic achievements are: 1) students do not master the effective learning skill, 2) they lack thinking maturity 3) they are too exam-oriented and achievement motivated and 4) they have serious conceptual problems. Specifically for mathematics, diagnostic tests of mathematics on entry to engineering programs showed major weaknesses even in students who seem appropriately qualified. The second major problem is that students fail to see the importance of mathematics in the engineering field that they have chosen, at least not until much later. Thus mathematics is considered redundant. The study hopes to find out the various misconceptions held by these students of mathematics, many of which remain undetected even until the end of their studies. The main purpose is to see whether students' mathematical thinking abilities vary according to gender, department, and pre-university background. It also aims to analyze the strength and weakness of these students in terms of the four specific mathematical thinking skills considered in this study.

3. Method

3.1. The Respondents

A sample of 174 first year engineering students from the 2005/2006 academic session, from the Department of Civil and Structures Engineering (C&S), the Department of Chemical and Process Engineering (C&P), the Department of Electrical, Electronics and Systems Engineering (EE&S), and the Department of Mechanical and Materials Engineering (M&M) were picked for the study. Respondents were also divided into three groups according to their pre-university backgrounds. These groups are 1) Matriculation students, 2) STPM students and 3) post-diploma students.

3.2. The Instrument

A questionnaire based on the Mathematical Thinking Classroom Assessment Techniques (Math CATs) taken from the CL-1: Field-tested Learning Assessment Guide (FLAG) website was used. The questionnaire focused on only four mathematical thinking skills namely, 1) Fault finding and fixing 2) Plausible estimation, 3) Convincing and proving and 4) Creating measures. The questionnaire is divided into two parts, the first being on demography while the second part consists of 5 multiple choice questions each representing a tool for assessing a particular mathematical thinking skill. The original tool that consists of open-ended type questions had to be modified into multiple choice type questions to ensure students would be willing to put some effort into answering the questions. However at the end of each question, students were given space to explain their answers or give comments. They were given the questionnaire during a regular class meeting in the final week of their second semester.

4. Results

4.1. Demography of respondents

Out of the total 174 respondents, 104 (60%) were male, and 70 (40%) were female. In terms of ethnicity, 107 (61%) were Bumiputra while 67 (39%) were of other ethnic backgrounds, including Chinese and Indians. Figure 1 depicts the percentage of respondents according to pre-university background. As depicted, the majority (64%) of the students involved in the study are post-matriculation students. Figure 2 depicts the percentage of students based on ethnicity.

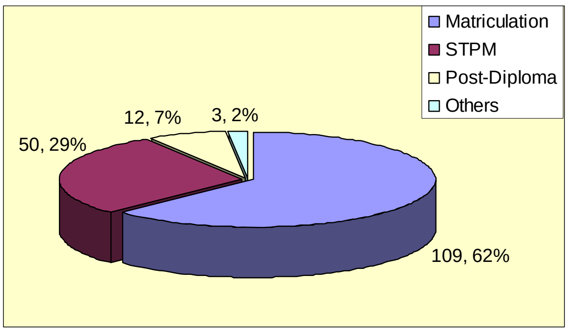


Fig. 1. Percentage of respondents based on pre-university background

4.2. Mathematical Thinking Skills

Of the four mathematical thinking skills, respondents displayed rather good abilities in the 'creating measure' skills. However for the other three skills, results were not encouraging. Details of the question and respondents' performance for each skill are discussed below.

4.2.1. Fault finding and fixing skill (Question 1)

The task offered students a number of mathematical mistakes that they were asked to diagnose. Students were given a graph of distance from start in meters versus time for 2 swimmers. It required students to analyze nine statements based on graphical interpretation, and deduce whether the statements were correct or erroneous. Students were also encouraged to explain the cause of the error, and rectify it. The graph representing the progress of the swimming race is shown in Figure 2, while the nine statements of commentary of the race are given in Table 1.

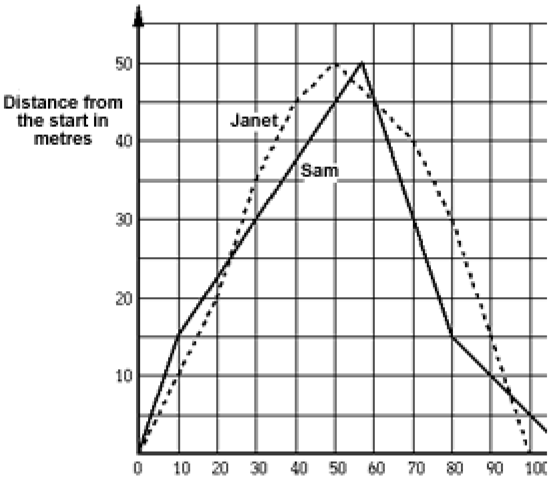


Fig. 2. Progress of swimming race.

The overall performance of students for this question is moderate. The output illustrates students did very well for items [1], [4], and [5] with percentages of correct answers of 85.6%, 88.5%, and 77.6% respectively. The percentage of correct answers is moderate for item [6] which is 46.6%. However, the majority of students answered incorrectly for items [2], [3], [7], [8], and [9]. The lowest score was obtained for item [9], with only 25.9% respondents giving the correct answer. In general, students showed reasonable competency in reading the position of a single swimmer. However, they seem to face a lot of problems when the motion involves comparison of speeds between two swimmers, and interpreting motions in the opposite direction, that is after the swimmers started making a turn towards the starting point. Statements [2] and [3] involve recognizing the scale of the graph before calculating the speed of motion. The wrong answers for these items are most probably due to failure to take into account the scale of the graph in calculating the speed of motion. Table 1 shows the percentages of correct answers and wrong answers for each item in question 1.

Table 1. Performance for question 1.

Statement	Correct answer	Wrong answer
1. Sam goes quickly into the lead.	**149 85.6%**	25 14.4%
2. He is swimming at 15m/s.	47 27%	**127 73%**
3. Janet is swimming at only 10m/s.	53 30.5%	**121 69.5%**
4. After 22 s, Janet overtakes Sam.	**154 88.5%**	20 11.5%
5. Janet swims more quickly than Sam from 25 s, until she turns at 50 seconds.	**135 77.6%**	39 22.4%
6. Sam overtakes Janet after 55 s, but she catches up again.	81 46.6%	93 53.4%
7. 5 seconds later, Janet is in the lead until right near the end.	65 37.4%	**109 62.6%**
8. Sam swims at a steady 30m/s after the turn, until 80 s, while Janet is gradually slowing down.	64 36.8%	**110 63.2%**
9. Sam wins by 10 s.	45 25.9%	**129 74.1%**

The overall performance of the respondents for question 1 is illustrated in Figure 3.

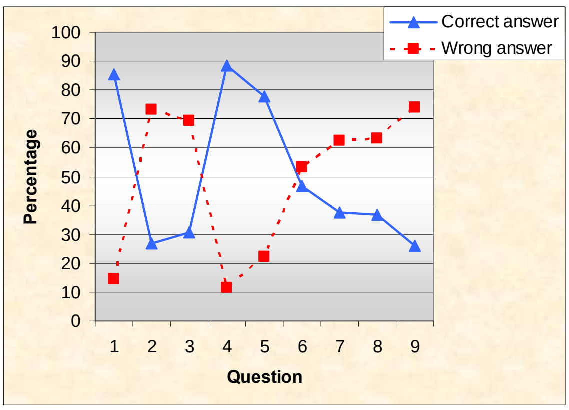


Fig. 3. Overall performance for each item in question 1.

Respondents' performance in this question were categorized as excellent, moderate or poor according to the number of correct answers to the nine questions as shown in Table 2.

Table 2. Categories of achievements for question 1

No. of correct answers	Category of Achievement
7 – 9	EXCELLENT
5 – 6	MODERATE
4 or less	POOR

Figure 4 shows that only 7% of the respondents achieved excellent result for this question. In comparison, 49% were moderate achievers while the remaining 44% were poor achievers.

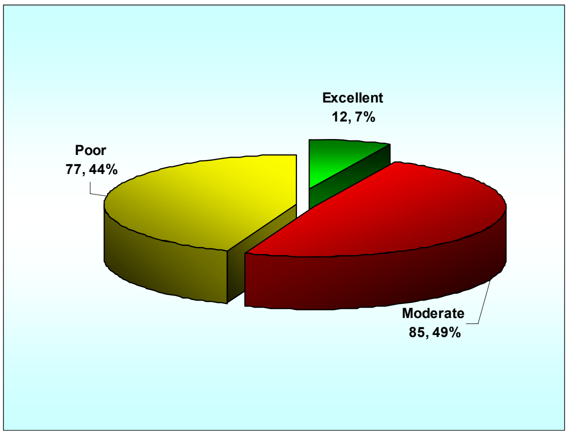


Fig. 4 Percentage of respondents according to categories of achievement for question 1.

In terms of gender, female respondents perform slightly better than the male respondents, while in terms of ethnic background, the Bumiputra respondents performed better than the non-Bumiputra respondents. Table 3 showed that 54.3% of the female respondents scored moderately, while the majority (49.0%) of male respondents scored poorly for question 1. Comparing ethnic background, Table 3 shows that the majority (57%) of the Bumiputra respondents scored moderately, while the majority (55.20%) of the non-Bumiputra respondents scored poorly. However, on the high end, the percentage of excellent scorers was higher among the non-Bumiputras compared to the Bumiputras.

Table 3. Percentages according to categories of achievement for question 1 based on gender and ethnic background.

	Excellent	Moderate	Poor
Male	5.80%	45.20%	**49.00%**
Female	8.60%	**54.30%**	37.10%
Bumi	5.60%	**57%**	37.40%
Non-bumi	9.00%	35.80%	**55.20%**

.

Comparing the results based on pre-university education background, some interesting results were observed. As evident from the chart in Figure 5, the Matriculation students performed better than the STPM students with the most (53.2%) of the Matriculation students getting moderate scores, whereas most (56.0%) of the STPM students scored poorly. All of the post-diploma students from other institution scored moderately. Even though none of the post-diploma students from polytechnic background attained excellent scores, the majority of them scored moderately for this question.

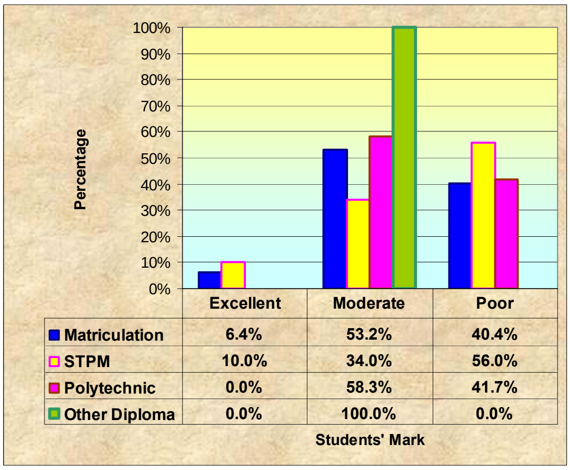


Fig. 5. Performance for question 1 based on pre-university education background.

4.2.2. Plausible estimation skill (Question 2)

For this mathematical thinking skill, students are involved in an activity central to modeling in science, where they are required to estimate something which at first glance seem impossible to answer without reference material, but which can be done by following a series of simple steps that only use common sense and simple arithmetic. It also requires students to practice arithmetic fluency, ability to handle large numbers, and conversions of units. The particular question given here involves estimating the thickness of a pile of papers. The question is as follows:

> Suppose you have a very large sheet of paper. You tear it in half and put one half on top of the other. You now have a stack of two sheets. You now tear the whole stack in half and place one on top of the other to make a new stack. You repeat this process, tearing 50 times. (Yes, I know it is impossible – just imagine you could). How high would the stack be? Make a sensible estimate, based on careful reasoning. Which answer is the best estimate?

I)	10 inches
II)	700 feet
III)	1000 miles
IV)	71 million miles

The best estimate is of course 71 million miles. Students' overall performance for question 2 was very poor. As observed from Figure 6, only 19.5% of the respondents answered the question correctly.

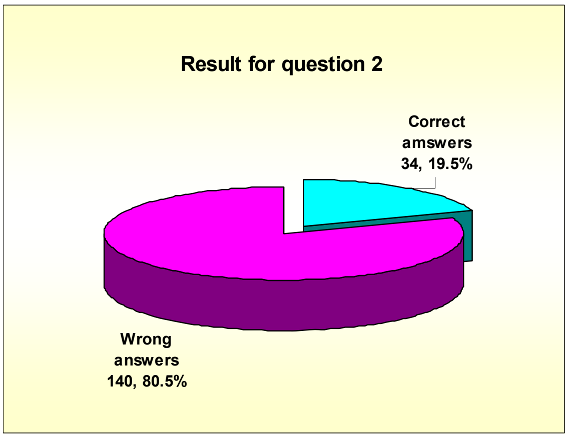


Fig. 6. Overall performance for question 2

Out of these correct answers, 85.3% respondents claimed that they performed actual calculation to arrive at the answer. On the other hand, those who answered incorrectly said that they were just guessing.

Table 4. Method used to answer question 2.

	just guessing	calculate
Correct answer	14.70%	**85.30%**
Wrong answer	**93.60%**	6.40%

Out of the 34 respondents, there were more males than females. In terms of ethnic background, there were more non-Bumiputras compared to Bumiputra respondents. Table 5 illustrates the distribution of respondents with correct answers based on gender and ethnic background.

Table 5. Distribution of respondents with correct answers based on gender and ethnics.

Group	Count	Percentage within group
Male	23	22.12%
Female	11	15.71%
Bumi	20	18.69%
Non-bumi	14	20.90%

As observed from Figure 7, in terms of pre-university education, the Matriculation students outperformed the other groups for question 2.

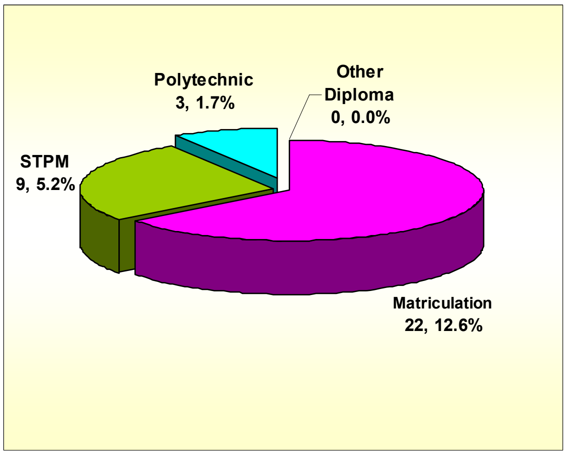


Fig. 7. Percentage of respondents getting correct answers based on pre-university education.

4.2.3. Convincing and proving skill (Questions 3&4)

These questions illustrate several kinds of proofs commonly encountered in mathematics. It intends to assess how well students are able to argue logically, by requesting students to evaluate 3 "proofs" and distinguish the correct from the flawed. Question 3 involves proving a true statement and question 4 involves proving a statement false using a counter-example. The first question is as follows:

Question 3

Here are 3 attempts at proving the following statement:

"*When you add three consecutive numbers, your answer is always a multiple of three.*"

Look carefully at each attempt. Which is the best 'proof'? Explain your reasoning as fully as possible.

The overall performance for this question is moderate. 15 respondents did not answer this question. Out of the remaining 159 respondents who answered, 63.5% respondents chose the correct answer which is Attempt 3. Figure 8 shows the percentage that chose the other two attempts.

Attempt 1

1 + 2 + 3 = 6	3 x 2 = 6
2 + 3 + 4 = 9	3 x 3 = 9
3 + 4 + 5 = 12	3 x 4 = 12
4 + 5 + 6 = 15	3 x 5 = 15
5 + 6 + 7 = 18	3 x 6 = 18

And so on.
So it must be true

Attempt 2
3 + 4 + 5
The two outside numbers (3 and 5) add up to give twice the middle number (4). So all three numbers add to give three times the middle number.
So it must be true.

Attempt 3
Let the numbers be n, n+1, and n+2
Since $n + (n+1) + (n+2) = 3n + 3 = 3(n+1)$
It is clearly true.

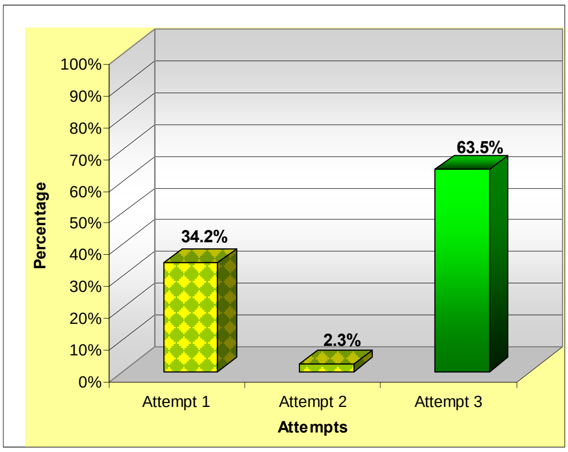


Fig. 8. Percentage of respondents based on choice of proof for question 3.

Table 6 shows the number of number of respondents giving the correct answer to question 3 based on gender and ethnic background. From the percentage within group, it is fair to say that the non-bumi students performed better than the bumi students for this question. In terms of gender, both males and females are equally good.

Table 6. Number of respondents with correct answers to question 3 based on gender and ethnic backgrounds.

Group	Count	Percentage within group
Male	62	59.62%
Female	39	55.71%
Bumi	56	52.34%
Non-bumi	45	67.16%

Figure 9 shows that the matriculation students performed better than thee other groups for question 3. However, the STPM students had a higher percentage within the group answering correctly.

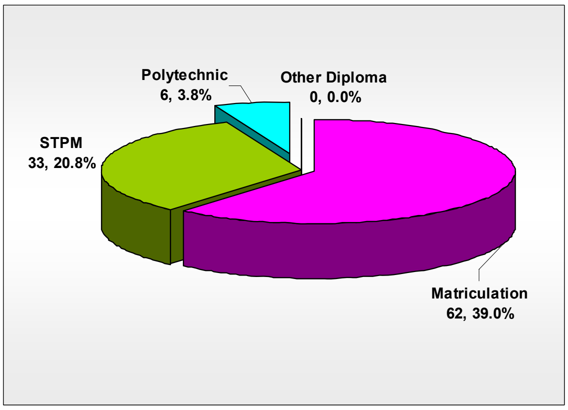


Fig. 9. Percentage of respondents choosing the correct proof for Question 3 based on pre-university education.

Question 4

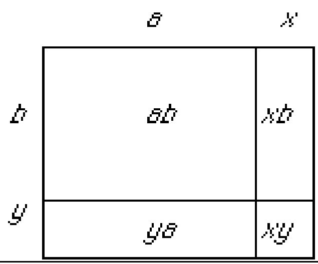
The question involves a false statement, with two attempts to prove the statement true and the third attempt prove the statement false using a counter example. The question is as follows:

Here are three attempts at proving the following statement:

"If you have two rectangles, the one with the greater perimeter will have the greater area?"

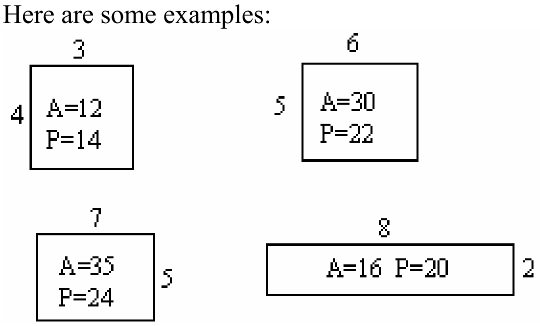
Which is the best 'proof'? Explain your reasoning as fully as possible.

Attempt 1
Suppose you have a rectangle with sides of length a and b. If you increase the parameter of the rectangle, you must increase these sides.
Suppose you increase the sides to lengths $a + x$ and $b + y$, where
$x\ ,\ \ y \geq 0$. (At least one of x or y >0 or you do not increase the perimeter).
The area obtained will be given by
$(a + x)(b + y) = ab + xb + ya + xy > ab$
So, increasing the perimeter must increase the area.
So, the statement is true.



Attempt 2

Here are some examples:

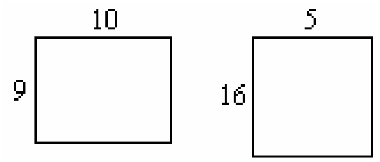


If you put them in a table, it is easy to see that the bigger the perimeter, the bigger the area.

Perimeter	14	16	22	24
Area	12	20	30	35

Attempt 3

Just look at these



The statement is clearly FALSE

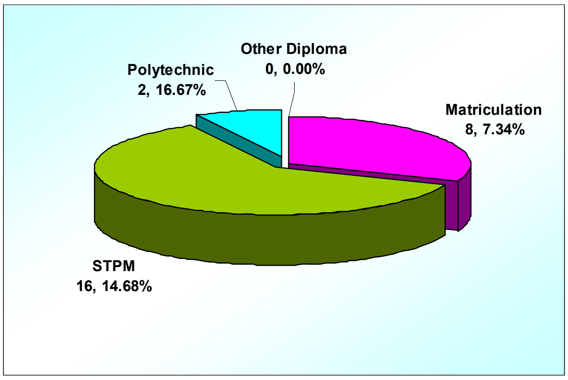


Fig. 10. Percentage of respondents choosing the correct proof for Question 4 based on pre-university education.

The overall performance of students for this question was very poor as sows in Figure 10, with only 16.6% students out of 157 who answered the question, choosing the correct proof, Figure 11 shows that majority of the students chose attempt 1 as their best proof. The correct answer is Attempt 3. As observed from Table 7, all groups of students across gender and ethnic performed equally badly for this question.

In terms of pre-university education, the STPM students scored better than the other groups for this question. Figure 12 depicts the number of correct answers from each group as well as the percentage within each group.

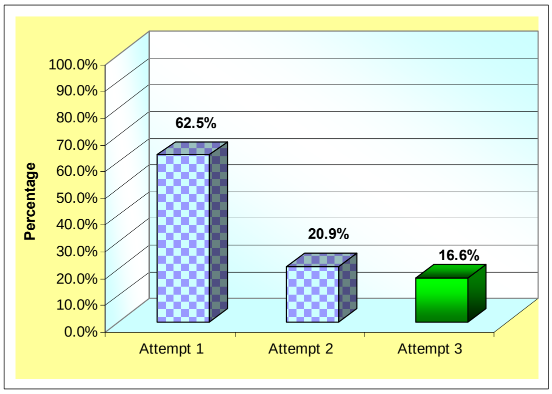


Fig. 11. Percentage of respondents based on choice of proof for question 4.

Table 7. Number of correct answers to question 4 based on gender and ethnic background.

Group	Count	Percentage within group
Male	20	19.23%
Female	6	8.57%
Bumi	10	9.35%
Non-bumi	16	23.88%

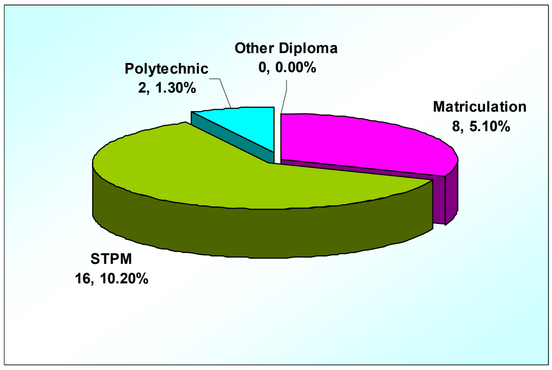


Fig. 12. Percentage of correct answers for question 4 based on pre-university education.

4.2.4. Creating measure skill (Question 5)

The task requires students to measure the concept of "crowdedness". They were given a set of six diagrams consisting of scattered points, and they were required to identify which of the picture were 'most crowded' and 'least crowded'. Q5(b) gives three possible ways of quantifying crowdedness. Students are required to choose the measure that will best fit their intuitive notion of crowdedness, enabling them to rank the pictures given from least crowded to most crowded. T This task intends to assess students' abilities to "mathematize" concepts and show students that there can be many different formal, quantitative measures of such concept.

The overall results for this question indicate that all respondents agree on the concept of most crowded and least crowded. For Q5(b), students were almost equally divided in terms of their choice of quantitative measure of crowdedness. The expected answer is measure no. II, that is, the *ratio of no of people to area of smallest circle surrounding them*, which takes into account the concept of density in measuring crowdedness. Of the three choices, this measure was the favourite with 40.2% responding choosing it as the best measure of crowdedness.

5. Discussion

For the fault finding and fixing skill, students showed competency in reading the position and speed of a single swimmer in one direction. However they seem to have problems when the motion involves comparison between two swimmers, and interpreting motions in opposite directions. For this skill, female students seem to be better than their male counterparts, and Bumiputra students outperform the non-Bumiputra students. In terms of pre-university education, the matriculation group was slightly better than the others.

Obviously, the majority of the students, do not yet have the plausible estimation skill, even though hints were given for calculation. This is not surprising however when we take into account the process of learning mathematics at school, where students are thought to believe that mathematical problems have neat, unique solutions, and that methods to solve problems will be provided to them. Maybe it is fair to conclude that for this particular problem, those students who got the estimation right really understood what they were doing. In terms of gender, ethnic background and pre-university education, students seem equally unequipped with this skill. However the Matriculation students displayed slightly more potential than the others. This could indicate their creativity and ingenuity which will only be truly reflected when they start going deeper into their respective engineering fields later.

For the proving and convincing skill, students showed moderate ability in proving a 'true statement', but fail to grasp the concept of proving a statement false using a counter example. It is very important for students to understand this concept because it is closely related to analytical thinking ability. Any practicing engineer must possess the ability to make a responsible decision based on abstract reasoning and analytical thinking. Here, the male and female students are equally good but the non-Bumiputra students did a little better than the Bumiputra students. The STPM students too performed a little better than the other groups. Their longer duration of study could be the contributing factor to this difference.

In general, the students showed an acceptable level of intuition on the concept of crowdedness. They also showed great potential in the ability to 'mathematize' this measure, with the highest percentage using the concept of 'density' as in choice no. II. This is their strength in potential and creativity, and the ability to use their imagination.

6. Conclusion

The study looks at four important mathematical thinking skills in order for students to think like mathematicians. These are *fault finding and fixing skill, the plausible estimation skill, the proving and convincing skill, and the creating measure skill.* In general, results suggest that students' abilities are varied across gender, ethnic background and pre-university education. Students demonstrated reasonable strength in fault-finding and fixing skill as well as creating measure skill. Weakness is detected in the plausible estimation skill. There were also indications of some misconceptions in the proving and convincing skill. Most students do not have the ability to disprove a statement using a counterexample. However these findings should not be taken as a measure of their performance at the end of the course. It merely points out what the students brought with them from their previous education, and it also gives some idea on how we could make them use their full potential. Early detection of any weaknesses or misconceptions can be corrected by making them realize their mistakes and by instilling the correct concepts. We cannot expect the students to do this on their own, especially when they are not pure mathematics students. However proper coaching using well-designed programs will certainly solve part of the problem.

References

Ahmed, P. K. and Rafiq, M. 1998. Integrated benchmarking: a holistic examination of select techniques for benchmarking analysis. *Benchmarking for Quality Management & Technology*. 5(3): 225-242.

Dodge, R. B. and McKeough, M. 2003. Internship and Nova Scotia Government experience. Education and Training. 45(1): 45-55.

Fink, A. and Kosecoff, J. 1998. How to conduct surveys: A Step-by-Step Guide. Second Edition. ISBN 0-7619-1408-0, London SAGE Publications, Inc.

Hill, N. 1996. Handbook of Customer Satisfaction Measurement. ISBN 0-566-07766-3. Hampshire: Gower Publishing Limited.

Knemeyer, A. M. and Murphy, P. R. 2002. Logistic internship: Employer and student perspectives. *International Journal of physical Distribution & Logistic Management.* 32(2): 135-152.

NPC, 2001. *A glossary of Productivity and Quality Terminologies*, National Productivity Corporation, Malaysia

Research innovative taxonomy towards understanding complex engineering behaviours

Mohd.Jamaludin bin Md.Noor, Ahmad Ruslan bin Mohd. Ridzuan

Faculty of Civil Engineering, Universiti Technologi MARA, 40450 Shah Alam, Malaysia; email: mohdjamaludin@yahoo.com
Ruslan bin Hashim
Faculty of Civil Engineering, Universiti Malaya, Malaysia.

Abstract

The Bloom's cognitive learning taxonomy defines the hierarchy in learning activities. However when there is a break in the logical link between the learning activities like failure in the application of a certain set of knowledge in explaining a certain behaviour then the activities become inapplicable and thereby dissolved the whole taxonomy. This already established the gap in knowledge. Then, a new set of knowledge is required which in turn demand a breakthrough research to reinstate the whole learning levels. A breakthrough research on the other hand requires a well structured mechanism that can lead to innovations or even inventions. Research innovative taxonomy which consists of hierarchy in research activities was developed during the research undertaken to understand complex soil behaviours which are wetting collapse and shallow landslide. The application of research innovative taxonomy is responsible for a research breakthrough where a realistic shear strength model has been successfully formulated. Then the application of research innovative taxonomy between comprehension and application level in the cognitive taxonomy has managed to produce relevant theoretical models that lead to the understanding of the mentioned complex soil behaviours. Thence the link between research innovative taxonomy and the Bloom's cognitive taxonomy will be described.

1. Introduction

The concept of soil shear strength was first introduced by Terzaghi (1936). Nevertheless there are still many shear strength related behaviours which are still not understood such as rainfall induced shallow landslide (Ramaswamy and Aziz, 1980; and Pitts, 1983; Ting, 1984; Brand, Premchitt and Phillipson, 1984; Brand, 1985, Brand, 1989) and wetting collapse or also known as inundated settlement (Blanchfield and Anderson, 2000). Wetting collapse is soil settlement under constant load when the foundation is inundated and the existing model (Terzaghi, 1943) can only indicate settlement when there is an increase in vertical loading. This is the problem when shear strength is not being incorporated in the model. On the other hand shallow landslide cannot be explained by conventional slope stability method (Taylor, 1948, Bishop, 1955; Bishop and Morgenstern, 1960). The application of the conventional slope stability method on partially saturated slopes always indicates failure whereas the slope is standing safely (Othman, 1989). This is because the existence of extra or apparent shear strength due to the presence of suction in the partially saturated zone is neglected. Apart from that with respect to rainfall induced slope failure, the type of failure encountered in the field (shallow failure) does not conform to the failure obtained from analysis (deep seated failure). On the other hand, the occurrence of shallow landslide used to be recognised as "local failure" among geotechnical engineers without offering any engineering explanation behind it. It is very much related to the infiltration of surface water into the soil whereas wetting collapse is caused by the rise of the groundwater table. Therefore these two cases involve the migration of water into partially saturated zone which changes the soil phase from being partially saturated to full or near saturation. Thence the application of the mechanics of unsaturated soil is very significant if those behaviours are to be understood.

Perhaps the key to these problems is the shear strength behaviour itself which is not fully understood. The approach of understanding these behaviours is very much related to the cognitive learning domain in Bloom's taxonomy. However a break is encountered due to the failure at application level which indicates a gap in knowledge. Innovative research taxonomy has been developed and employed to a research undertaken to unveil the knowledge in order to close the gap and thereby reinstate the continuity between the learning activities.

The first objective of this research is to determine the most appropriate soil shear strength behaviour for both saturated and unsaturated conditions. The second and third objectives are to develop respective theoretical models to explain the wetting collapse and shallow landslide respectively. Nevertheless, the first objective is of most importance since it is the key to the understanding those complex behaviours which has become the interest to many geotechnical researchers. However even the task to determine a good representative soil shear strength model is already very complex and challenging. Fredlund et al. (1995) and Vanapalli et al. (1996) only attempted to fit equation for shear strength behaviour relative to suction without considering the behaviour relative to net stress. Even that their contribution in that respect are still opened for improvements. Alternative approach has been tried through critical state theory like the model introduced by Alonso et al. (1990) and Wheeler and Sivakumar, (1995). Nonetheless the shortcoming of those models has been noted by Wheeler et al. (2003). All these challenges reflect the level of difficulties inherent in this subject. In fact the effort of understanding wetting collapse phenomenon has begun when the effective stress equation for partially saturated soils was introduced by Bishop (1959). Nevertheless the equation was criticised by Bishop et al. (1960) and Jenning and Burland (1962) when logical explanation of the wetting collapse behaviour cannot be achieved. The introduction of a more precise and comprehensive soil shear strength model by Md.Noor and Anderson (2006) compared to those introduced by Fredlund et al (1978), Fredlund et al (1995) and Vanapalli et al. (1996) is hoped to unfold the entanglement in this matter.

The first shear strength equation which was introduced by Terzaghi (1936) is a linear relationship between shear strength and effective stress, $(\sigma - u_w)$ for saturated soil condition. However, tests on clays (e.g. Bishop, 1966), sands (e.g. Fukushima and Tatsuoka, 1984), and gravels (e.g. Charles and Watts, 1980) have shown that the angle of shearing resistance (ϕ') increases as the confining pressure decreases. In other words there is a non-linear drop in shear strength as effective stress approaches zero. Then the concept of shear strength was extended to partially saturated soils which involved two independent stress state variables, net stress $(\sigma - u_a)$ and suction $(u_a - u_w)$. by Fredlund et al, (1978) where they proposed a planar surface in $\tau:(\sigma - u_a):(u_a - u_w)$ space, which is referred to as extended Mohr-Coulomb space. The problem with these shear strength models is that they do not have the attribute of steep drop in shear strength when the soil is close to saturation which was first discovered by Escario and Saez (1986). They presented experimental evidence that showed that the variation of shear strength relative to suction was non-linear and this was confirmed by authors such as Gan et al. (1988), Escario and Juca (1989) and Toll et al. (2000). The planar type shear strength model assumed that shear strength increased indefinitely with suction but it was later realised that shear strength decreases as suction increases from residual suction. Residual suction, $(u_a - u_w)_r$, is the optimum value of suction which produced the maximum apparent shear strength where beyond this apparent shear strength start to diminish (Fredlund and Zing, 1994).

Then aim of the research innovative taxonomy is to provide a simplistic mechanism towards producing innovative research findings. The application of this research innovative taxonomy has contributed towards achieving the best representative soil shear strength model (Md.Noor and Anderson, 2006) for both saturated and partially saturated soils. Unable to achieve this has become the stumbling block to the understanding of many soil behaviours. Subsequently the application of the shear strength model has lead to the discovery of soil settlement framework and the slope stability equation which allowed for understanding of wetting collapse and shallow landslide behaviour. Therefore the characteristic of the taxonomy which is responsible for the research breakthrough will be elaborated.

2. Research Innovative taxonomy

The research innovative taxonomy has been developed during the undertaking of very significant and complex research goals which are the formulation of the soil settlement framework in understanding the wetting collapse behaviour and the formulation of slope stability equation that can model shallow landslide. It started off by identifying the right form of the curved surface shear strength envelope followed by the need to define the shape by some formulations or mathematical equations. The structure of the research innovative taxonomy in reference to the cognitive or mental skill or knowledge domain introduced by Bloom (1956) is shown in Figure 1. It is designed with the intention geared towards producing innovation in research findings and thereby reinstates the cognitive domain regarding the referred subject. The failure of applying the current state of knowledge to explain certain soil behaviour has forced the break in the cognitive taxonomy at the comprehension level without able to reach the application level. When this happen the current state of knowledge is no longer applicable and alternative framework or theory is required. In other words the whole cognitive domain on the subject has been dissolved and a new set of domain has to be reinstated base on the new set of knowledge. The linear shear strength envelope of Terzaghi (1936) or the planar type of shear strength envelope of Fredlund et al. (1978) has to be upgraded or innovated to suit the real soil shear strength behaviours reported in the literatures.

At this stage, the gap in knowledge is already established. In order to identify the new set of knowledge the research innovative taxonomy has to start with a fresh literature review followed by critical thoughts in order to conceptualising, synthesising and evaluating the information gathered with the consideration of possibilities. It needs to be concluded with a pattern for understanding that is base on evidence. This is followed by further data collection besides those acquired from literatures by conducting a new set of laboratory tests in order to reinforced the understanding of the gap.

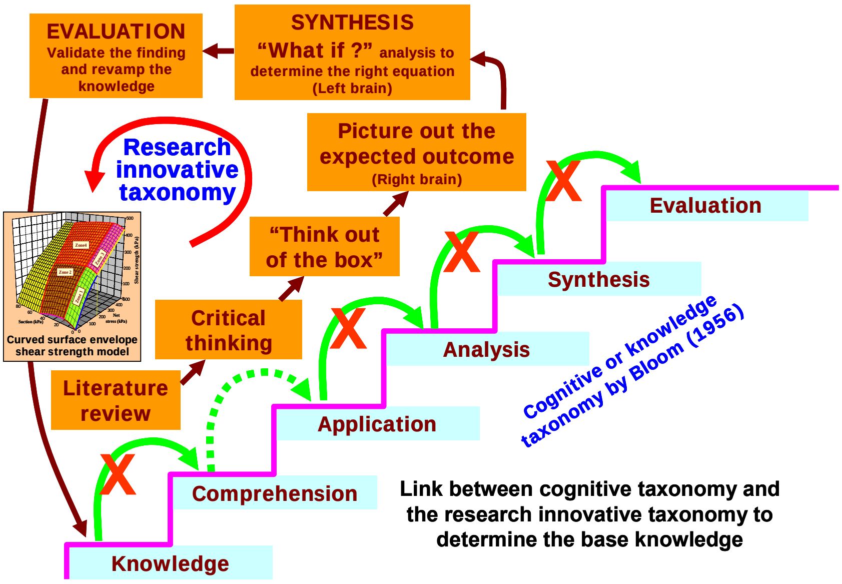
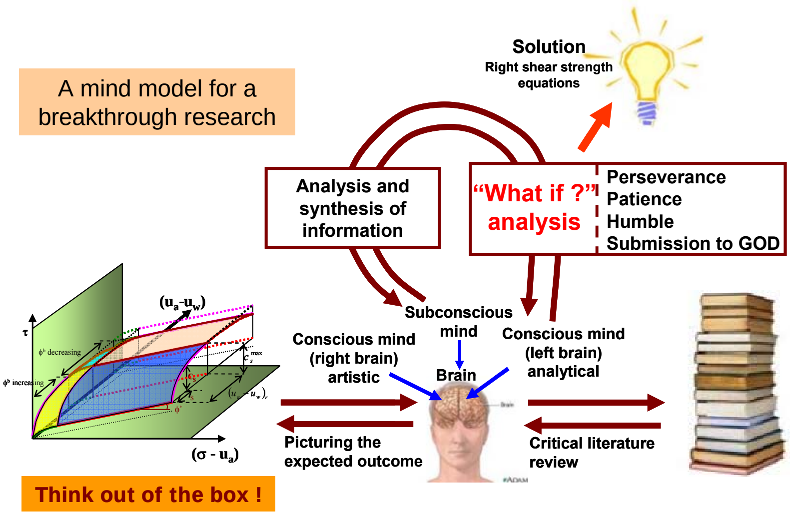


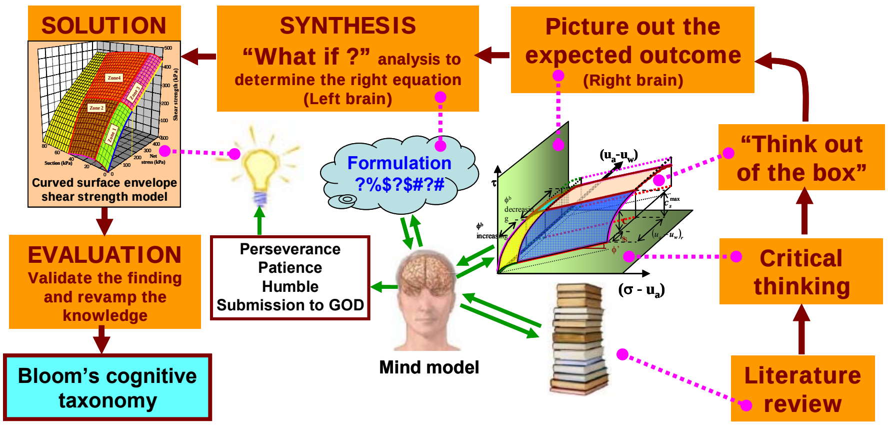
Figure 1. Research innovative taxonomy in reference to the cognitive taxonomy of Bloom (1956).

The next step in the proposed research innovative taxonomy is the "think out-of-the-box". This is actually a proverb which means to look at the problem outside the conventional way and requires openness to a new way of seeing the problem and the willingness to explore. This is where innovation begins. There is no point of approaching the problem through conventional methods or according to the in-the-box thinkers since the failure in applying the knowledge is already known.

Subsequently the most important step in this taxonomy chain is the step to picture out the outcome targeted. In this way the power of the right brain of high creativity and intuitive will be fully exploited. This concept of approaching a problem is also known as "begin with end in mind". When the expected outcome has already successfully being map out into a picture the creativity potentials in the subconscious mind will start working. It will assess the problem at wholes rather than at parts. The advantage of triggering the subconscious mind to work is that it will never stop even though when the researcher stopped. Through some patience and perseverance in pursuing the matter, by god willing sooner or latter the working of the subconscious mind will lead to the solution. This role of the subconscious mind is always associated in any breakthrough research whereby at this stage of the research innovative taxonomy, the power of the conscious mind in the left brain is equally active. They are in fact working together where the facts or information gathered by the conscious mind through the critical literature review are always being analyse and synthesise by the subconscious mind. The latter is taking place without realisation which can be categorised as in accidental ongoing process. The interaction between the conscious and the subconscious mind in processing the information is illustrated by a mind model in Figure 2(a) and its link between research innovative taxonomy is illustrated in Figure 2(b). By god's willing through some patience, perseverance, humbleness and submission to god the solution will be arrived through multiple "What if?" analysis which is the subset of synthesis activity.



(a) Mind model: The interaction between the conscious and the subconscious mind in processing the information that may lead to the solution.



(b). Interaction between mind model and research innovative taxonomy.

Figure 2. Research innovative taxonomy and mind model.

The "What if ?" analysis is a structured trial an error process towards fitting the shape of the shear strength surface envelope with appropriate equations. This is probably the most difficult part in the innovation chain. Failure at this stage will leave the knowledge hanging and consequently lead to failure in achieving research objectives. Any developed concept or method derived from the pictured model can only be applied qualitatively without a strong and firm quantitative solution. However if through a strong spirit towards success which make the researcher very perseverance in action, patience at work, humbleness and full submission to god and the solution is stroke, the next step is to evaluate the solution.

Evaluation is the final stage of the research innovative taxonomy. At this stage the mathematical model need to be verified against the gathered data. When verification is achieved then its applicability can be checked through the Bloom's cognitive learning taxonomy especially at the application level. In case if there is a failure at this level then the research innovative taxonomy cycle has to be reactivated and this process has to be repeated until all the cognitive learning levels are complied.

In order to accomplish the whole knowledge acquiring process which involved both the cognitive and the research innovative taxonomy which is in this case are understanding the behaviour of shallow landslide and wetting collapse, theoretical models must be developed to test the applicability of the newly introduced soil shear strength mathematical model of Md.Noor and Anderson (2006). This is the intermediate activity between comprehension and application as illustrated in Figure 3. When the theoretical models managed to rationalise these soil behaviours by utilising the curved surface envelope shear strength model then the whole learning process can be accomplished up to evaluation stage in Bloom's (1956) taxonomy.

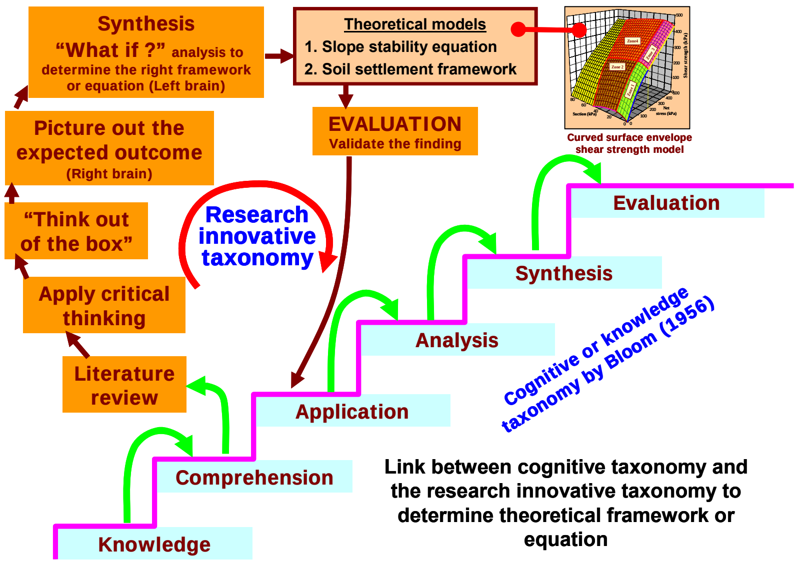


Figure 3 Development of theoretical models in the research innovative taxonomy to test the applicability of the new set of base knowledge.

3. Critical Review on the Reported Shear Strength Behaviour with Respect to Suction and Effective Stress

In the research innovative taxonomy it is very important that critical thinking is applied while carrying out the literature review. Not all the reported or the published data are correct. The data need to be thoroughly scrutinised and try viewing them from a different perspectives.

Figure 4 shows data for fine ash tuff obtained by Gan and Fredlund (1996) and shows the shear strength decreasing with increase in suction beyond residual suction. They noted that this behaviour is exhibited at low (< 20 kPa) net confining stress, and at higher net stress (>100 kPa) the shear strength appears to approach a constant value. A close inspection of their data points also indicates a slight reduction in shear strength at higher levels of suction, even under higher net confining pressure. Similar behaviour has been reported by Toll et al. (2000) for coarser soils.

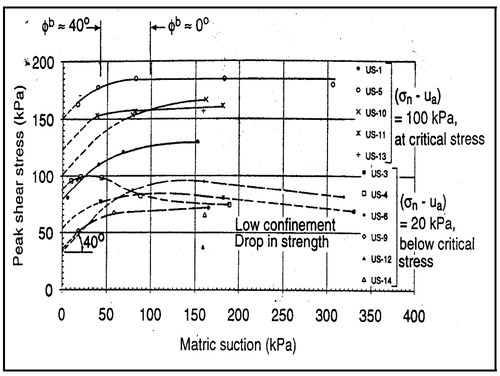


Figure 4. Non-linear behaviour of shear strength with suction

Figure 5 shows an example of non-linear peak shear strength behaviour with effective stress with the angle of shearing resistance (ϕ') decreasing at a decreasing rate with increasing confining pressure and only tending to become constant at high values of confining pressure. Salman (1995) has specifically conducted triaxial tests at low stress levels on saturated specimens of coarse sand and fine gravel and he found a similar increase in ϕ' as the net confining pressure approached zero. It can therefore be concluded that for saturated soils the shear strength variation with effective stress is a combination of non-linear behaviour at lower effective stresses and linear behaviour at higher effective stresses. The effective stress at which the behaviour changes from non-linear to linear is called the transition effective stress ($(\square - u_w)_t$).

Thence it can be concluded that the existing linear shear strength models of Fredlund et al. (1978) do not exactly replicate the soil shear strength behaviour and an alternative and more representative mathematical model is demanded.

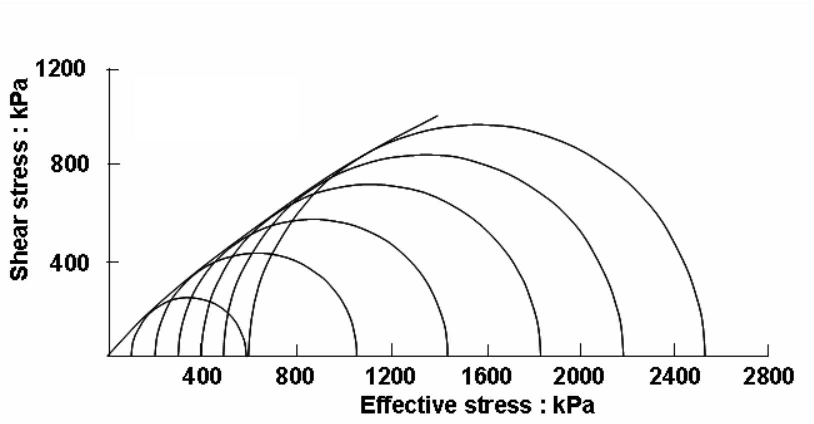


Figure 5 Curvilinear Mohr-Coulomb envelope (from Indraratna et al., 1993)

4. Mapping out the expected outcome

From the critical literature review conducted in Section 3 the anticipated appropriate shape of the shear strength envelope can be summarised as follows.

1. Shear strength varies linearly with effective stress at high confining pressure but curves down to zero at zero effective stress (Bishop, 1966 ; Charles and Watts, 1980; Indraratna et al. 1993 and Salman 1995).
2. Shear strength drops steeply as suction approaches zero but showed a gradual drop as suction increases from residual suction. The latter becomes more gradual as effective stress increases (Escario and Saez, 1986; Escario and Juca, 1989; Gan and Fredlund, 1996 and Toll et al. 2000).

Again, the out-of-box approach or the unconventional way of defining the anticipated shear strength surface envelope needs to be applied in mapping out the form of the targeted surface envelope. The previous model (Fredlund et al., 1978) is a simple plane surface envelope and base on the combination of the gathered shear strength behaviour relative to net or effective stress and suction, it seems that the envelope is a curved surface as shown in Figure 6. Since the shape is slightly complex therefore it is wise to divide it into four sections as illustrated in the figure. Again this is the unconventional way or out-of-the-box technique of defining the envelope and moreover this technique will slightly simplify the difficulties of finding the representative equations. The ability to map out the picture of the expected outcome is the artistic power of the right brain. At this point of the research innovative taxonomy the activity of the subconscious mind is actively triggered and focus towards the revealed picture. When the subconscious mind is focus it becomes coherent in analysing and synthesising the information.

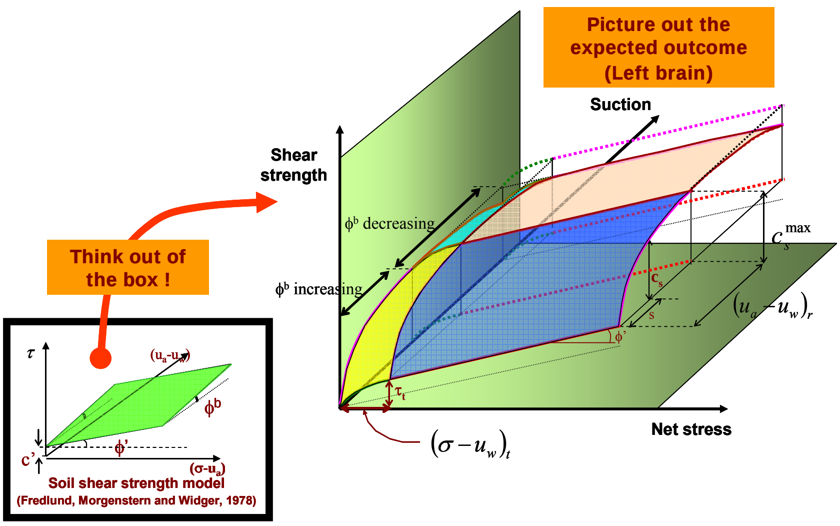


Figure 6. Mapping out the expected shape of the realistic soil shear strength envelope by applying the out-of-the-box concept.

5. Unconventional Mathematical Shear Strength Model

Subsequently the "What if?" analysis is conducted and at this point of the research innovative taxonomy it relies on the analytical capability of the left brain. At this juncture also the interaction between the conscious and the subconscious mind is a cyclic to and fro process as illustrated in Figure 2. Through a continuous and perseverance trial, god willing the solution which is the mathematical formulation of the envelope will be strike.

It is difficult to achieve a close approximation to experimental data if the curved shear strength envelope with respect to either net stress or suction is expressed by a single equation. Therefore the proposed warped surface model divides the surface envelope into four zones and the dividing lines correspond to residual suction $(u_a - u_w)_r$ and transition effective stress (□ − $u_w)_t$ as shown in Figure 7. This is again the "out-of-the-box" concept. The variation of shear strength is defined by the curves OA and AB with respect to suction, and by the curve OC and the line CD with respect to effective stress at saturation.

The shear strength equations are formed by adding the equation for the strength due to suction of unsaturated soil to the equation for the strength of the saturated soil. The equation for apparent shear strength (c_s) due to the presence of suction (Equation 1) represents the curve OA and is valid for suction from zero up to residual suction.

$$c_s = \frac{(u_a - u_w)}{(u_a - u_w)_r}\left[1 + \frac{(u_a - u_w)_r - (u_a - u_w)}{(u_a - u_w)_r}\right] c_s^{\max} \qquad \text{Equation 1}$$

where $c_s^{\max}$ is the maximum apparent cohesion.

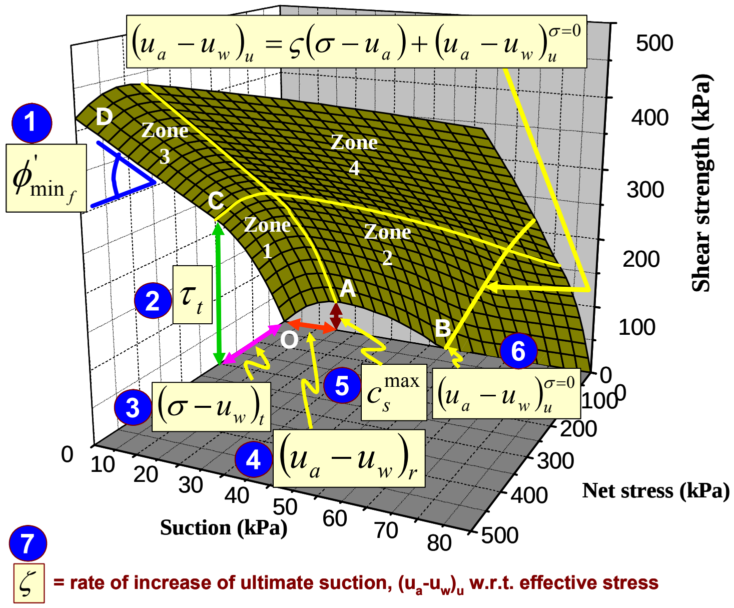


Figure 7 The seven shear strength parameters need to define the curved surface envelope.

The equation for apparent shear strength for suction greater than residual up to the ultimate suction (Equation 2) represents the curve AB in Figure 7. Ultimate suction is the value of suction at which the apparent shear strength has become zero.

$$c_s = c_s^{\max}\left[\frac{(u_a - u_w)_u - (u_a - u_w)}{(u_a - u_w)_u - (u_a - u_w)_r}\right] \times \left[1 - \frac{(u_a - u_w)_r - (u_a - u_w)}{(u_a - u_w)_u - (u_a - u_w)_r}\right] \qquad \text{Equation 2}$$

Since ultimate suction, $(u_a - u_w)_u$, varies with net stress then the ultimate suction in Equation 2 is defined by Equation 3 assuming a linear variation.

$$(u_a - u_w)_u = \zeta(\sigma - u_a) + (u_a - u_w)_u^{\sigma'=0} \qquad \text{Equation 3}$$

where □ is the rate of change of ultimate suction with respect to net stress and is constant, and $(u_a - u_w)_u^{\sigma'=0}$ is the ultimate suction when the net stress is zero.

The equation for shear strength at saturation (Equation 4) represents curve OC and is valid for effective stresses from zero up to the transition effective stress.

$$\tau_f = \frac{(\sigma - u_w)}{(\sigma - u_w)_t}\left[1 + \frac{(\sigma - u_w)_t - (\sigma - u_w)}{N(\sigma - u_w)_t}\right]\tau_t \qquad \text{Equation 4}$$

The term N in Equation 4 is given by Equation 5.

$$N = \frac{1}{1 - \left[(\sigma - u_w)_t \dfrac{\tan\phi'_{\min_f}}{\tau_t}\right]} \qquad \text{Equation 5}$$

The effective minimum friction angle at failure, $\phi'_{\min_f}$, is the angle of the linear section of the graph (i.e. CD) from the horizontal and τ_t is the transition shear strength illustrated by the parameter no. 2 in Figure 7.

The equation for shear strength at saturation (Equation 6) represents the line CD in Figure 7 and is valid for effective stresses greater than the transition effective stress

$$\tau_f = (\sigma - u_w)\tan\phi'_{\min_f} + \left[\tau_t - (\sigma - u_w)_t \tan\phi'_{\min_f}\right] \qquad \text{Equation 6}$$

The value of N given by Equation 5 makes the gradient of the curve OC at point C equal to the gradient of the linear section CD ensuring a smooth transition between the non-linear and the linear sections of the graph for shear strength at saturation.

The shear strength equations (Equation 7 -10) representing each of the four zones of the surface envelope shown in Figure 7 are formed by combining the corresponding shear strength equations representing the saturated and unsaturated conditions.

$$\tau_f = \frac{(\sigma - u_a)}{(\sigma - u_w)_t}\left[1 + \frac{(\sigma - u_w)_t - (\sigma - u_a)}{N(\sigma - u_w)_t}\right]\tau_t + \frac{(u_a - u_w)}{(u_a - u_w)_r}\left[1 + \frac{(u_a - u_w)_r - (u_a - u_w)}{(u_a - u_w)_r}\right]c_s^{\max}$$

Equation 7

Valid for Zone 1 where suction $0 \geq (u_a - u_w) \leq (u_a - u_w)_r$ and net stress $0 \geq (\sigma - u_a) \leq (\sigma - u_w)_t$.

$$\tau_f = \frac{(\sigma - u_a)}{(\sigma - u_w)_t}\left[1 + \frac{(\sigma - u_w)_t - (\sigma - u_a)}{N(\sigma - u_w)_t}\right]\tau_t + c_s^{\max}\left[\frac{(u_a - u_w)_u - (u_a - u_w)}{(u_a - u_w)_u - (u_a - u_w)_r}\right]\times\left[1 - \frac{(u_a - u_w)_r - (u_a - u_w)}{(u_a - u_w)_u - (u_a - u_w)_r}\right]$$

Equation 8

Valid for Zone 2 where suction $(u_a - u_w)_r \geq (u_a - u_w) \leq (u_a - u_w)_u$ and net stress $0 \geq (\sigma - u_a) \leq (\sigma - u_w)_t$.

$$\tau_f = (\sigma - u_a)\tan\phi'_{\min_f} + \left[\tau_t - (\sigma - u_w)_t \tan\phi'_{\min_f}\right] + \frac{(u_a - u_w)}{(u_a - u_w)_r}\left[1 + \frac{(u_a - u_w)_r - (u_a - u_w)}{(u_a - u_w)_r}\right]c_s^{\max}$$

Equation 9

Valid for Zone 3 where suction $0 \geq (u_a - u_w) \leq (u_a - u_w)_r$ and net stress $(\sigma - u_a) \geq (\sigma - u_w)_t$.

$$\tau_f = (\sigma - u_a)\tan\phi'_{\min_f} + \left[\tau_t - (\sigma - u_w)_t \tan\phi'_{\min_f}\right] + c_s^{\max}\left[\frac{(u_a - u_w)_u - (u_a - u_w)}{(u_a - u_w)_u - (u_a - u_w)_r}\right]\times\left[1 - \frac{(u_a - u_w)_r - (u_a - u_w)}{(u_a - u_w)_u - (u_a - u_w)_r}\right]$$

Equation 10

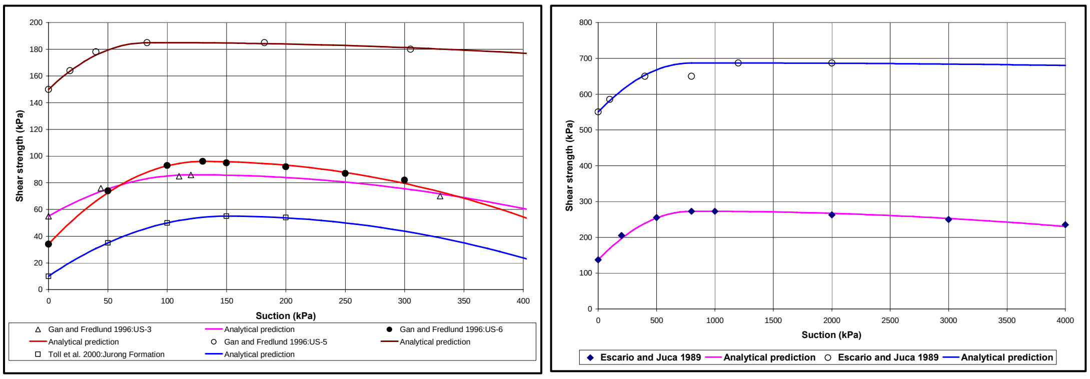
Valid for Zone 4 where suction $(u_a - u_w)_r \geq (u_a - u_w) \leq (u_a - u_w)_u$ and net stress $(\sigma - u_a) \geq (\sigma - u_w)_t$.

Therefore a total of six equations (i.e. Equations 3, 5, 7, 8, 9, 10) are required to completely define the extended Mohr-Coulomb shear surface envelope shown in Figure 7. The equations form a smooth transition between the shear strength zones.

6. Evaluation: Model Validation against Published Data

The final step in the research innovative taxonomy is the evaluation activity. If the proposed shear strength equations are managed to be validated against experimental data, then the outcome, which is the soil shear strength model can be adopted as a new knowledge to be the basis for the new set of cognitive learning taxonomy. However, even though if this is successful then this new knowledge is still yet to be tested its applicability in the cognitive taxonomy.

The performance of the shear strength equations (i.e. Equations 7–10) representing the four zones in Figure 7 is totally dependent on the performance of their component equations (i.e. Equations 1, 2, 4, 6). Therefore the accuracy of the component equations in predicting the respective saturated and the unsaturated shear strength behaviour governs the accuracy of the Equations 7 - 10. The model was validated by comparing published experimental data with the prediction by the Equations 1 and 2 for the unsaturated shear strength behaviour with respect to suction, and Equations 4 and 6 for the saturated shear strength behaviour. Figures 8 and 9 present the predictions of these equations using the shear strength parameters given in Tables 1 and 2. The points are the reported data and the lines represent the predictions. The excellent match indicates the applicability of the model to define the actual soil shear strength behaviour. And the next step is to test its applicability to explain various soil behaviours in the cognitive taxonomy when curved surface envelope shear strength is applied in the respective framework (i.e. soil settlement framework and slope stability equation).



(a) Data from Gan and Fredlund (1996) and Toll et al. (2000).

(b) Data from Escario and Juca (1989).

Figure 8. Performance of Equations 1 and 2 in predicting the unsaturated shear strength variation with suction.

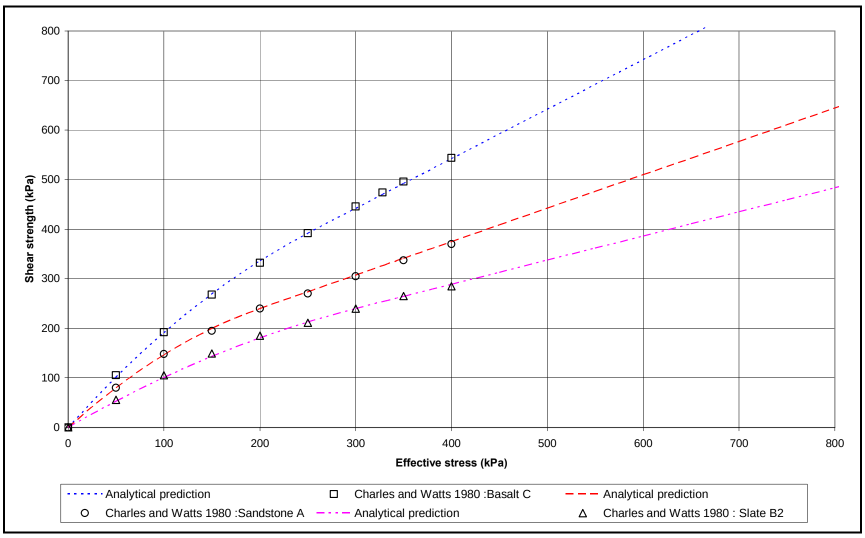


Figure 9. Performance of Equations 4 and 6 in predicting the saturated shear strength behaviour obtained (from Charles and Watts, 1980).

Table 1. Shear strength parameters (kPa) used in Equations 1 and 2.

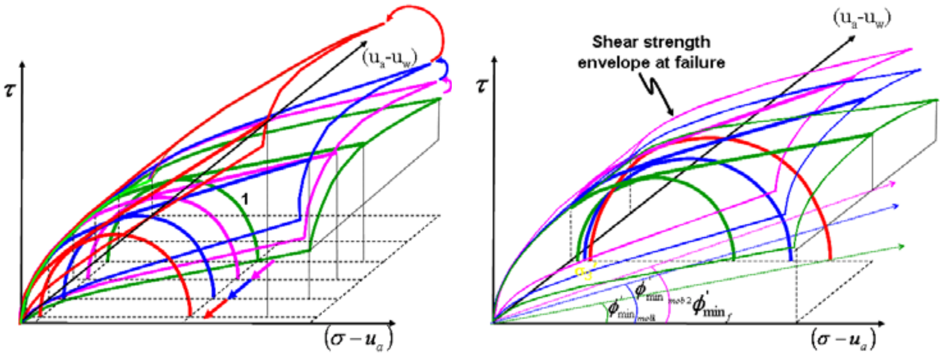
Soil type	Net confining pressure	Residual suction	Ultimate suction	Maximum apparent shear strength
Madrid clayey sand (Escario & Juca 1989)	600	800	15000	137
Madrid clayey sand (Escario & Juca 1989)	120	800	6500	136
Decomposed fine ash tuff, Hong Kong (Gan & Fredlund 1996)	20	120	430	31
Decomposed fine ash tuff, Hong Kong (Gan & Fredlund 1996)	20	130	460	62
Decomposed fine ash tuff, Hong Kong (Gan & Fredlund 1996)	100	83	750	35
Residual soil from Jurong formation, Singapore (Toll et al. 2000)	50	150	450	45

Table 2. Shear strength parameters (kPa) used in Equations 4 and 6 for the analytical prediction of the experimental data reported by Charles and Watts (1980).

Soil type (max. particle size 38mm)	Transition effective stress	Transition shear strength	Effective minimum friction angle (deg.)
Basalt C	250	392	45
Sandstone A	200	240	34
Slate B2	300	240	26

7. Applicability of the soil settlement framework to explain wetting collapse.

A soil settlement framework applying the curved surface shear strength model is developed to explain the wetting collapse behaviour. The framework is called “Rotational multiple yield surface framework” where the mobilised shear strength envelope is acting as the yield surface and it rotates about suction axis when the soil undergo settlement (Md.Noor and Anderson, 2007). However, some researchers have opted for other type of approach in assessing this behaviour like the critical state model of Alonso et al. (1990) because the uniqueness of the mobilised shear strength envelope as yield surface it is not realise. Referring to Figure 10, the diameter of the semi-circles is representing the magnitude of vertical stress or load. The Mohr circle will enlarge when the vertical stress is increased through increase in the loading and the Mohr circle will be pulled towards the frontal plane while maintaining its diameter when suction decreases through the rise of water table or infiltration. And the state of stress equilibrium is when the Mohr circle is at the point of touching with the envelope. Whenever the Mohr circle extends higher than the existing envelope there is a state of stress equilibrium which will trigger settlement that simultaneously rotates the envelope until stress equilibrium is reinstated. The process of wetting collapse and loading collapse are illustrated in Figure 10(a) and (b) respectively. By engaging the curved surface shear strength envelope in this manner the framework is able to explain the occurrence of wetting and loading collapse especially the former which cannot be explain previously. This will then bridge up the gap in the cognitive taxonomy and thereby accomplish the whole process of research innovative and cognitive taxonomy.



(a) Wetting collapse (b) Loading collapse

Figure 10. Soil settlement model: The pictures were mapped out in the "picture out the expected outcome activity" in the research innovative taxonomy.

8. Applicability of slope stability equation to simulate shallow landslide.

Another advantage of the curved surface shear strength envelope will be demonstrated when its application in a newly develop slope stability equation is able to simulate the shallow landslide. The new slope stability equation (Md.Noor, 2007) is as shown in Equation 11.

$$FOS = \frac{\sum s_i \times R}{\sum W_i \times x_i} = \frac{\sum \{\tau_i \times \beta\} \times R}{\sum \left[\gamma_{dry} \times V_i + \theta_{ave} \times 9.81 \times V_i\right] \times x_i} \qquad \text{Equation 11}$$

The shear strength, τ in the numerator will take up one of the four shear strength equations in the curved surface envelope shear strength model and the selection depends on the value of net stress and suction at the referred depth. Thus the real shear strength magnitude will be applied in the analysis. The denominator will account for the increase in the disturbing moment as the soil become heavier when infiltrated. The effect of rainfall infiltration towards triggering slope failure is already well accepted. This factor is neglected in the conventional slope stability method where the soil weight is assumed to be constant. The advantage of considering the real shear strength magnitude and the actual soil weight when it becomes wetter is that it is able to simulate the real condition that exist within the soil and thence allowed the simulation of the shallow landslide. When the real field behaviour i.e. shallow landslide is able to be simulated by incorporating the effect of infiltration then the stability factor produced by the Equation 11 would be a real representative state of stability of a slope under consideration. The shallow type of landslide used to be regards as a complex behaviour which cannot be understood through the conventional slope stability method. Then the coupled between the new curved surface shear strength model and the new slope stability equation has successfully explained the occurrence of shallow landslide and thereby give a clear indication of its applicability. This would then bridge up the comprehension and application learning activity in the Bloom,s (1956) cognitive taxonomy.

9. Conclusions

The conclusions that can be drawn from this study are as follows.

1. The application of research innovative taxonomy has helped to achieve a good representative soil shear strength model which forms the base knowledge for a new cognitive taxonomy.
2. The application of research innovative taxonomy between comprehension and application learning activities in the Bloom's (1956) cognitive taxonomy has helped to discover the new soil settlement framework and new slope stability equation which allowed the understanding of wetting collapse behaviour and shallow landslide respectively. It is the steep drop in shear strength near saturation exhibited in the curved surface envelope shear strength model that governs these complex soil behaviours.
3. The application of the cognitive Bloom's (1956) taxonomy is able detect the gap in

knowledge and forms the best platform to test the validity of a newly developed knowledge and newly developed framework that can explain a specific soil behaviour.

4. The research innovative taxonomy and the cognitive Bloom's (1956) taxonomy can be a useful tool for an ambitious researcher to help achieving a breakthrough in research.

References

Alonso, E.E., Gens, A., and Josa, A. (1990), "A constitutive model fro partially saturated soils." Geotechnique, 40(3), 405-430

Bishop, A. W. (1966). "The strength of soils as engineering materials." *Geotechnique*, 16(2), 91-130.

Bishop, A.W. (1955). "The use of the slip circle in the stability analysis of slope." Geotechnique, 4(2).

Bishop, A.W. (1959). "The principle pf effective stress." Teknisk Ukeblad, 106(39), 859-863.

Bishop, A.W. and Morgenstern, N.R. (1960). "Stability coefficients of earth slope." Geotechnique, 10.

Blanchfield, R., and Anderson, W.F. (2000). "Wetting collapse in opencast coalmine backfill." Proceedings of the ICE Geotechnical Engineering, London: 139-149.

Bloom, B.S. (1956). "Taxonomy of educational objectives." Handbook 1: The Cognitive Domain, David McKay Co Inc.

Brand, E.W. (1985). "Predicting the performance of residual soil slopes." (Theme Lecture). Proc. 11th Int. Conf. SMFE, San Francisco, (5), 2541-2578

Brand, E.W. (1989). "Landslides: Extent and Economic significance." Proc. 28th Int. Geological Congress: Symposium on landslides, Washington D.C., 303-322

Brand, E.W., Premchitt, J. and Phillipson, H.B. (1984). "Relationship between rainfall and landslides in Hong Kong." Proc. 4th Int. Symp. Landslides, Toronto, (1), 377-384.

Charles, J. A., and Watts, K. S. (1980). "The influence of confining pressure on the shear strength of compacted rockfill." Geotechnique, 30(4), 353-367.

Escario, V., and Juca, J. (1989). "Strength and deformation of partly saturated soils." *12th. International Conference on Soil Mechanics and Foundation Engineering*, Rio de Janeiro, 3, 43-46.

Escario, V., and Saez, J. (1986). "The shear strength of partly saturated soils." *Geotechnique*, 36(3), 453-456.

Fredlund, D. G., and Xing, A. (1994). "Equations for the soil-water characteristic curve." *Canadian Geotechnical Journal*, 31(4), 533-546.

Fredlund, D. G., Morgenstern, N. R. and Widger, R.A. (1978). "The shear strength of unsaturated soils." *Canadian Geotechnical Journal*, 15(3), 313-321.

Fukushima, S., and Tatsuoka, F. (1984). "Strength and deformation characteristics of saturated sand at extremely low pressures." *Soils and Foundations, JSSMFE*, 24(4), 30-48.

Gan, J. K. M., and Fredlund, D. G. (1996). "Shear strength characteristics of two saprolitic soils." *Canadian Geotechnical Journal*, 33, 595-609.

Gan, J. K. M., Fredlund, D. G., and Rahardjo, H. (1988). "Determination of the Shear Strength Parameters of an Unsaturated Soil Using the Direct Shear Test." *Canadian Geotechnical Journal*, 25(3), 500-510.

Indraratna, B. and Salim, W. (2002). "Modelling of particle breakage of coarse aggregates incorporating strength and dilatancy" *Proc. of Instn of Civil Engrs, Geotechnical Engineering*, 155(4), 243-252.

Indraratna, B., Wijewardena, L. S. S., and Balasubramaniam, A. S. (1993). "Large-scale triaxial testing of greywacke rockfill." *Geotechnique*, 43(1), 37-51.

Md.Noor M.J. (2007). "Slope Stability Method Incorporating the Curved Surface Envelope Shear Strength Model." Proceeding of the South East Asia Conference, Kuala Lumpur, Malaysia.

Md.Noor M.J. and Anderson (2007). "A Qualitative Framework for Loading and Wetting Collapses in Saturated and Unsaturated Soils." Proceeding of the South East Asia Conference, Kuala Lumpur, Malaysia.

Md.Noor M.J. and Anderson W.F. (2006). "A comprehensive shear strength model for saturated and unsaturated soils." Proceeding of the 4th International Conference on unsaturated soils. Arizona, USA.

Othman, M.A. (1989). "Highway Cut Slope Instability Problem in West Malaysia." Ph. D. thesis, Bristol University, UK.

Pitts, J. (1983). "The form and causes of slope failures in an area of west Singapore

Island." Singapore Journal Trop. Geog., (12), 162-168.

Ramasamy, S.D. and Aziz, M.A. (1980). "Rain induced landslides of Singapore and their control." Proc. Int. Symp. Landslides, New Delhi, (1), 403-406.

Salman, T. H. (1995). "Triaxial behaviour of partially saturated granular soils at low stress levels." *Ph.D. Thesis*, University of Sheffield, Sheffield.

Taylor, D.W. (1948) "Fundamentals of soil mechanics." John Wiley, New York.

Terzaghi. K. (1936). "The shear resistance of saturated soils." Proceedings for the 1st International Conference on Soil Mechanics and Foundation Engineering (Cambridge MA), 1, 54 – 56

Terzaghi. K. (1943). "Theoretical soil mechanics." New York, Wiley Publications.

Ting, W.H. (1984). "Stability of slopes in Malaysia." Proc. Symposium Geotechnical aspects of Mass and Material Transportation, Bangkok, 119-128

Toll, D. G., Ong, B. H., and Raharjo, H. (2000). "Triaxial testing of unsaturated samples of undisturbed residual soil from Singapore." *Proceedings of the Conference on Unsaturated Soils for Asia, Singapore*, Balkeema, 581-586.

Vanapalli, S.K., Fredlund, D.G., Pufahl, D.E. and Clifton, A.W. (1996). "Model for the prediction of shear strength with respect to soil suction." Canadian Geotechnical Journal, 33(3), 379-392

Wheeler, S.J. and Sivakumar, V (1995). " An elasto-plasticity critical state framework for unsaturated silt soil." Geotechnique, 45(1), 35-53

Wheeler, S.J., Sharma, R.S., and Buisson, M.S.R. (2003) "Coupling of hydraulic hysteresis and stress-strain behaviour in unsaturated soils." Geotechnique, 53(1), 41-54