Overcoming Students Obstacles in Multivariable Calculus through Blended Learning: A Mathematical Thinking Approach

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Abstract

Multivariable calculus is one of the most difficult courses for undergraduate students in many fields of studies. This study used blended learning as a teaching and learning environment to support students' mathematical thinking and to help students in overcoming their obstacles in the learning of multivariable calculus. The main purpose of this study is to determine the impact of blended learning on the students' learning of multivariable calculus and in overcoming students' obstacles. The results revealed that blended learning is an adequate environment since it provides sufficient tools that support students' mathematical thinking powers to overcome their obstacles in learning multivariable calculus.

Keywords: Blended Learning, Mathematical Thinking, Multivariable Calculus, Students' Obstacles

1. Introduction

Calculus particularly multivariable calculus is one of the most important parts of mathematics syllabus for undergraduate students. It is offered as prerequisite course to other advanced mathematics courses and even other courses. However, for most students calculus specifically multivariable calculus is one of the most difficult courses in their fields of study (Eisenberg, 1991; Tall, 1993; Artigue and Ervynck, 1993; Yudariah and Roselainy, 2001; Willcox and Bounova, 2004; Kashefi, Zaleha, and Yudariah, 2010, 2011a, b). Many students will struggle as they encounter the non-routine problems where the solution methods are not obvious and require them to use problem solving processes.

Various difficult areas in the learning of multivariable calculus have been identified. In a study of multivariable calculus, Kashefi, Zaleha, and Yudariah (2010, 2011a, b) found that for many students, finding the domain and the range, sketching the graph, finding the differences between d and ∂ in partial derivatives, and the problems related to multiple integral were the most difficult parts of multivariable calculus. The important reasons of students' difficulties were: students' idiosyncrasy attributed from previous mathematical experience, the negative effect of students' mathematical knowledge construction, students' poor prior knowledge on basic calculus or lack of practice, selecting inappropriate representation of three worlds of mathematics, the transition from one world to another world of mathematics, algebraic manipulation, and memorizing.

Many methods have been applied to support students to overcome their difficulties in mathematics. Improving students' learning through the enhancement of their problem solving and mathematical thinking skills as well as through using technological tools to support conceptual understanding and problem solving methods are now thought to be more appropriate to enable students to cope with the mathematics needed for their real world problems. Researchers, by promoting mathematical thinking with or without computers, try to support students understanding of mathematical concepts and to help them solve real problems in face-to-face classroom situations (Dubinsky, 1991; Yudariah, 1995; Watson and Mason, 1998; Mason, 2002; Tall, 1986, 1995, 2004; Roselainy, 2009). Although some work has been done and reported on modelling mathematical thinking in face-to-face

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multivariable calculus classes, a review of the literature indicates that very few studies have been carried out which have been focused on the integration of strategies to invoke mathematical thinking explicitly in blended learning environment.

In the study of Multivariable Calculus, Kashefi, Zaleha, and Yudariah (2012a, b) adopted the theoretical foundation of Gray and Tall (2001) and Tall (2004) and used frameworks from Mason, Burton, and Stacey (1982), Watson and Mason (1998) and the works of Lumsdaine and Lumsdaine (1995) to develop the mathematical pedagogy for multivariable calculus classroom practice. They highlighted some strategies to support students to empower themselves with their own mathematical thinking powers and help them in constructing new mathematical knowledge and generic skills, particularly, communication, teamwork, problem solving, and technology skills through blended learning environment (Kashefi, Zaleha, and Yudariah, 2012a).

The blended learning environment give students the opportunities to benefit from both the face-to-face and the e-learning instruction. Classroom tasks, Assessments, Computer and web aide, and Strategies are the elements of the model of blended learning mathematics which is used as a guide to classroom instruction. They designed the classroom tasks in special manner based on mathematical thinking. In fact, designing prompts and questions used in order to initiate mathematical communication between the students and lecturer. Furthermore, synchronous and asynchronous web communication facilities such as chat, email, and discussion board supported the students' oral and written communication. By doing group assignments and presentations as a team, not only they could support the students' team work but they can also encourage discussion and sharing of ideas among the students. Working in pairs, small group, critical thinking and problem solving, students' own examples, doing assignments, reading and writing in the face-to-face and web environment are other strategies of this method.

By using the web environment, the resources is prepared in the most sufficient ways for it to be used in face-to-face class and in laboratory session (as online and offline). In this environment, students can have access to lecture notes, web-based interactive educational tools, animations, videos, forums module, chat module, journal module, assignments, assessments, survey and feedback. In addition, it will also help the students to find more information about content and questions, and to submit assignments, projects and laboratory reports.

In this study, we used this model that conceptualized a framework for supporting mathematical knowledge construction and invoking students' mathematical thinking powers and generic skills through a blended learning environment. The main goal of this study is to determine the impact of blended learning multivariable course on the students learning of multivariable calculus specifically in overcoming students' obstacles.

2. Method

One class of 62 first year engineering students enrolled in multivariable calculus at Islamic Azad University of Kermanshah (IAUKSH) in Iran in the fall semester of 2010 was selected for this study. The first-named author, with more than 8 years experience of teaching multivariable calculus, taught this class. The multivariable calculus offered by IAUKSH is a three credit undergraduate course and covers functions of several variables, partial derivatives, multiple integrals, vector functions and vector calculus. These topics were taught over a period of 14 weeks with 3 meeting hours per week consisting of 2 hours face-to-face and a 1 hour laboratory session. In the lecture session, the mathematical concepts were introduced to the whole class. After the students had established a general idea of the concept, they then proceeded to the laboratory session. In the laboratory session, online activities directed students to perform interactive mathematics tasks, and to post messages and questions on the discussion board.

Data were collected through students' written solution and semi-structured interviews. Based on the preliminary study and literature review, eight problems were selected from the topics that students had difficulties. Some of the problems were given either in the quiz, test, or final exams. These problems were as follows.

1. Find the domain and range for the function: $f(x,y)=\sqrt{64-4x^2-y^2}$ and sketch the domain. (Quiz)
2. (i) Suppose $f(y,z)=9-\mathrm{y}^2-\mathrm{z}^2$.

a. Find and sketch the domain.
b. Determine the range.
c. Sketch the graph of function.

(ii) Sketch the graph of $x^2 + \frac{y^2}{4} - \frac{z^2}{9} = 1$. Does the graph represent a function? (Test)

3. (i) Determine the domain and range of the following function:

$f(x, y) = \ln\sqrt{1 - x^2 - y^2}$.

(ii) Find f_{xy} for the function. (Test)

4. Evaluate the integral $\int_0^1 \int_y^1 \sin(x^2)\, dxdy$ by reversing the order of integration. (Test)

5. Find the limit if it exists

$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2 + y^4}$. (Final exam)

6. Find the absolute extrema of the function $f(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = -\frac{3}{5}x + 5$. (Final exam)

7. Find the volume of the solid bounded by the graphs of $z = 9 - y^2$, $x + z = 9$, $x = 0$ and $z = 0$. (Final exam)

8. Let G be the wedge in the first octant cut from the cylindrical solid $y^2 + z^2 = 1$ by the plans $y = x$ and $x = 0$. Evaluate $\iiint_G z\, dV$. (Final exam)

Moreover, several students were selected for the semi-structured interviews based on their responses to the written assessments. During the sessions, the reasons of their responses especially their capabilities and difficulties in solving the problems were uncovered. The common questions in the semi-structured interviews that were asked from the selected students were: what they do to find to find the answer of the problem, how they find it, and what were their difficulties to solve the problems.

Based on the students' written solutions and their interviews to the assessments in face-to-face environment, the participants' mathematical errors are identified and classified based on mathematical thinking approach. For data analysis, Miles and Huberman's (1994) qualitative analysis method was adopted as the main framework in analyzing the data obtained from the students' responses. In order to analyze the students' responses to the open ended questions, three stages of this method were used: data reduction, data display, and conclusion drawing. The analysis of the students' errors in the assessments was based on Peng and Luo (2009) framework. This framework includes two separate dimensions namely the nature of mathematical error and the phrases of error analysis. However, the nature of mathematical error was modified based on the scheme described by Mason (2002) on mathematical thinking approach.

3. Findings and Discussion

Most students could find the domain of function correctly for Problems 1 (75%), Problem 2(i)-a (93%), and Problem 3(i) (48%). Solving different examples and problems, and by using computer tools and animations were the important methods that the students believed supported them in finding the domain. The results revealed that students' poor prior knowledge about the concept of function and poor written communication were the major reasons of their difficulties. Figure 1 represents a typical student's response as a typical response of those who wrote the domain as $D_f = \{(x, y)|\, x^2 + y^2 \le 1\}$ incorrectly.

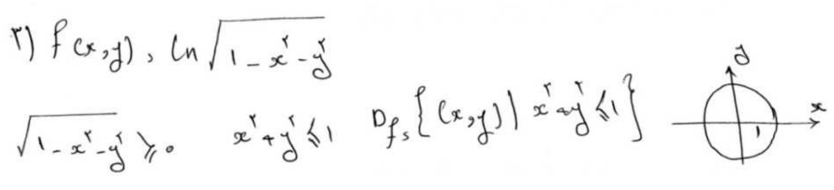

Figure 1. A typical student's attempt in finding the domain of $f(x, y) = ln\sqrt{1 - x^2 - y^2}$

Majority of the student sketched the graph of domain in Problems 1 (72%) and Problem 2(i)-a (86%). They believed that using computer tools in teaching the techniques of sketching the graph was the effective method that supported them in sketching the graph of domain, especially in 3-dimansions (kashefi, Zaleha, and Yudariah, 2011b). Figure 2 represents a student's response, as a typical student's response, to the quiz problem in which the student sketched the graph of domain in 3-dimenssions. For most students, poor prior knowledge was also the important reason of their difficulties in sketching the graph of domain.

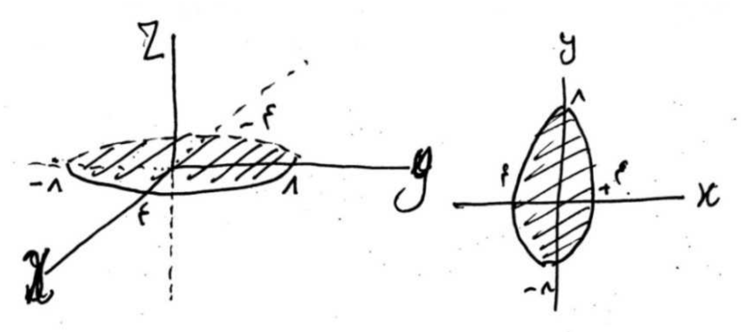

Figure 2. A typical student's attempt in finding the domain of $f(y,z) = \sqrt{64 - 4y^2 - z^2}$

Most of the students were also able to find the range of Problem 1 (70%), Problem 2(i)-b (71%), and Problem 3(a) (52%). Most of these students could find the range of *f* correctly by selecting an appropriate representation of mathematics world (the symbolic world). Using computer tools in teaching the topic in the class was other reason of their success in finding the range. However, poor prior knowledge and algebraic manipulation were still the reasons of the students' difficulties in finding the range for non-routine problems.

In sketching the graph of Problem 2 part (i)-c, 68% of the students were able to successfully sketched the graph and for part 2(ii), 80% of the students were successful. Figure 3 represents a typical student's response in finding the domain, the range, and sketching the graph of $f(y,z) = 9 - y^2 - z^2$. The student found the domain as $D_f = \{(y,z)|\ y,z \in R\ \}$ and sketched it correctly. This student was not only able to find the range as $R_f = \{x|\ x \leq 9\}$ but also able to sketch the graph of *f*. Using computer tools for sketching the graph of two-variable functions helped the students in understanding how to sketch the different traces and to make link among them for sketching the graph. In fact, the student could sketch the graph of surface correctly by using an appropriate representation of mathematics world (the symbolic world) and the transition from this world to the embodied world of mathematics. This shows that students could make a link among the different traces for sketching the graph of surface. In addition, using animations to show different graphs of two-variable functions and surfaces in respect to different variables was another reason that helped them in sketching the graph correctly. For few students, the symbolic insight strategy also helped them to remember the six common types of quadric surfaces and to sketch the graph of surface correctly. However, the results revealed that students' inflexibility in handling different symbols and their poor prior knowledge were the reasons of students' difficulties in sketching the surfaces of Problem 2 part (i)-c. In addition, sketching was the reason of the students' inability in solving Problems 4.

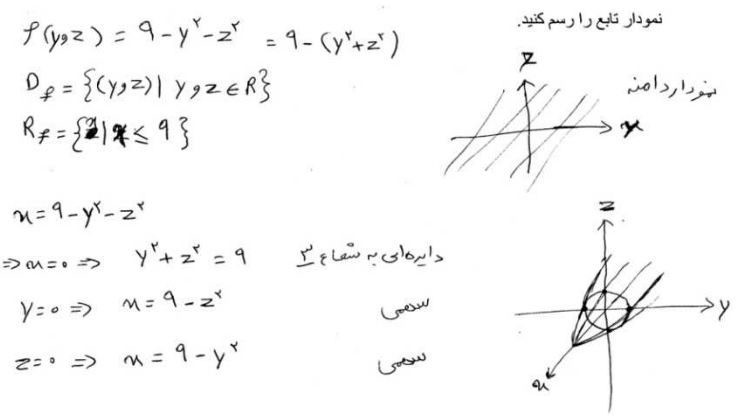

Figure 3. Amir's attempt in finding the domain and range and in sketching of the graph of $f(y,z) = 9 - (y^2 + z^2)$

Most students were able to solve Problem 3(ii) that is related to partial derivative; however, their poor prior knowledge was the reason of difficulties. Majority of the students could also solve Problem 5 on the limit. Using the different paths in students' solutions showed that most of them knew the condition of the existence of limit at a point. Figure 4 represents a typical student's response in which he solved it correctly. By testing the different paths such as $y = 0$ and $x = 0$, $y = x^2$, and $x = y^2$ this student found that for $x = y^2$, the limit of $\frac{xy^2}{x^2+y^4}$ is $\frac{1}{2}$, which is a different number from the limits of it that obtained from other paths. Memorizing the process and solving the problem without sketching the paths and poor algebraic manipulation were also the reasons of their difficulties.

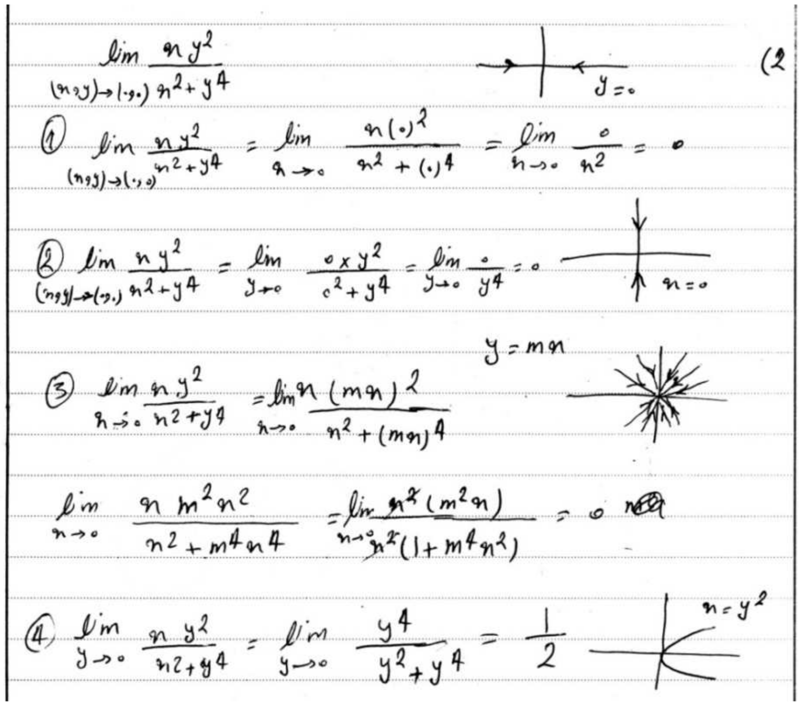

Figure 4. A typical student's response to Problem 4

Almost half of the students solved Problem 6 about finding the absolute extrema. Most students could not solve this problem because they tend to confuse this problem with the local extremum problem. Poor prior knowledge and algebraic manipulation were other reasons of the students' difficulties. In solving this problem, writing the limits of integration based on the region of integration and poor integration techniques were also the major reasons of students' difficulties in solving Problems 7 and 8. Figure 5 shows a typical student's response in which the student incorrectly found the limits as $\int_0^y \int_0^{\frac{\pi}{4}} \int_0^{y^2+z} dz\, dy\, dx$.

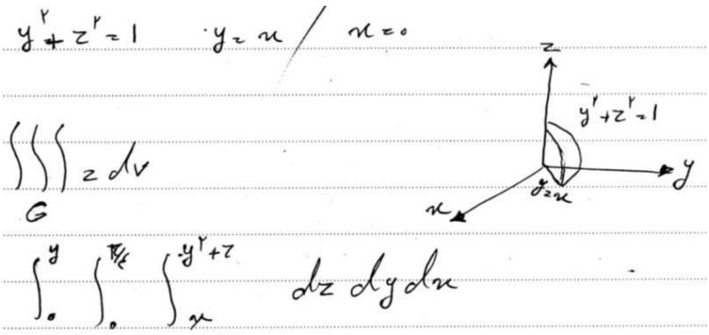

Figure 5. A typical student's response to Problem 4

4. Conclusion

Most students showed their ability to overcome the difficulties in solving problems that reported in the previous study. Students' solutions to written assessments and interviews revealed the impact of the blended learning multivariable calculus course in reducing the students' difficulties. However, algebraic manipulation and poor prior knowledge on some topics in basic calculus were the reasons of why some students still have difficulties in solving problems (Kashefi, Zaleha, Yudariah, and Roselainy Abd. Rahman, 2012b). These difficulties were with the concepts. Students' responses also confirmed the effectiveness of course on students' learning. It seems that in this method, it is necessary to use new strategies and tools when teaching students with a wide variance in preparation and abilities.

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