Applying Constructionism and Problem Based Learning for Developing Dynamic Educational Material for Mathematics At Undergraduate University Level

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Abstract

As a result of changes in society and education, assumptions about the knowledge of entrants to university have become obsolete. One area in which this seems to be true is mathematics. This paper presents our research aiming at tackling with this problem by developing digital educational material for mathematics education, which will be student-driven, dynamic, and multimodal. Our approach will be supported by the theories of Constructionism and PBL. The impact of its use will be evaluated in university settings. It is expected that the evaluation will demonstrate an improvement in student engagement and understanding and consequently in student performance.

Keywords: Mathematics, Teaching and Learning, Problem Based Learning, Constructionism, Dynamic;

"In every department of physical science there is only so much science, properly so-called, as there is mathematics." –
Immanuel Kant

1. Introduction

The social and political dimension of mathematics education is generally acknowledged (Valero & Zevenbergen, 2004). Moreover, the importance of mathematics education to other educations, technical and non-technical, is indisputable. In some of these educations (e.g. engineering, media technology, biology), mathematics is not considered as a core subject, but it is still very essential for other core subjects. In this case, mathematics provide a kind of "toolset", which is indispensable in order to deal with challenges of core subjects.

Despite its importance, performance of many undergraduate university students in mathematics education is poor (Timcenko, 2009; Mustoe, 2001; Bialek & Botstein, 2004; Brent, 2004). Various causes of poor performance in undergraduate mathematics are identified in literature (Kessel & Linn, 1996). One of them is the lack of necessary background in the subject, which is deteriorated by the fact that over the last years an increased number of students are taking time out from studies after completing high school - often referred to as a "gap year" (Martin, 2010). A gap year makes the transition from high school to higher education harder and aggregates the so-called "transition problems". Moreover, wider participation in higher education has brought students with very different backgrounds to university classes, transforming them to a really heterogeneous group of people. Finally, changes in the specification of qualifications for high school students have also made traditional assumptions about mathematics knowledge of entrants to university courses obsolete (Kitchen, 1999; Kagesten, 1999; Greene & Foster, 2003).

Poor performance in mathematics has been recognized as one of the main causes of dropout at university level (Akinsola, & Tella, 2007). Difficulties in the exposition and development of mathematical ideas create difficulties in performing well in core subjects and thus developing a sense of general failure in undergraduates. Moreover, performing poor in mathematics lowers self-esteem and increases the anxiety level of students, making them more prone to drop out of university (Bennett, 2003).

This paper presents our ongoing research aiming at tackling with the problem of poor performance in mathematics by developing digital educational material for undergraduate university students. We are particularly interested in undergraduate university studies where mathematics is not a core subject, but it is fundamental for coping with core subjects. We focus on developing dynamic and multimodal material for mathematics teaching and learning, where the level of interaction and the way to present knowledge will be adjusted to student's cognitive ability. This digital material will be used according to a teaching and learning method combining constructionism (Papert & Harel, 1991) and Problem Based Learning (PBL) (Boud, 1998). In order to improve the design of the proposed approach, we are conducting exploratory research, by following a group of students during their mathematics courses. This kind of research will also enable us to better identify students' deficiencies in mathematics.

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Finally, we are currently conducting focus groups with students and interviews with teachers in order to evaluate the effect of our approach on teaching and learning of mathematics at undergraduate university level.

2. Methods

Our research is based on the principle that the development of tools for mathematics education is strengthened in several ways when based on a theoretical perspective. Adopting a theory in teaching or learning of mathematics should be part of an attempt to understand how mathematics can be taught and learned and what an educational tool can do to help in this process. It is not assumed that a theory is a statement of truth. Rather, a theory is used to provide explanations of phenomena that are observed in teachers who are trying to communicate their knowledge in mathematics or in students who are trying to construct their understandings of mathematical concepts and to suggest directions for pedagogy that can help in these processes.

2.1. The Theory of Constructionism

The theory of constructionism is inspired by the constructivist theory that individual learners construct mental models to understand the world around them. According to the principles of constructivism, learning environments should support multiple perspectives or interpretations of reality, knowledge construction, context-rich, experience-based activities (von Gesersfeld, 1991; Dubinsky & McDonald, 2001). However, constructionism holds that learning can happen most effectively when people are also active in making tangible objects in the real world. In this sense, constructionism is connected with experiential learning, and builds on Piaget's epistemological theory of constructivism (Wadsworth, 1996).

Papert defined constructionism in (Papert, 1986) as follows: "*The word constructionism is a mnemonic for two aspects of the theory of science education underlying this project. From constructivist theories of psychology we take a view of learning as a reconstruction rather than as a transmission of knowledge. Then we extend the idea of manipulative materials to the idea that learning is most effective when part of an activity the learner experiences as constructing a meaningful product.*"

Constructionism has been primarily used in science and mathematics education (Papert, 1980). In order to support the constructionist approach to teaching and learning, a number of programming languages have been created (e.g. Logo) (Harel, 1991). Moreover, because of its nature it has been combined with digital technologies in order to implement interactive educational methods (Kafai & Resnick, 1996; Laborde, et al, 2006).

2.2. Problem Based Learning (PBL)

PBL is a student-centered pedagogy in which students learn through the experience of problem solving (Kolmos, Fink, & Krogh, 2004). Learning begins with a problem to be solved, posed in such a way that students need to gain new knowledge before they can solve the problem, and thereby learning both thinking strategies and domain knowledge. The goals of PBL are to help the students develop flexible knowledge, effective problem solving skills, self-directed learning, effective collaboration skills and intrinsic motivation (Hmelo-Silver, 2004).

PBL supports group work. Working in groups, students identify what they already know, what they need to know, and how and where to access new information that may lead to resolution of the problem. This procedure enhances content knowledge while simultaneously fosters the development of communication, problem-solving, critical thinking, collaboration, and self-directed learning skills. PBL may position students in a simulated real world working and professional context which involves policy, process, and ethical problems that will need to be understood and resolved to some outcome. By working through a combination of learning strategies to discover the nature of a problem, understanding the constraints and options to its resolution, defining the input variables, and understanding the viewpoints involved, students learn to negotiate the complex sociological nature of the problem and how competing resolutions may inform decision-making.

PBL represents also a paradigm shift from traditional classroom/lecture teaching. The role of the instructor in PBL (known as the tutor) is to facilitate learning by supporting, guiding, and monitoring the learning process. The tutor must build students' confidence to take on the problem, and encourage the students, while also stretching their understanding. Therefore, the role of the teacher is to guide and challenge the learning process rather than strictly provide knowledge.

PBL was first introduced in the medical school program at McMaster University in Hamilton, Ontario, Canada in the late 1960s (Neville, 2009). Since then various universities and other educational institutes have adopted PBL as a model of teaching and learning. In Aalborg University, Denmark, since its establishment in 1974, all university programs have been based on PBL, also referred to as "PBL - The Aalborg model" (Kjærsdam & Enemark, 1994). The PBL - Aalborg Model has become both nationally and internationally recognized and a trademark for Aalborg University.

2.3. Combining Constructionism with PBL

Our efforts in developing educational material for mathematics take place in a PBL context, namely in the Media Technology program at Aalborg University Copenhagen. The Media Technology program is structured in modules and organized as a problem-based study. A module is a program element or a group of program elements, which aims to give students a set of

professional skills within a fixed time frame specified in ECTS credits, and concluding with one or more examinations within specific exam periods. The program is based on a combination of academic, problem-oriented and interdisciplinary approaches and organized based on work and evaluation methods that combine skills and reflection, such as: lectures, classroom instruction, project work, workshops, exercises (individually and in groups), teacher feedback, reflection.

Mathematics courses in our department are conducted in the form of lectures, which are followed by exercise sessions in groups. The lectures are given by a professor, who follows a transmission teaching model and uses slides projected on the wall for communicating the content of the curriculum. After these lectures, the students have to work in groups in hand-in assignments, based on the notion of PBL group work. The assignments are typical math exercises that are solved by hand. In some cases (e.g. numerical integration, linear algebra), students are asked to hand in a solution in a computer program (i.e. Matlab), along with the by-hand solution. During the exercise time, there are two teaching assistants, who support the students during this process but only when the students ask for it. In order to complete successfully the course, the students have to pass an individual written examination.

Our current research effort focuses on building educational material for transforming assignments for mathematics education in a PBL environment at Media Technology, according to the constructionism stance. Constructionist learning involves students drawing their own conclusions through creative experimentation and the making of social objects. Therefore, we intent to introduce new learning activities, where students can experiment with pre-made digital mathematical visualizations but also are asked to create their own in order to solve course assignments. An important aspect of our research is therefore the combination of PBL group work, where students are considered to be active agents who engage in social knowledge construction, with constructionist activities, where students are building mathematical concepts through development of computer applications.

The combination of constructionism and PBL has resulted in a framework for the use and concrete scope of the proposed material for mathematics teaching and learning. According to this framework, our approach has the following characteristics:

- A part of our educational material presents fundamental mathematical concepts relative to the curriculum at hand in a dynamic way. This material will provide students with opportunities to develop visualization skills, and to explore mathematical concepts.
- A part of our educational material presents partially the solution to mathematical problems and requires the learners (students) to act in order to complete it. According to the principles of constructionism, in this case the students are supported in order to feel that they have ownership of the overall problem.
- Following the PBL approach, the learning activities of the proposed material are related to a larger task. This allows the students to see the connection between activities and thus discover the purpose of their learning.
- The tasks included in our learning activities are in line with students' cognitive ability. In order to achieve this we are currently conducting exploratory research, as it was aforementioned. As constructionism suggests, we aim at increasing the students' self-esteem and making learning valuable.
- Our approach realizes a dynamic educational method by allowing and encouraging the students to test ideas against different views in different contexts. This has been proved to enhance student performance and engagement (Borman & Sleigh, 2011). Students are also encouraged to obtain solutions to self-selected problems, by creating mathematical models and investigating mathematical relations dynamically. This procedure has been proved to help students gaining a deeper insight (Naftaliev & Yerushalmy, 2011).
- The suggested educational material is also multimodal supporting different modes of interaction with its user (teachers and students). We employ auditory-visual interaction between the educational material and its users.
- The proposed material will be used by teachers and it will enable them to implement interactive teaching. A part of this material will be also used by student groups during exercise time.

This framework serves a set of concrete requirements in our design of digital technologies for mathematics education.

2.4. Design

Our research aims at developing dynamic and multimodal digital educational material, which will support teaching and learning of mathematics at undergraduate university level. We are mainly interested in undergraduate university studies where mathematics is not a core subject, but it is very fundamental for performing well in core subjects of the study. For such an approach to be successive, it is important that it takes into account undergraduates' actual deficiencies in mathematics. Therefore, one of our goals is to gain familiarity with the actual background of undergraduates in mathematics and acquire significant insight into their attitude towards mathematics. Furthermore, taking into account the PBL method, we seek information on how students get along in their groups during mathematics problem solving, what meanings they give to their actions, and what issues concern them. We are also interested in investigating social phenomena without explicit expectations.

In order to achieve the aforementioned goals, we are currently conducting exploratory research. Exploratory research is flexible and can address research questions of all types. It will enable us to formulate the requirements framework during the design of the proposed educational material. This framework is needed in order to define the best research design but also the content of our material in relation to general topics of mathematics undergraduate university education. With the purpose of gathering exploratory data, we decided to investigate how the "Mathematics for Multimedia Applications" course is taught for Media Technology students, and observe groups of students taking this course. Media Technology is a study that fits perfectly to

our statement of focusing on studies where mathematics is crucial for mastering core subjects (e.g. "Computer Graphics Programming and Rendering", "Description of Dynamical Input and Output Media Applications").

The "Mathematics for Multimedia Applications" course is given during the second (spring) semester and introduces mathematics needed for media technology applications. More specifically the course covers basic elements of Calculus, Trigonometry, Geometry, and Algebra. At the beginning of the semester we asked for volunteer student groups for collaboration. One group composed of seven students, three females and four males, responded to our request. We are currently conducting overt, direct observations of ten lectures and of ten exercise sessions. These observations span the whole semester. During these observations we are gathering data on individual behaviors of students and teachers (the professor and teaching assistants), interactions between them, and how digital technologies could contribute in improving teaching and learning for this course. These findings will be incorporated in the future steps of our design.

3. Implementation

The field of mathematics has benefited from technology throughout its history. Mathematical tools have advanced from the ancient counting tools to digital technology tools of today. Digital technologies made their appearance in mathematics education in the 1970s. Since then, computers equipped with increasingly sophisticated software, graphics calculators, and web-based applications offering virtual learning environments have changed the mathematics teaching and learning (Oldknow, Taylor, & Tetlow, 2010).

Our scope is to develop digital tools that will be used as educational material (for teaching and learning). With use of this education material, we want to engage students in building their own visualizations of mathematical concepts. In order to enable students to create such kind of visualizations, we use mathematics specific software, i.e. GeoGebra and Matlab. GeoGebra is open source dynamic mathematics software for teaching and learning mathematics from middle school through undergraduate university level (Hohenwarter, Preiner, & Yi, 2007). We chose GeoGebra because it combines the ease-of-use of dynamic geometry software with some basic features of computer algebra systems, and thereby helps to bridge concepts from geometry, algebra, and calculus. Moreover, GeoGebra can be used for active and problem-oriented teaching, which is important for PBL. Finally, in GeoGebra students can easily create constructions from scratch on their own and alter mathematical assignments in order to reach the solution, which is very fundamental to our application of constructionism. For the representation of mathematics in 3D, we use Matlab, which is a high-level language and interactive environment for numerical computation, visualization, and programming. We chose Matlab since it provides built-in 3-D plotting functions, as well as volume visualization functions. Furthermore, it is a tool that students in Media Technology (but also in studies we are focusing on) usually know how to use.

As a first step, we have implemented a set of applets with use of the aforementioned mathematics software that was introduced in the "Mathematics for Multimedia Applications". The applets were used during the lectures by the professor, in order to better explain and visualize mathematical concepts. The same applets accompanied by questions about their content were available to students during their exercise time. In order to answer the questions the students should interact with the applets. An example of such an applet is shown in Figure 1.

The main characteristics of the proposed educational material are based on the basic principles of both constructionism and PBL. Moreover, it presents knowledge to undergraduate university students and engages them in a dynamic and interactive way. The reflection on using our material for teaching of mathematics combined with exploratory research on student background in and attitude towards mathematics will provide us with valuable knowledge in order to improve our design of educational material. Our ultimate goal is to implement and evaluate our approach within a specific university study (Media Technology), while defining a generic framework for implementing digital educational material that can be used to other university studies, where mathematics is not a core subject but it is essential for other core subjects.

4. Evaluation

In order to evaluate the first version of our educational material we are conducting focus group discussions with the group of students we observe during this semester. We chose focus group discussions rather than interviews because focus group discussion produces data and insights that would be less accessible without interaction found in a group setting. Listening to others' verbalized experiences stimulates memories, ideas, and experiences in participants, which could not be achieved through a one-on-one interview. During these sessions one of our research team moderates the focus group discussion on students' experience with specific course assignments.
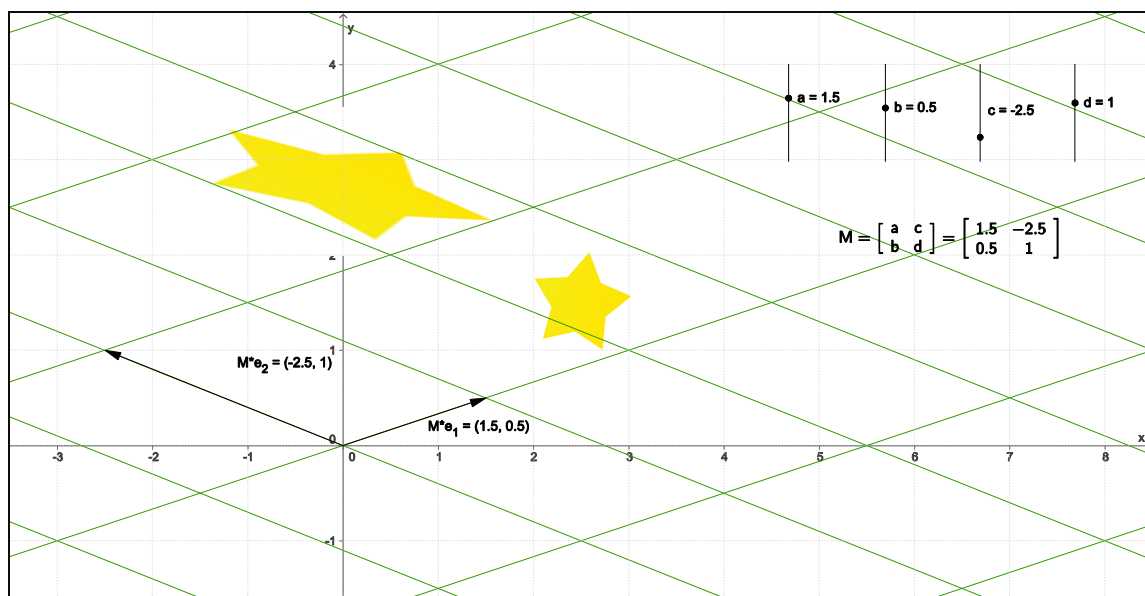
Figure 1. Visualizing Linear Transformations:
Students can change the matrix of the linear transformation by using the sliders, and observe how the grid and image are transformed

Moreover, we are conducting interviews with the professor and the teaching assistants involved in the "Mathematics for Multimedia Applications" course. Although teaching assistants play a subsidiary role in course, we believe that their contribution to our evaluation is valuable. By having the opportunity of supporting the students while they are working on their assignments, they get insight into which parts of the curriculum are challenging for students. Moreover, the current assistants have been also students in Media Technology and therefore have an overall perception of both teaching and learning of mathematics in this study. During these interviews we aim at getting feedback on how students worked in their assignments and whether and how they used the developed applets. Based on this feedback, we will be able to crosscheck the data we are gathering during the focus group discussions with students. Moreover, we seek proposals on improving our intervention based on their experience and observations.

5. Conclusion

The research presented in this paper will result in an educational model for mathematics teaching and learning, which will be evaluated in an undergraduate university setting. On one hand, we expect that the introduction of this material in teaching practice will enable teachers to combine constructionism pedagogical ideas with PBL, to teach mathematics in a wider context, and to adopt a student based strategy. The dynamic aspect of our approach will also contribute to teachers communicating their knowledge in a more effective manner. On the other hand, it is expected to help students assimilate mathematical knowledge and overcome their deficiencies regarding mathematics. An important aspect of our approach is that the level of interactivity of the developed educational material should be adjusted to university undergraduate students' cognitive ability. If the learning tasks evolve fast and are complicated compared to students' abilities, then they would not have any added value. On the other hand, if learning activities evolve slow or are trivial, students would easily lose interest. For defining the challenges that students face in mathematics, we are conducting observations in actual university classes, and interviews with teachers and students. This will also enable us to gain insight on problematic areas and adapt our research on dealing with them. Finally, we are able to develop student based solutions, which is very crucial if one wishes to improve student engagement and performance.

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