Improving the Teaching of Engineering Mathematics Using Action Research

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Abstract

Our experience showed that students struggled with new concepts in Engineering Mathematics and faced difficulties with the need to use various mathematical techniques to solve problems. Thus, we embarked on a study to improve our teaching and students' learning. We designed teaching strategies to support students' development of mathematical knowledge and problem solving as well as communication and team working skills. We employed an action research perspective as various methods within this stance can ensure flexibility in responding to the dynamics of interaction between the teachers and their students. This paper presents an account of our study and highlights various issues in the use of action research.

Keywords: Action research, engineering mathematics, teaching, learning.

1. Introduction

We have seen how hard our students worked but they still had difficulties in understanding the concepts taught. They worked on many problems to increase their understanding but still found non-routine questions difficult. We saw that these students could not 'see' the 'general' through the particulars nor 'see' the particulars in the general when dealing with mathematical concepts and examples. They could not see the essential features of a technique or recognize these when presented in different forms (Roselainy et. al., 2010).

We reasoned that a more effective method was needed to support students' learning. Thus, we decided to implement teaching integrated with strategies to invoke students' mathematical thinking powers so as to effect changes in engineering students' attitudes as well as increase their facilities with mathematical ideas. In doing so, we decided to investigate changes in teaching practice and the effects of implementing teaching integrated with mathematical thinking strategies on students' learning behaviour. We chose to use action research and case study methods and thus, the direction and focus of the research was also examined and re-examined after perusal of findings during various stages of the study. The research sought to answer the following research questions: (i) What are the factors that affect students' learning behaviour?; (ii) What are the students' attitudes and perceptions towards Calculus?; (iii) What changes are required in the teaching practice?; (iv) What are suitable strategies to invoke students' use of their own mathematical thinking powers?; and (v) What are the changes in students' attitudes and learning behaviour? However, this paper will focus on the research methodology and discusses issues related to its implementation.

2. Research Implementation

The research was conducted in three different phases, over a number of years, from 2001 to 2008, where actions in each subsequent phase were determined by findings from the previous one. During that time, there were also changes in the academic and administration situations due to review of students' entry requirements, the curriculum

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as well as professional obligations. A brief description of the research process will be given with general explanation and justifications of the decisions and changes made during each phase.

2.1. Overview

The first phase of the study covered a general period from November 2001 to November 2004. We studied four groups of students from the Industrial Design Course (SRI) from the Faculty of Mechanical Engineering, which we called Block 1. We taught two groups of students Basic Calculus and the other two groups, Calculus II. An early decision was to follow students through two mathematics course to study the effect of the teaching strategies. However, due to the changing conditions of entry and professional responsibilities, these students became a group that was 'atypical' during the research period. They were part of a students' body that came in with *Sijil Pelajaran Malaysia* (SPM, Malaysian Certificate of Education), a qualification equivalent to the GCE 'O' Levels in the UK. The entry qualifications to UTM were upgraded to post-SPM qualifications such as *Sijil Tinggi Pelajaran Malaysia* (STPM, Malaysia Higher Education Certificate), Matriculation certificates as well as relevant Diploma qualifications. They were also taught in Bahasa Malaysia (Malay language).

These changes meant that we had to extend our study to include the newer intake and this led us to the second phase which consisted of a study of two groups of students whom were taught Engineering Mathematics in their second year and in the second semester of 2005/06. These were the Block 2 students. These students were taught in English and were following the new curriculum for engineering education. Both blocks of students were taught by the same lecturer. Conclusions made from the observations and interviews highlighted the importance students put on 'rapport' with the lecturer. We were concerned whether the students' responses were influenced by the personality of the lecturer involved. Thus, we decided on a third phase of study which consisted of two more groups of students but whom were taught by the other two lecturers. They were also studying Engineering Mathematics and were from the Faculty of Electrical Engineering in the first semester of the academic session 2007/2008. All the students were taught using the same teaching strategies that were developed. In this phase, the lecturer who was not involved in the teaching observed the classes and conducted interviews with student volunteers.

2.2 Studying the Natural Setting – using action research

Ainley (1999) takes the view that teaching and research could be complementary and she advocated that a teacher doing research must be able to recognize how the research informs on teaching, and when experience as a teacher also informs the research. The action research process has several features that were appropriate to our research concerns. Firstly, action research is an intervention in personal practice with a commitment to educational improvement (Mcniff, Lomax & Whitehead, 1996). The teacher becomes the subject and object of research but with a greater awareness of the actions that are being carried out. It means that the teacher must investigate her actions and motives systematically, be critical of her interpretations and findings and be more open to alternative viewpoints. We wanted to present a study that was realistic in describing the teaching and learning environment. We had at least 15 years' teaching experience each, and as we had chosen a methodology whereby we were both the subjects and instruments of research, it was necessary to be alert to the duality of roles, teacher and researcher, to view the teaching and learning setting as,

> " … *a learning situation in which the researchers have to understand their own actions and activities as well as those of the people they are studying*." (Burgess, 1982)

In the following account of research techniques that were chosen and used, we sometimes use the singular pronoun to simplify our presentation. The following techniques were chosen to achieve the goals of working with subjects in their natural setting.

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2.1.1. Participant Observation

In this research, the researcher was also the teacher, thus she was a full participant of the classroom but not in the same manner as the students. The researcher was researching her own practice as well as observing the students' behaviour in the class. Students were also informed about the research process. The objectives of the research were shared with them and they were reassured that any incidents or quotes that would be used would be anonymous to protect their identities.

It became important to distinguish between the various observations notes that were taken during the research period. Mason (2002) suggested that the act of noticing was an important element of practice. He described three different levels of noticing: ordinary, marking and recording. In addition, he put forward the notion of disciplined noticing as an attempt to be systematic and methodical and suggested several elements to enhance the chances of noticing. These were: keeping accounts, developing sensitivities, recognising choices, preparing and noticing, labelling and validating with others. By adopting the ideas of disciplined noticing, it has enabled the researcher to heighten her awareness of researching her own practice. Thus in writing the accounts of the events, personal opinions and judgments were not included in the descriptions of the classroom situation or other interaction with the students. These were written separate from class records and were clearly identified. There was also the difficulty in choosing events to write about. In the beginning, everything was considered worthy to be noted but during the later stages, there was an element of choice and the reasons for these choices must be made clear as much as possible. The notes were read through regularly, reflected on and were used to identify students' responses to the teaching strategies, the presentation of the mathematical contents and techniques and to deliberate over changes that may be needed.

2.2.2 Critical Reflection

A teacher engaging in action research brings with her all her understanding and experiences of the teaching and learning situation and thus has to reflect on and examine these prior knowledge and all assumptions to be able to increase her awareness and insight into the research. Kemmis and McTaggart (1998) explained it as, "*plan, act, observe and reflect*". It was important to alternate the action with critical reflection, which can be about the data and the interpretations made on it, a critique of the methodology to improve it and an examination of assumptions about factors that influenced the study. Dick (2000) called the alternating action with reflection as the spiral nature of action research. He considered that the spiral provided the two main advantages of the method, which were, firstly, each turn of the spiral offered chances to test the interpretations that was developed against the data collected and secondly, with each turn, plans were developed to test the assumptions that guided them in action. This very process set the condition for flexibility and responsiveness for effective change. We adopted Griffiths & Tann (1992) ideas about the importance of uncovering personal theories in reflective practice and making them explicit. They felt that this particular process has not been given due attention in other theories on reflection. They suggested five different levels of reflection to support theorising that were grouped into two major categories, i.e. **Reflection-in-action**: likely to be personal and private. Two levels were relevant here, (1) Act-react (Rapid reaction) – at this level, reaction was immediate, and (2) React-monitor-react/ rework-plan-act (Repair) – at this level, although there was pause for thought, it was on 'the spot' and very quick. The other category was **Reflection-on-action**: likely to be interpersonal and collegial which consisted of the other three levels, i.e., (3) Act-observe-analyse and evaluate-plan-act (Review). At this level, thought and reflection were going on after actions were completed; (4) Act-observe systematically-analyse rigorously-evaluate-plan-act (Research). At this level, observation became systematic and sharply focused; (5) Act-observe systematically-analyse rigorously-evaluate-re-theorise-plan-act (Re-theorising and reformulating). This was the level of abstract, rigorous reflect which was formulated and reformulated.

It was at the last two levels that engagement with public theories became more prominent. Their framework opened up ideas on the roles of personal theories and public theories in teaching practice and more importantly, how theory and practice were related in education. They also discussed the differences in language used to articulate personal theories compared to the abstract terminology of public theories. They felt that it was essential to recognise the importance of public theories without losing sight of the importance of personal theories embedded in practice. We found Griffith & Tann's framework helpful in offering an efficient and practical guide for a teacher to undertake reflective practice as well as increase her awareness of the research aspect of her work. In particular, reflecting upon our personal ideas and theories about students' difficulties and constantly examining and comparing it with public theories was an important initial step in embarking on this study. Our ideas were examined against published literature as well as with other colleagues so as to gain a wider perspective on issues emerging. We had discussed and lamented on our students' mathematical difficulties for years and we found findings of certain published research works that seemed to support our concerns. These assumptions were identified and were used in the first instance to design and develop the teaching strategies. Public theories on specific students' difficulties with mathematical concepts, how students learn, suggested strategies to overcome these difficulties were also referred to in creating the teaching strategies that were implemented. However, at every stage, the teaching acts, students' responses, various issues emerging from the students-teacher interaction were reflected upon thus ensuring that the cycles of '*planning-implementation-review-modify*' were continually carried out. External influences (organisational, administrative, etc) were also considered as sometimes these provided information about changes to some initial assumptions about the students studied. Additionally, experiences within the research cycle sometimes necessitated changing certain notions about various aspects related to the students' such as abilities, prior knowledge, motivation, behaviour and attitudes.

2.2.3 Observation of Colleagues' Teaching

For the first two phases, only one lecturer used the strategies in her classes. However, we worked collaboratively in implementing the research and held numerous discussions about the progress of the work, reviewing our assumptions and evaluating the teaching strategies and observing the class. In the class, tasks were given in worksheets for students to work on. However, during the first phase, our results showed that students preferred to have documented tasks to use in class and for their own future reference. We decided to compile the worksheets and turn it into a workbook. Thus, beginning in the academic session 2005/2006, a workbook was designed to help students work on the mathematical tasks but an early version of the workbook was only ready for use in the academic session 2006/2007. In the third phase, the other two lecturers carried out the same teaching strategies and used the workbook in their classrooms. The first lecturer became the observer in this period. The decision to observe their teaching was to add objectivity to the evaluation of the teaching strategies and investigate whether personality was a significant factor in the success or failure of the new teaching. The observer role was made known to the students from the beginning whereby her contact with the students stemmed only from the research. Students' group discussion was observed, their work in the workbook was studied as well as some interviews were conducted with willing participants.

2.2.4 The Research is concerned with Participants' Views of Teaching and Learning

An important consideration of the research perspective was to find out what the students felt about the teaching and learning process, in particular, what they thought of about their mathematics learning and their supposed difficulties. Some of the methods adopted towards these objectives are as follows. (i) **Interviews –** Interviews was only conducted with students who volunteered and were of two types. The first type were aimed at finding out what students thought about their life at university, their learning and the teaching specifically about our teaching. We used unstructured and non-directive in the mode of 'informant' interviews (Powney & Watts, 1987). The second type was more specific interviews about their mathematical learning and had included some problem solving sessions. The interviews were conducted in a conversational style and the students were given some freedom to impose on its structure though we had determined a general agenda. In the second type, the interviews were semi-formal and were focused on the mathematics that students have learnt and they were asked to solve some problems.

(ii) **Conversations:** Although some researchers regarded informal interviews as conversations or discussions (Woods, 1986), we have decided to differentiate the two forms of data collection. Even in the most informal interviews, some prior arrangement was made to set the occasion whereas conversations would usually occur more naturally and would not be limited to any specific time, place or topics. There were several different forms of conversation or discussions that occurred: (a) group conversations or discussions during group work where we were passive listeners; (b) discussions where we might join in but keeping to the flow without steering the discussion, (c) conversations that were started because we had some questions or was seeking some explanations or information and (d) conversations initiated by the students themselves, within the classroom or at other times. In most cases, the discussions were about the mathematics but as the semester progress, we were getting more cases of students coming up to talk about their personal problems which were related to their finances, emotions, family or studies. The drawback in these sessions was in taking down notes as in so doing; it disrupted the situation thus any notes were usually made after the events. (iii) **Questionnaires** – the students were given two questionnaires, (1) the course evaluation questionnaire given out by the University and (2) a course evaluation questionnaire that we prepared ourselves. The University's questionnaire was regularly given out to all students at the end of the semester in which the students were to evaluate the lecturer's presentation and course materials. Some questions were also asked about the lecturer's relationship with the students. In the questionnaire we prepared, questions were more specific about the strategies that were used. Students were also asked to evaluate the progress of their learning.

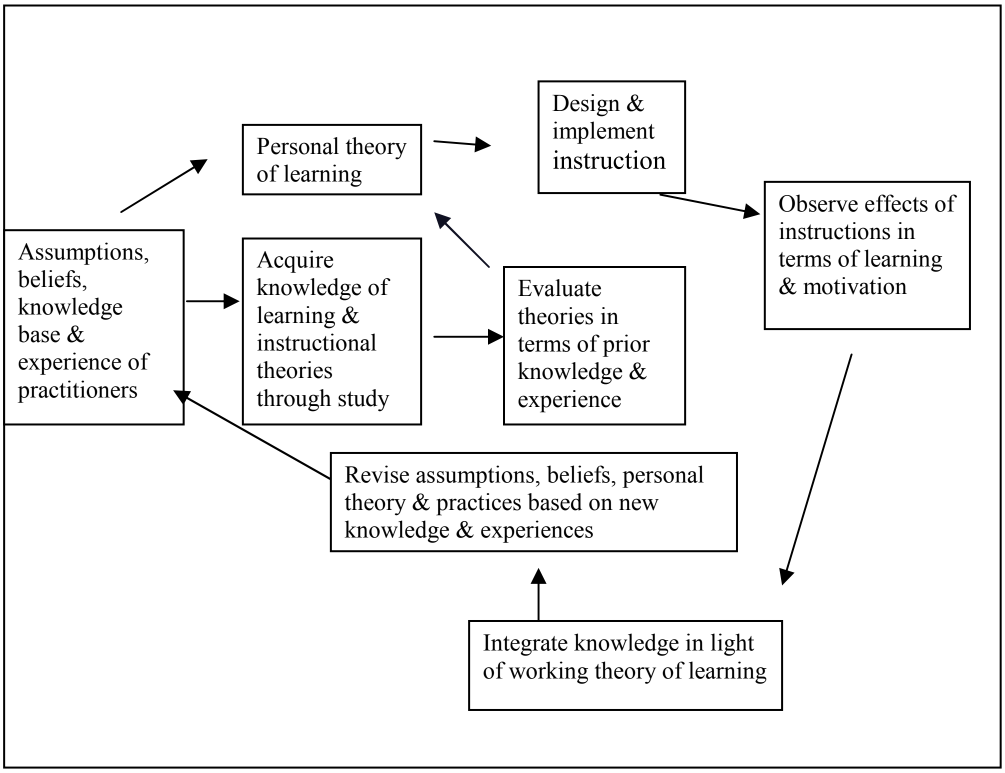
2.2.5 The Research is Concerned with Students' Development in the Learning of Mathematics

The main purpose of the study was to find out if implementing teaching designed to encourage students' use of their own mathematical thinking was able to effect changes in their attitudes and learning behaviour. Data was also collected that would display the students' facility with the mathematical concepts and techniques they were learning. Supportive data were collected using the following means. (i) **Diagnostic test** – a diagnostic test on pre-calculus topics was administered at the beginning of the study to the first group. The test was designed to identify students' conceptual understanding or lack of it. However, in subsequent semesters, we were working with students who had higher mathematical qualifications upon entry, thus we decided that we wanted to identify their problem solving skills that should have been part of their prior knowledge. We designed a simple problem-solving sheet on basic concepts in Basic Calculus; (ii) **Problem solving questionnaire –** the questionnaire consisted of 7 questions on differentiation and integration which the students would have studied at school and again in the first semester at university. This was given to students in second phase; (iii) **Students' class worksheets** – consisted of questions specially structured to help students recognise mathematical thinking powers; (iv) **Students' examples** – a collection of students' work that might show how they work on problems and help in the evaluation of the effect of the teaching acts; and (v) **Observations and conversations** – with students as they work on the mathematical problems.

2.2.6 Development of Strategies and Teaching Materials

We needed a pedagogical approach that would help us focus on students' development of mathematical knowledge and competencies. Various research findings were studied so as to identify explanations and suggestions associated with students' learning, encompassing areas such as: mathematical knowledge construction and curriculum development (Eisenberg, 1991; Md. Nor Bakar, 1991; Tall & Razali, 1993; Tall, 1996, Artigue, 1991, Schwarz et al, 2002); psychology of mathematics education (Skemp, 1987; Gray & Tall, 1994; Schoenfeld, 1989; Tall, 1995; Gray et al, 1999; Gray & Tall, 2001); motivation (Schoenfeld, 1989; Anthony, 2000); frameworks for classroom implementation of mathematical thinking strategies (Mason et al, 1982; Watson & Mason, 1998; Mason, 1999; Mason & Johnston-Wilder, 2004). We then designed teaching strategies to support and encourage students' use of their own thinking powers, enhance their mathematical skills development and other outcomes that were deemed important for engineering undergraduates such as communication and team working. However, the following model gives a succinct representation of the process of preparing the teaching strategies and their implementation. The model was adapted form Driscoll (1994). The model depicts different phases that occurred during the planning, designing and implementation stages of the teaching strategies. However, we were researching our own practice thus this cycle of activities were within the cycles of the research process.

Figure 1: Process of Designing and Implementing Teaching Strategies



One of the main objectives of the teaching acts implemented was to shift students' awareness gradually from rote learning towards understanding the procedures and recognising their own mathematical powers. To encourage students to practise mathematical communication, they were asked to work in pairs or in small groups. Working in this manner has helped to lessen their anxieties and provide a non-intimidating environment for students to communicate their mathematical ideas verbally. A more detailed description of the strategies used and tasks designed is given in Roselainy (2009).

3.0 Analysis of Data

In literature on qualitative data analysis, conducting analysis simultaneously with data collection was considered one of its most prominent characteristics (Becker, et al, 1968; Glaser and Strauss, 1967). Dick (2000) used the metaphor of a spiral to describe action research and that each turn contained data collection, data analysis, action planning, action and evaluation. The research was carried out based on a cycle of activities, '*planning-implementation-review-modify*', and thus at every stage data was collected, analysed and contributed towards further action planning. Actions were then implemented, reviewed and evaluated. Thus data was interpreted as they were collected and turned into action plans which were then acted on. Results of the action were reviewed immediately and informed the next turn of the cycle. Many conclusions were captured during the actions. However, we felt that it was necessary to allocate time to review the whole process and this phase of further reflection, much in the nature of

Griffith & Tann's level 3 reflection, was a time to look at the cycles of research, the actions implemented, conclusions made based on analysis of data within the teaching period and identify any other relevant factors and issues. The courses in UTM follow the semester system and thus the break in between semesters was a natural period to undertake this overall reflection so as to plan for the next period of teaching. Several analytical methods that were common to many qualitative research perspectives were used in this study (Miles & Huberman, 1994).

We were concentrating on the appropriateness of the teaching strategies used, students' learning behaviour and our own behaviour. Thus at any one time, the focus of attention would be shifting from one or other of these concerns. Analysis carried out during the research would also influence and direct the focusing and refocusing of teaching and research acts that were carried out. Here, we had presented the strengths of an action research perspective but later, we will present an evaluation of the method based on our experience as there were difficulties in conducting research on your own practice.

4.0 Research results

We decided to use a model adapted by Norman and Prichard (1994), which was first suggested by Cocking and Chipman (1988). They used the basic model in their research project that looked at the cognitive obstacles in the learning of Calculus and identified three major components of the teaching and learning situation, which were, Entry Mastery, Student Motivation and Opportunity to Learn. However, Norman and Prichard were primarily concerned with the Entry Mastery category but we were focused on the other two components: **Opportunity to Learn** and **Students' Motivation**. We felt that we had no influence in changing the Entry Mastery component but only gathered data to identify the students' mastery of their prior mathematical knowledge. In the following discussion, we will identify the factors that shaped and affected our teaching practice. We did propose some additional elements to the categories of Opportunity to Learn and Students' Motivation in our version of the model (Roselainy et al, 2010).

Focusing on factors influencing teaching practice, we identified, (i) **Teachers' Own Attitudes and Motivation:** We found that our attitudes and motivation were also affected by the dynamics and events during the teaching sessions from within the classroom, interaction with students and colleagues out of it, the organisational and administrative components of the course implementation. For example, in Phase 1, our colleague had to work with groups of students who were unmotivated due to various factors such as belief about the importance of mathematics, their course and their ability. Students' unresponsiveness was also affecting her responses. In that phase, we found that we had to address students' self esteem and beliefs about the relevance of their courses. In a later period of Phase 1, she had students who were 'very happy' to be in her class, which of course, made her happy as well when conducting the lessons. There must be some conscious effort to maintain objectivity in our responses to students' personal problems or problems with the mathematics. (ii) **Teaching Acts and Task Design –** We will highlight the various factors that were needed to develop or learn to sustain a learning environment that would promote thinking and communication. These factors were, (a) **How to be a good facilitator of active learning?** In the beginning of the research, we realized that good intentions alone was not enough, we needed skills to facilitate students' learning. We needed to consciously think of ways to change old habits that were no longer suitable to support the changes that we wanted to effect. One particular way was to engage in the mathematical tasks ourselves, so as to be in the position of the learner and thus experienced the process of learning. However, students still preferred lectures; we had to think of ways to negotiate for change. Thus, we assigned some time for lectures and introduce sessions for students to engage with mathematical tasks during the classroom session. To promote communication, a short session to encourage students' reflection on their work was also introduced. The reflection was sometimes verbal or in written form. Thus the class interaction was usually of the nature, '*lecture – activity – review – activity – reflection – summary*'. Depending on the classroom situation, all or some of these were carried out. It was obvious that new skills had to be developed to support the teaching acts that we wanted to implement. (b) **To pay attention to the students' responses, both verbal and non-verbal.** As the teaching became more student centred, we had to take more notice of students' responses. We were introducing a way of teaching that required more students' participation and the students were not accustomed to such an environment. Some responded favourably but others appeared unresponsive but we soon realised that they were responding but through non-verbal mannerisms. It was easier to respond to their verbal contributions; their comments, criticisms, requests, inquiries; but much harder to interpret their non-verbal responses. Again, we found that we were continually negotiating and mediating for change and had to ensure certain conditions to gain trust and coax students to participate in our class. We had to be less judgemental when working with the students. In terms of non-verbal responses, we decided that if and when we did make a judgement of what they meant by their gestures or body language, we would state these explicitly and request that the students verbalise their needs and feelings. We had explicitly stated that there will be no recrimination or punishment for honesty and these approaches had helped to foster better teacher – to – student communication and vice-versa. (c) **Spontaneous coping skills to deal with students' mathematical and affective difficulties**. This study did not focus on the social and cultural factors affecting students' learning within the classroom setting. We had many students who expressed their gratitude to a supportive teacher which implied certain issues in the way the teaching and learning culture has developed but this is not within the research focus. We can only change the way we interact with our students and encourage them to be themselves without sacrificing courtesy and good manners. By using the teaching strategies, we have focused on students' mathematical abilities but we would always emphasise and draw the students' attention to their own awareness and achievements and refuse to accept credit for their own efforts and hard work. A student once said, "*Madam, you saved my life*" to the first lecturer and she replied, "*no, you saved your life*" in reference to the fact that he passed his Calculus II paper. To further support students to be independent learners and increasing their confidence; the teaching strategies, classroom settings and social interaction were designed and implemented towards these very objectives but to those students who needed some personal or pastoral care, this was duly given but within the limits that we were comfortable with. Students who needed more help were referred to the Students' Support Services for more professional help.(d) **Presentation of tasks –** The tasks were designed to allow students to work on mathematical thinking. However, changes were made as to how they were presented to students as we worked through the different phases. In each task in the first phase, we focused on the mathematical powers of specialising and generalising to increase students' awareness of the mathematical processes. However, the students needed frequent 'prompting' to be able to work through the tasks and wanted to be able to refer to their work for review and revision. In addition, they preferred 'official' notes provided by the lecturer as they also needed assurance that their work would be graded and what was taught was to be examined. Thus in phase 2, we provided students with worksheets so that the topics were presented in a structured manner and the prompts and questions were made explicit that it would be easier to refer to and to use during class work. The worksheets were collected for assessment for some of the work. They also used a textbook for extra reading and as a source of tutorial questions. However, we were only using the first three topics for the research and work was carried out in the more traditional mode for the other topics. In phase 3, we used the workbook that would allow students to work using the strategies consistently for all the topics. Several other features were also added in the book to promote a learning environment that would encourage students' engagement with the mathematical tasks, increase students' mathematical communication and group work.

4 Discussion

Criticisms against qualitative research focus on its subjectivity as a source of bias in the data reported or in any accounts produced thus we have to confront the issues of reliability, validity and generalisability of action research.

(i) Reliability, Validity and Generalisability

What are the criteria for assessing action research? Researchers take different positions in discussing these issues. Some researchers suggested that these concepts are irrelevant in a qualitative study (Stenbacka, 2001), whilst some offered different criteria deemed more suitable such as consistency or dependability and applicability or transferability (Lincoln & Guba, 1985). On the other hand, Dick (2000) maintained reliability and validity are still useful concepts in determining that the information collected is trustworthy and that the description given is based on some degree of objectivity. In this study, we used the ideas offered by Herr and Anderson (2005) which linked the goals of action research to their validity criteria. Their criteria were developed from their experience in insider action research studies. Table 1 shows the link between goals of action research, the criteria of validity and some comments that describes the meaning of the terms. The table was modified from the original table presented in Herr and Anderson (2005, p 55) meanwhile the comments are summaries of the explanations given.

Table 1: Validity criteria in action research

	Goals of action research	**Validity criteria**	**Comments**
1	The generation of new knowledge	Dialogic and process validity	The determination of the 'goodness' of research through peer review.
2	The achievement of action-oriented outcomes	Outcome validity	The extent objectives of study were met and the problems resolved.
3	The education of both researchers and participants	Catalytic validity	The understanding of all who were involved in the research was increased and they were moved to some action of change.
4	Results that are relevant to the local setting	Democratic validity	The accounting of multiple perspectives and interests.
5	A sound an appropriate research methodology	Process validity	The inclusion of multiple perspectives and the determination of what constituted as suitable evidence of the study's assertions.

These criteria will allow the methodology and the results of this study to be evaluated to test for trustworthiness and relevance. Various methods to support reliability and validity were used and briefly described as follows. (a) The research methods implemented have been clearly described at the beginning of the research, they are reviewed regularly and discussions on the modifications are also presented; (b) Triangulation – multiple strategies were used to collect data so that they can be checked and compared; (c) Reflexivity – we have attempted to address and identify our subjectivity by including our reflections as part of the cycles of research process. Assumptions and decisions made during the study are clearly identified as well as the basis for selecting informants, giving their description and the context of the setting chosen; (d) Peer validation – Peer reviews from other lecturers who have experience of teaching the mathematics subjects and similar students was sought so as to balance our subjectivity.

Could the results obtained be generalised to other situations? The research objectives of this study have excluded the idea of generalisation in the perspective of applying the research to other contexts. It is a study that seeks to describe a teaching and learning situation, enhanced the understanding of such a situation and also seeks to improve teaching practice. However, Mason (2002) put forward the idea of robustness that can be connected to generalisability. Robustness refers to "*the variety of conditions under which sensitivities inform action effectively, and the variety of people who recognise and resonate with proposed distinctions*" (Mason, 2002 (b), p 188). He contends that an import consideration for other researchers is how to identify the aspects that can inform future actions. Using such an idea of generalisability, we believe that there are certain aspects of this research that can be used in other teaching and learning research, for example, the teaching strategies described, the manner of how relationships with the students were developed and used to negotiate learning.

(ii) Review of the research process

In hindsight, there are certain aspects of the methods that we would have modified such as: (1) Observations – we would have co-opted colleagues to observe my teaching and my classes on a periodic basis so as to garner objective evaluation of what was happening in my class much earlier in the research; (2) Recording of observations

– we thought that the use of videos or tape recorders was intrusive as students become more aware of their views being recorded. However, current trends in 'reality shows' showed that if the equipment is used consistently, students might forget that they are there. Perhaps, it would have helped to have these forms of records of classroom events to compare with our notes. We have to admit that we have much influence in deciding what to record and report. Nevertheless, by making explicit the reasons of choice, we hope that it will put the report in appropriate context.

5 Conclusion

We decided to embark on a formal research undertaking in carrying out improvements to our teaching practice and by doing so; it enhances our awareness of all the activities that contributed to the preparations and implementation of our teaching. We also became much more sensitive to our students' learning behaviour and gradually became better at noticing and recording what we noticed. Definitely, we will be better teachers because of the experience. We chose action research methodology because it encapsulates the process of learning that a teacher herself must go through and acknowledge to be a better teacher. A teacher who is a learner herself will understand and support better learning behaviour in her students.

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