BLENDED LEARNING ENVIRONMENT TO DEVELOP PERSONAS AND THEMES IN ENGINEERING STUDENTS USING MATHEMATICAL ORIENTED ACTIVITIES

AISHA MAHMOOD

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Engineering Education)

School of Graduate Studies
Universiti Teknologi Malaysia

AUGUST 2015

THESIS STATUS DECLARATION LETTER

August 2015

Librarian
Perpustakaan Sultanah Zanariah
UTM, Skudai
Johor

Sir/Madam,

CLASSIFICATION OF THESIS AS RESTRICTED

Blended Learning Environment to Develop Personas and Themes in Engineering Students using Mathematical Oriented Activities

AISHA MAHMOOD

Please be informed that the above mentioned thesis entitled *"Blended Learning Environment to Develop Personas and Themes in Engineering Students using Mathematical Oriented Activities"* be classified as RESTRICTED for a period of three (3) years from the date of this letter. The reasons for this classification are

(i) Unpublished Data

(ii) Unpublished Findings and Results

Thank you.

Sincerely yours,

Dr Mohd Fauzi Othman
Senior Lecturer,
Center For Artificial Intelligence and Robotics (CAIRO)
Universiti Teknologi Malaysia
Jalan Semarak, 54100
Kuala Lumpur
Tel: +603-26913710
Hp: +60197120085
Fax:+603-26970815

DEDICATION

To

My Dearest and Nearest Aunt Ms. Koukab Tasnim Butt,

My Parents,

My Lovely Kids, Faaiz Khan and Rida Fatima

And

All My Wonderful Friends & Family

ACKNOWLEDGEMENT

In the name of Allah, The Most Gracious and The Most Merciful

All praise and thanks are for Allah who has made this journey interesting and meaningful for me. This study has been made possible through the cooperation, support, and help of several people and departments. I express my sincere gratitude especially to my supervisors, Dr. Mohd. Fauzi Bin Othman from the Faculty of Electrical Engineering (FKE), and Associate Professor Dr. Yudariah Mohammad Yusof from the Faculty of Science, Department of Mathematical Sciences. Their continuous guidance, patience, support, and encouragement over the years are deeply appreciated. I also acknowledge the financial assistance for this research by the MJIIT at the Universiti Teknologi Malaysia in the form of Research Student Grant. The Centre for Engineering Education provided a platform for a continuous professional development and knowledge construction throughout my research journey.

I also want to thank the Director of the Centre for Engineering Education, Associate Professor Dr. Khairiyah Mohd Yusof who helped me interact with pioneers in the engineering education and provided me with the opportunity to expand my vision and helped me in changing my perception about engineering education. I also want to thank Dr. Fatin, Dr. Helmi, Dr. Fadzil, Dr. Narina, Dr. Dayang, Dr. Hafiz, Dr. Roselainy, Dr. Aminah, Dr. Zaleha and Dr. Ismail Said for their critical review and constructive feedback throughout my research studies.

I would also like to acknowledge the encouragement given to my research by my mentors from US, Monica Cardella (Purdue), Lisa Benson (Clemson University), and Rebecca Bates (Minnesota State University), and my peer researchers from US especially Dr. Farrah Fayyaz (Purdue) who have all helped me make sense of my research struggles and, also, Dr. Jennifer Turns (University of Washington) who helped me make sense of my reflective practice and guided me how to improve it.

I am especially indebted to the research assistants who helped me in collecting the data, transcribing the videos, and digitizing all the research documents. I also thank all the students of the first year engineering program who participated in this study.

I would also like to thank all my peer researchers, lecturers, assistant professors, and professors who were always there with their constructive feedback to help me improve the quality of this research.

In the end, I would like to thank all my friends and family for their unconditional love and support throughout this journey.

ABSTRACT

There is not only an emergent need to implement innovative pedagogies but also to understand in more depth what actually happens in engineering classrooms and how to accelerate the rate at which research on students provides influence on teaching practices. The growing trend in higher education based on previous studies, highlighted the potential of blended learning in supporting mathematical thinking among fresh engineering students. This research is designed to develop and implement a blended learning environment using a well-practiced problem solving strategy integrated with selected MIT-BLOSSOMS modules and investigated its implications by developing student personas and emergent themes of engineering students. The study starts by knowing the students, their current knowledge state and what they have already experienced relating to mathematical thinking and learning. A web-based, artificially intelligent Assessment and Learning in Knowledge Spaces (ALEKS) system is used to know the students' current knowledge state. Classroom observations and focus groups were used to investigate the emergent themes whereas written activity responses were analyzed to show the activation of mathematical thinking processes in conceptual embodiment and operational symbolism. Findings highlight the emergent themes of met-befores, met-afters, implications of blended learning and challenges whilst problem solving. The results show that blended learning can support "horizontal mathematization" during problem solving activities by manipulating students' conflicting met-befores, increasing their diligence during problem solving and improving student-teacher relationship. The student personas are developed as a potential pedagogical tool to communicate the vital research findings to the Community of Practice (CoP) and have the potential to develop empathy among engineering educators. This research is transferable and replicable to tertiary as well as secondary education by modifying the blending options on the spectra of time, space, technologies, pedagogy, format, courses, participants and complexity of the problem solving activities accordingly.

ABSTRAK

Terdapat keperluan yang berkaitan dengan pelaksanaan pengajaran inovatif untuk memahami dengan lebih mendalam apa yang sebenarnya berlaku di dalam kelas kejuruteraan dan bagaimana untuk mempercepatkan kadar di mana kajian mengenai pelajar memberi pengaruh ke atas amalan pengajaran. Kadar peningkatan yang semakin meningkat dalam pendidikan tinggi berdasarkan kajian sebelum ini, menekankan potensi pembelajaran digabungkan dalam menyokong pemikiran matematik di kalangan bakal pelajar kejuruteraan. Kajian ini bertujuan untuk membangunkan dan melaksanakan persekitaran pembelajaran yang digabungkan dengan menggunakan penyelesaian masalah strategi yang diamalkan, disepadukan dengan modul MIT-BLOSSOMS telah dipilih dan disiasat implikasinya bagi membangunkan aktiviti yang berorientasikan penyelesaian masalah dalam pemikiran matematik. Kajian utama dimulakan dengan mengenali pelajar, mengetahui keadaan pengetahuan semasa pelajar dan memahami apa yang telah para pelajar pelajari berkaitan dengan pemikiran dan pembelajaran matematik sistem pintar berasaskan sesawang yaitu Pentaksiran dan Pembelajaran dalam Ruang Pengetahuan (ALEKS) digunakan untuk mengetahui keadaan pengetahuan semasa pelajar. Pemerhatian di dalam bilik darjah dan kumpulan sasaran digunakan bagi mengenal pasti faktor-faktor yang menyumbang kepada pembentukan karakter pelajar, manakala tindak balas bertulis dari pelajar dianalisa bagi mengetahui kadar pemahaman dan proses pemikiran matematik pelajar dalam bentuk konsep dan simbolik. Penemuan kajian mengetengahkan faktor-faktor yang menyumbang kepada pembentukan karakter pelajar adalah berdasarkan faktor *met-befores*, *met-afters* dan implikasinya kepada pembelajaran dicampur dan cabaran manakala penyelesaian masalah. Hasil kajian menunjukkan bahawa pembelajaran dipadukan boleh menyokong *'horizontal mathematization'* semasa aktiviti penyelesaian masalah dengan memanipulasikan konflik *met-befores* pelajar, meningkatkan ketekunan mereka semasa menyelesaikan masalah dan memperbaiki hubungan guru dan pelajar. Personaliti pelajar dibangunkan sebagai alat yang berpotensi untuk menyampaikan hasil penyelidikan penting kepada Komuniti Amalan (CoP) dan mempunyai potensi untuk membangunkan pemahaman dan rasa untuk dikongsi di kalangan pendidik kejuruteraan. Kajian ini boleh dipindah milik dan boleh diulangi untuk pengajian tinggi dan juga pendidikan menengah dengan mengubah pilihan pengadunan pada spektrum masa, ruang, teknologi, format, kursus, peserta dan kerumitan masalah aktiviti menyelesaikan sewajarnya.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

ALEKS	-	Assessment and LEarning in Knowledge Spaces
AR	-	Action Research
BL	-	Blended Learning
BLOSSOMS	-	Blended Learning Open Source Science Or Math Studies
CoP	-	Community of Practice
EER	-	Engineering Education Research
F2F	-	Face-to-Face
ITL		Inductive Teaching and Learning
MIT	-	Massachusetts Institute of Technology
MOE	-	Ministry Of Education
MOOC	-	Massive Open Online Course
MTL	-	Mathematical Thinking Lab
OER	-	Open Educational Resource
PS	-	Problem Solving
PSA	-	Problem Solving Activity
P12	-	Preschool to Grade 12 (Equivalent to Secondary Education)
RQ	-	Research Question
RU	-	Research University
STEM		Science, Technology, Engineering and Mathematics
UTM	-	Universiti Teknologi Malaysia

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

There is an emergent need associated with the implementation of innovative pedagogies to understand in more depth what is actually happening in engineering classrooms in the context of new learning environment. Before making in-depth inquiries, it is needed to know “what knowledge, skills, and attitudes do learners bring to their engineering education that influences what (and how) they learn in a new learning environment?” and then “how do learners progress from naïve conceptions and partial understandings to richer knowledge and skills that facilitate innovative thinking?” (EERC, 2006). It is further needed to comprehend the emerging themes in a new learning environment besides knowing the variance of knowledge, skills, and attitudes of engineering students in different scenarios/situations (EERC, 2006).

Following the trend of blended learning in engineering education, first an interpretive action research is selected from the pool of emerging methodologies in engineering education research, and then employed in this study. A blended learning environment is developed and implemented for developing student personas and emergent themes using mathematical thinking oriented context-rich problem solving activities for first year engineering students. Mathematical thinking oriented problem solving is an essential component in the skill set required for future engineers (Broadbridge and Henderson, 2008). Sometimes, engineers join the workplace with inadequate mathematical thinking and problem solving skills. That is because the teaching emphasis is on content mastery rather than learning mathematical thinking processes and problem solving strategies (Alpers, 2010; Cardella, 2007a; Ferri, 2012;

Gainsburg, 2006; Goold and Devitt, 2013; Trevelyan, 2009). To deal with the above issue, a well-practiced evidence-based problem solving strategy by Mason et.al. (2010) is first integrated with selected MIT-BLOSSOMS modules to develop problem-solving activities followed by their implementation to create a blended learning environment. Mixed-ability students practiced the problem solving strategy to solve the context-rich problems in collaborative groups. The research process is assisted by understanding the human innate abilities to think mathematically, knowing the students, their current knowledge state and their prior experiences related to mathematical thinking and then implementing a blended learning environment conducive to context-rich problem solving. The action research is conducted followed by monitoring and evaluating the activated mathematical thinking processes and resulted in some interesting and emergent themes during this study. Blended learning in this research successfully activated embodied mathematical thinking processes thus supported students in horizontal mathematization and affected students' met-befores in a supportive way. The instructional approach not only evidenced the improved problem solving skills of the students at all ability levels but also the improved engagement of all the students. One of the main outcomes is the evidence-based student personas presented as a potential pedagogical tool to transfer implications of this research to Community of Practice (CoP) that includes engineering and mathematics faculty, junior researchers and postgraduate students. The trajectory of the practitioner as a researcher is also captured through detailed descriptions that will be a valuable contribution towards bridging the research and practice gap through this research. The narratives during the transformation from practice to praxis showed struggles of the researcher in the way to become a reflective teacher and action researcher. This research also has the potential to make impact on P12 (secondary) engineering education by reporting the status of mathematical thinking and problem solving skills of the students leaving P12 (secondary education) and joining engineering education. It is thus suggested to revamp the instruction at secondary level to help students in entering the engineering program with adequate skills (Tolbert and Cardella, 2013).

This chapter will further provide the background of the problem, statement of the problem, research objectives, research questions, importance of the study, and thesis outline. In the next section, the background of the study is described in the context of engineering education.

1.2 Background of the Problem

The developing knowledge on effective teaching and learning, evolving social and global needs, and sprouting intents and anticipations of stakeholders make it vital that we change the way we educate our future engineers (Siddiqui, 2014). Engineering expertise of a civilization always maintained its significance for a sustaining modern economy and its progress towards future advancements, whereas the inclination towards engineering as a career has diminished in Western as well as in Eastern countries (Becker, 2010; Elliott, 2009; Forfás, 2009; King, 2008; McKinsey, 2011; Organisation for Economic Co-Operation and Development, 2010).

In recent times, the emerging concern to drive the efforts for improving the science, technology, engineering, and mathematics (STEM) education has become wide-ranging from under-representation of minorities and issues of high attritions of students from STEM majors to the broader problems related to the quality of education and the shifting emphasis from teacher-centered to learner-centered (Adams et al., 2011; Seymour, 2002). In the new century, there is an utter need to train and equip engineers in such a way that they can function effectively in an altering context of the engineering profession (Sheppard et al., 2008). Technological advancements and rapidly changing global economy with their associated challenges resulted in engineers working globally (Lynn and Salzman, 2009). The major change in the culture of how people think, act, and perceive their roles professionally and personally is essential to address the sustainability challenge (Sterling, 2004).

Traditional ways of engineering education are not aligned with today's needs for training engineers (Duderstadt, 2010). Tomorrow's engineers should be more flexible and creative to address the changing world demands and that is only possible through the transformation of engineering education (Bransford, 2007; Chubin et al., 2008; Duderstadt, 2008; EERC, 2006; National Academy of Engineering, 2005; National Science Foundation, 1995). The engineering curricula and teaching and learning practices need to be changed to attract and retain students with diverse talents and backgrounds in engineering education, for providing engaging learning experiences to the students and to prepare them for work in the new realisms (Siddiqui, 2014).

Goold and Devitt (2013) also shared similar concerns specific to the role of mathematics for engineering education. It is also highlighted that practising engineers use broader mathematical thinking rather than what they have been taught through the syllabus (Alpers, 2010; Cardella, 2007a; Gainsburg, 2006; Goold and Devitt, 2013; Trevelyan, 2009). Moreover, it is evident that major engineering practices depend on the engineers' mathematical thinking skills, like contextual and prior experiences, reasoning and justification of inferences, and designing new solutions (Gainsburg, 2006). Problem solving, including working collaboratively on complex problems, critical thinking, complex data analysis, numerical reasoning, and appropriate applications of technology are valuable for employers (English 2002).

The literature is reviewed on various efforts in improving the mathematical thinking and problem solving skills among engineering students (Abdul Rahman and Mohammad Yusof, 2008; Abdul Rahman and Mohd Yusof, 2002; Abdul Rahman, 2008, 2007; Abdul Rahman et al., 2010, 2007, 2005; Baharun et al., 2008, 2007; Borovik and Gardiner, 2006; Broadbridge and Henderson, 2008; Ismail and Kasmin, 2008; Kashefi, 2012; Mohammad Yusof and Abdul Rahman, 2004, 2001; Mohammad Yusof and Tall, 1999; Mohammad Yusof et al., 1999). The previous studies highlighted the difficulties of engineering students in manipulating concepts, coordinating multiple procedures, manipulating symbols in a flexible way, answering non-routine problems, lacking problem solving skills, and the students' inability to select and use appropriate mathematical representations. Therefore, there is still a room to develop learning environments conducive to mathematical thinking and problem solving at undergraduate level (Bergsten, 2007) and addressing the low level of engagement in the classroom (Fritze and Nordkvelle, 2003; Smith et al., 2005).

The persistent gulf between research and practice (Finelli et al., 2014; Fink et al., 2005; Smith, 2000; Turns et al., 2013) has also become a major concern in engineering education research. Therefore, future research should not only focus on exploring the emergent themes during an innovative classroom practice to foster the mathematical thinking skills among future engineers but should also devise an effective way to minimize the research-practice gap. In the next section, the researcher formulated a problem statement by focusing on the research gaps from related literature and by following the trend of blended learning environment and by evaluating the needs and demands of engineering education research.

1.3 Statement of the Problem

Keeping in view the perspective "the evolving challenges facing engineers, and how engineering education must adapt to suit these needs" (Fortenberry, 2006), "the engineering profession is calling for new and better kinds of learning by engineering students. Accomplishing this, requires new and better kinds of teaching and curricula, which in turn requires engineering faculty to think about teaching and learning in more scholarly ways" (Fink et al., 2005). It is also needed to "get on with the task of making deep and solid inquiries into learning processes, using the best methods we can bring to bear to advance scientific knowledge and understanding of learning from the variety of research perspectives that are available" (Anderson et al., 2000). Moreover, "the emergence of a new research trend that attempts to develop better understanding of the nature and processes of teacher change and the factors that affect these processes" (English, 2002) should also be in focus.

During the transition from secondary education (P12) to engineering education, students are expected to be equipped with adequate mathematical thinking skills so that they can undergo rigorous design thinking processes afterwards (Tolbert and Cardella, 2013). However, the lack of resources and didactic teaching during P12 (secondary education) hinder their development of mathematical thinking processes and thus students join engineering programs with insufficient mathematical thinking skills (Mahmood et al., 2012). On top of that, the similar methods of teaching mathematics at tertiary level stress on the content of mathematical theory rather than the motivations and thoughts that underlie this content (Mamona-Downs and Downs, 2002). Moreover, a disconnection perseveres between "theories of individual mathematics learning" and the "teaching and learning practices in the classroom" Kress (2011b). Kress (2011a) also argued that "explorations around what happens in the black box of mind have not fully resolved the daily problems faced by students and teachers" in the real classroom whereas, Goos, Galbraith, and Renshaw (2002) emphasize that, "given our incomplete understanding of mathematical thinking, we need further research on mathematics learning in authentic environments before continuing to make changes in the classrooms." Kress (2011a, p. 194) specifically mentioned that more research is needed to improve teaching "practice of mathematics by exploring the social dimension of learning (which complements theories that explain individual cognitive processes)." That is a way to "develop better curriculum materials, refine pedagogy, and improve the structuring of classroom environments." Serious considerations should be given to find ways to enhance the process of mathematical thinking, even if some sacrifice in content may be needed to achieve this (Mamona-Downs and Downs, 2002). The technological advancement and educational research have also developed to a level that raise a demand to introduce the emerging strategies and techniques of teaching and learning even at first year in an engineering program. Students should learn more what is presently customary the "process of mathematical thinking" rather than the "product of mathematical thought" (terms borrowed from Skemp, 1971 as cited by English, 2002)**.**

Some local researchers have also attempted to enhance engineering students' mathematical competency through mathematical thinking (Baharun et al., 2007), enhance mathematical thinking through active learning in engineering mathematics (Abdul Rahman et al., 2007), change teacher and student's attitudes towards calculus through mathematical thinking (Abdul Rahman, 2008), recognize a student's struggle through mathematical knowledge construction (Abdul Rahman et al., 2005), translate learning theories into practice in enhancing a student's mathematical learning (Abdul Rahman, 2007), change attitudes towards university mathematics through problem solving (Mohammad Yusof and Tall, 1999), facilitate thinking and communication in Mathematics (Baharun et al., 2008), cultivate mathematical thinking in differential equations through a computer algebra system (Zeynivandnezhad, 2014) and employ blended learning to cultivate mathematical thinking in multivariable calculus (Kashefi, 2012). Various issues and challenges emerge from the above research initiatives, such as different students' learning styles, their typical beliefs and attitudes, insufficient prior knowledge, insufficient problem solving skills, inappropriate selection and use of mathematical representations, poor conceptual knowledge, poor symbolic manipulation skills and difficulties in answering non-routine problems. Some other researchers reported issues like exam-oriented culture, insufficient assessment methods, lack of resources, and the minimal role of technology in supporting mathematical thinking (Rahman et al., 2012a, 2012b; Tall, 1998). However, the optimal ways to improve students' mathematical thinking and problem solving skills are not well understood yet. Many instructors and commentators place the poor performance of fresh engineering students in problem solving to a deficit of knowledge base and/or conceptual understanding in mathematics (Gupta and Elby, 2011). The future recommendations are to use pedagogical and technological tools to improve problem solving and mathematical thinking skills in new learning environments (Bersin, 2004; Bourne et al., 2005; Garrison and Vaughan, 2008; Graham and Dziuban, 2008; Güzer and Caner, 2014; Inoue, 2010; Kaur, 2013; Picciano, 2007).

Understanding the underpinning human abilities to think mathematically, knowing the students' current knowledge state and their prior experiences related to mathematical thinking (Tall, 2013), are the key factors that need to be understood before understanding how future engineers learn to think mathematically. The traditional learning environments are not supportive for mathematical thinking and problem solving due to the lecture based teaching of mathematics at undergraduate level (Bergsten, 2007). Instead of active learning, the students are passive learners with low level of engagement in the class (Fritze and Nordkvelle, 2003; Smith et al., 2005). Therefore, mathematics is viewed as a non-creative subject with minimum social engagement and collaboration (Alsina, 2002; Weber, 2004), less affective and non-supportive to higher-order thinking (Breen and O'Shea, 2011; Dubinsky and McDonald, 2001; Leron and Dubinsky, 1995). However, by providing a new environment for learning to cultivate mathematical thinking explicitly, the in-depth understanding is needed, of what actually happens, specifically when innovative pedagogies are implemented in the real engineering classrooms (Light and Case, 2011).

The one end of continuum of mathematical thinking and learning practices is a didactic or constructive way of teaching in the classroom and the other end is “a synchronous broadcast model” (Bourne et al., 2005) so that lectures can be viewed immediately or recorded for future playback. Same level of interaction as in typical classrooms can be achieved through synchronous online systems. However, it is more difficult to implement constructivist approaches (Bourne et al., 2005) to implement in the fully online practices supporting mathematical thinking and its associated challenges (Rahman et al., 2012a, 2012b; Sam and Yong, 2006; Tall, 1998). Some researchers (Bourne et al., 2005) predicted that the online education and traditional on-campus education will become more blended or integrated to entertain factors like time, space, attitude, disparity in knowledge, and cognitive demands whereas Kashefi (2012) suggested the use of blended learning for engineering mathematics to support the mathematical thinking of new students joining engineering education. Bridging research and practice in engineering education can also help the engineering educators to advance their research in the guided direction to fulfil the futuristic workplace demands. The potential of blended learning to activate mathematical thinking processes during context-rich problem solving activities should be investigated to inform the scholarship of teaching (Harun, 2012; Hull et al., 2013; Kashefi et al., 2013, 2012; A Mahmood et al., 2013; Mohammad Yusof et al., 2012; Sam et al., 2009) and to develop new pedagogical tools like student personas to bridge research-practice gap and improve teaching practices (Adlin and Pruitt, 2010; Elliott, 2005; Faily and Flechais, 2011; Goodwin, 2008; Grudin and Pruitt, 2003; Nielsen, 2013; Turns et al., 2013; Wikberg Nilsson et al., 2010). However, the lack of framework persists in developing and implementing blended learning for supporting mathematical thinking. We also have insufficient knowledge of what themes would emerge and how differently students learn in different teaching and learning scenarios.

The driving force in conducting this research is to investigate the potential of blended learning to develop student personas and emergent themes while supporting mathematical thinking processes besides developing problem solving expertise among first year engineering students. This empirical research will get the insights of new learning experiences of first year engineering students during their context-rich problem solving activities utilizing open educational resources. The emergent themes and student personas while activating the mathematical thinking processes during problem solving activities through blended learning will guide the practitioner how to improve further or influence future teaching and learning experiences, in turn, improving the mathematical thinking skills among prospective engineers.

In short, by implementing innovative pedagogies in the real engineering classrooms through blended learning to support mathematical thinking of prospective engineers during problem solving activities, the in-depth investigations in the form of emergent themes are essential of what actually happens during the new learning experience. It is also required to develop the engineering students' personas as potential pedagogical tools to accelerate the rate of translating the research into practice.

1.4 Research Objectives (ROs)

The following are the research objectives of this study:

1. To develop and implement a blended learning environment using mathematical thinking oriented problem-solving activities.
2. To develop engineering students' personas and emergent themes while investigating the implications of blended learning on students' mathematical thinking during problem solving activities.

1.5 Research Questions (RQs)

This research is conducted to answer the following questions:

1. What knowledge (mathematics), skills (mathematical thinking and problem solving) and prior experiences do students bring along that influence how they learn to think mathematically in a blended learning environment?
2. What would be the process to develop, and implement a blended learning environment that incorporates a well-practiced problem

solving strategy and a pedagogical tool supporting engineering students' mathematical thinking, learning, and problem solving skills?

3. What are the emergent themes translating into the implications of the blended learning on the students' mathematical thinking and learning during problem solving activities?
4. What would be the process to develop the students' personas to describe archetype students in different scenarios (the Classroom and the MTL) and illustrate the activation of their mathematical thinking processes in embodied and symbolic world of mathematics?

1.6 Importance of the Study in the context of Engineering Education

The importance of this study is highlighted by relating the ROs and RQs with respective engineering education research areas and strands of inquiry as shown in Table 1.1.

The educational importance will be achieved by not only developing and implementing but also unfolding the potential of blended learning to improve the current mathematical thinking and problem solving skills among prospective engineers. The pragmatic importance is related to utilizing and/or producing innovative ideas, resources, and tools to introduce and encourage non-traditional teaching methods in engineering mathematics classroom, and to improve a practitioner's learning about her own practice involving integrating, implementing, testing, and disseminating such materials and methods. The professional importance is emphasized by welcoming assistance and cooperation from our colleagues from mathematics education, and to work with them in an open, inclusive, collaborative, and practice-based research environment to improve the overall quality of engineering education and to inform the Community of Practice (CoP) on how to use the student personas as pedagogical tool in their own complex contexts.

From this study, the engineering educator-cum-researcher will have the opportunity to extend her existing professional development experiences to further meet the engineering educator’s needs. That would also help her to draw “future recommendations” for refined pedagogy, improved curriculum materials, and the structuring of the classroom environment to fulfil the needs of first year engineering students in helping them to become better mathematical thinkers.

Table 1.1: Importance of the study in the context of engineering education by relating the ROs and RQs with respective engineering education research areas and strands of inquiry (EERC, 2006)

RO	RQ	EER Area	Strand of Inquiry
RO1: To develop and implement a blended learning environment using mathematical thinking oriented problem solving activities ***(RO1 is targeted in the Chapter #4 of this thesis)***	RQ1: What knowledge (mathematics), skills (mathematical thinking and Problem solving) and prior experiences do students bring along that influence how they learn to think mathematically in a blended learning environment?	Area 2: Engineering Learning Mechanisms	Knowing our Students (the variance of knowledge, skills, and attitudes of students) [What knowledge, skills, and attitudes do learners bring to their engineering education that influences what (and how) they learn?]
	RQ2: What would be the process to develop, and implement a blended learning environment that incorporates a well-practiced problem solving strategy and a pedagogical tool supporting engineering students' mathematical thinking, learning, and problem solving skills?	Area 3: Engineering Learning Systems	Designing (Developing and implementing) learning environments Teaming and Collaborative Learning
RO2: To develop engineering students' personas and emergent themes while investigating the implications of the blended learning on students' mathematical thinking during problem solving activities ***(RO2 is targeted in the Chapter #5 of this thesis)***	RQ3: What are the emergent themes translating into the implications of the blended learning on students' mathematical thinking and learning during problem solving activities?	Area 2: Engineering Learning Mechanisms	The learning progressions (trajectories) of learners and their educational experiences that develop knowledge, skills and identity necessary to be an engineer. [How do learners progress from naïve conceptions and partial understandings to richer knowledge and skills that facilitate innovative thinking?]
	RQ4: What would be the process to develop the students' personas to describe archetype students in different scenarios (the Classroom and the MTL) and illustrate the activation of their mathematical thinking processes in embodied and symbolic world of mathematics?	Area 2: Engineering Learning Mechanisms	The variance of knowledge, skills, and attitudes of students in different scenarios (situations)

1.7 Operational Definitions

The following terms and constructs have specific meaning in this thesis as given below:

Action Research: is a form of self-reflective problem solving, which enables practitioners to better understand and solve pressing problems in educational settings. The action (what you do) aspect of action research is about improving practice. The research (how you learn about and explain what you do) aspect is about creating knowledge about practice. The knowledge created is the knowledge of one's practice (McNiff and Whitehead, 2010).

Blended learning: is the integration of online with face-to-face learning in the form of mathematical thinking oriented problem solving activities in a planned, pedagogically valuable manner.

Community of Practice: is a group of people sharing similar problems, concerns, and passion about a topic who interact with each other on regular basis to improve their knowledge base and expertise in the related area (Wenger et al., 2002).

Constructivist teaching: is based on the conjecture that learning occurs if students are actively engaged in their knowledge construction. The role of the teacher is that of 'guide on the side' and a facilitator during that learning (Heinze, 2008).

Constructivism: "recognizes that knowledge construction about the social world and ourselves is reliant on human perception, convention, and social experience rather than just reflecting an external reality (Elliott, 2005).

Didactic teaching: occurs when knowledge is 'imposed' on the learner. The role of the teacher is that of the 'sage on the stage'. It is opposite of constructivist teaching" (Heinze, 2008).

Empathy: is the feeling as a result of understanding and sharing another person's emotions and experiences. "It is a basic process of social observation, where whatever observed are purposive actions rather than raw physical objects and behaviour from which action is inferred (Elliott, 2005). In this research, the empathy is not just a feeling, rather it is a skill to effectively participate in teaching and learning practices.

Face-to-face: is a mode of interaction between individuals in an environment based on their physical presence. So the body language and other non-verbal communication clues can serve as an effective way that interaction (Heinze, 2008).

Learning: is an enduring change in behavior, or in the capacity to behave in a given fashion, which resulted from practice or other forms of experiences (Schunk, 2012, p. 3).

Mathematical Thinking: is a specialized function distinctive from generalized thinking. It is best seen as a continuous, cyclical process of cognition in which a person strives to make sense of a vast sea of sensory data, map the mathematical world, attend to social convention while coping with individual differences in the beliefs of every mathematical thinker and finally extending his/her choices.

Met-after: is a new structure that students will develop in their brains as the effect of new experience of blended learning related to mathematical thinking, learning and problem solving.

Met-before: is a current structure that students have in their brains as a result of experiences they have met before related to mathematical thinking, learning and problem solving.

Nodes: are used to conceptually represent codes during the process of data analysis using QSR NVivo 10 software program (Heinze, 2008).

Node tree: is a tree hierarchy showing the logical composition of nodes in the NVivo. Root of the tree is placed at the top in the tree node diagrams as used in this study. An automatically assigned unique number in QSR NVivo software identifies a node. For example if a node is located within the third tree, seventh branch, tenth twig and fourteenth leaf then its node number will be (3 7 10 14) (Heinze, 2008).

Pedagogy: is an art and science of teaching based on specific assumptions related to learning processes.

Persona: is an evidence-based description of a person within the context of Engineering Mathematics I Class and the Mathematical Thinking Lab (MTL), whose pertinent characteristics and challenges are of importance in this research. The use of personas is said to be a human behaviour, based on the presumption that first year engineering students join engineering education along with their prior experiences that can be either supportive or conflicting in learning new concepts and skills in different and new learning environments.

Sense making: is developing understanding of a situation, context, or concept by connecting it with existing knowledge. (NCTM, 2009)

1.8 Thesis Outline

This section will outline the details of all the chapters. Figure 1.1 also elicits the whole research process in terms of constituent components and their placement in this thesis under respective chapters.

Chapter 1: Introduction

In this chapter, the researcher started with the introduction of this research and described what, why and how this research is needed to be conducted. Then the background of problem, statement of problem, research objectives, research questions, and importance of the study are discussed.

Chapter 2: Literature Review

After introducing the chapter, the researcher explained the role of mathematics in engineering education. The key concepts and ideas are then discussed under the headings of blended learning, mathematical thinking, and mathematical thinking as problem solving. Then the researcher explained HPL meta-framework followed by the theoretical framework adapted from the three worlds of mathematical thinking for this research. Before describing the research paradigm and methodology considerations, a brief introduction of student personas is also provided.

Chapter 3: Research Methodology

The researcher introduced the chapter followed by a comparison of her philosophical assumptions with different research paradigms. The qualitative research process comprising epistemology, theoretical perspective, and methodology are then discussed. After rationalizing the choice of the action research methodology, its cycle and process are described, followed by the data types and data collection techniques. Then the researcher explained the research paradigm-implementation process, the research setting, the participants of the research, the researcher's background, and the research method-implementation process. The data collection, the two staged data analysis is then discussed followed by the integration process of problem solving strategy with BLOSSOMS modules to develop a blended learning environment conducive to mathematical thinking. The persona development process is then described followed by their problem solving activity response analysis. The discussion is closed by presenting the way in which the quality of the research is addressed.

Chapter 4: Developing and Implementing Blended Learning Environment

After introducing this chapter, the initial idea of the research, reconnaissance, and initial planning followed by preliminary action research cycle and pilot action research cycles 1 and 2 are described in detail. The researcher then described the details of "knowing the respondents" and "knowing their current knowledge state" in the main study. The initial diagnosis and discussion followed by the description of the main action research cycles I and II are given in detail.

Chapter 5: Emergent Themes and Student Personas

In this chapter, introduction is followed by emerging themes of this research. Students' met-befores and the challenges whilst problem solving are first discussed. Then the impact of blended learning as students' met-afters, diligence during mathematical problem solving and student teacher relationships are discovered and reported. The evidence-based students' personas are then discussed followed by the scenarios for problem analysis and idea development. The modified rubric to assess the activation of mathematical thinking processes based on pre-identified deductive coding scheme is then discussed followed by written activity response analysis of selected personas. The discussion is closed by presenting the results of problem solving activity response analysis for all the personas.

Chapter 6: Discussions

Introduction is followed by discussions in accordance with the research objectives and questions. Making sense of researcher's reflective practice, challenges faced during the study; and limitations and delimitations of the study are also discussed in this chapter.

Chapter 7: Conclusions and Future Recommendations

After drawing the conclusions, the implications of this research and future recommendations are presented in this last chapter.

.

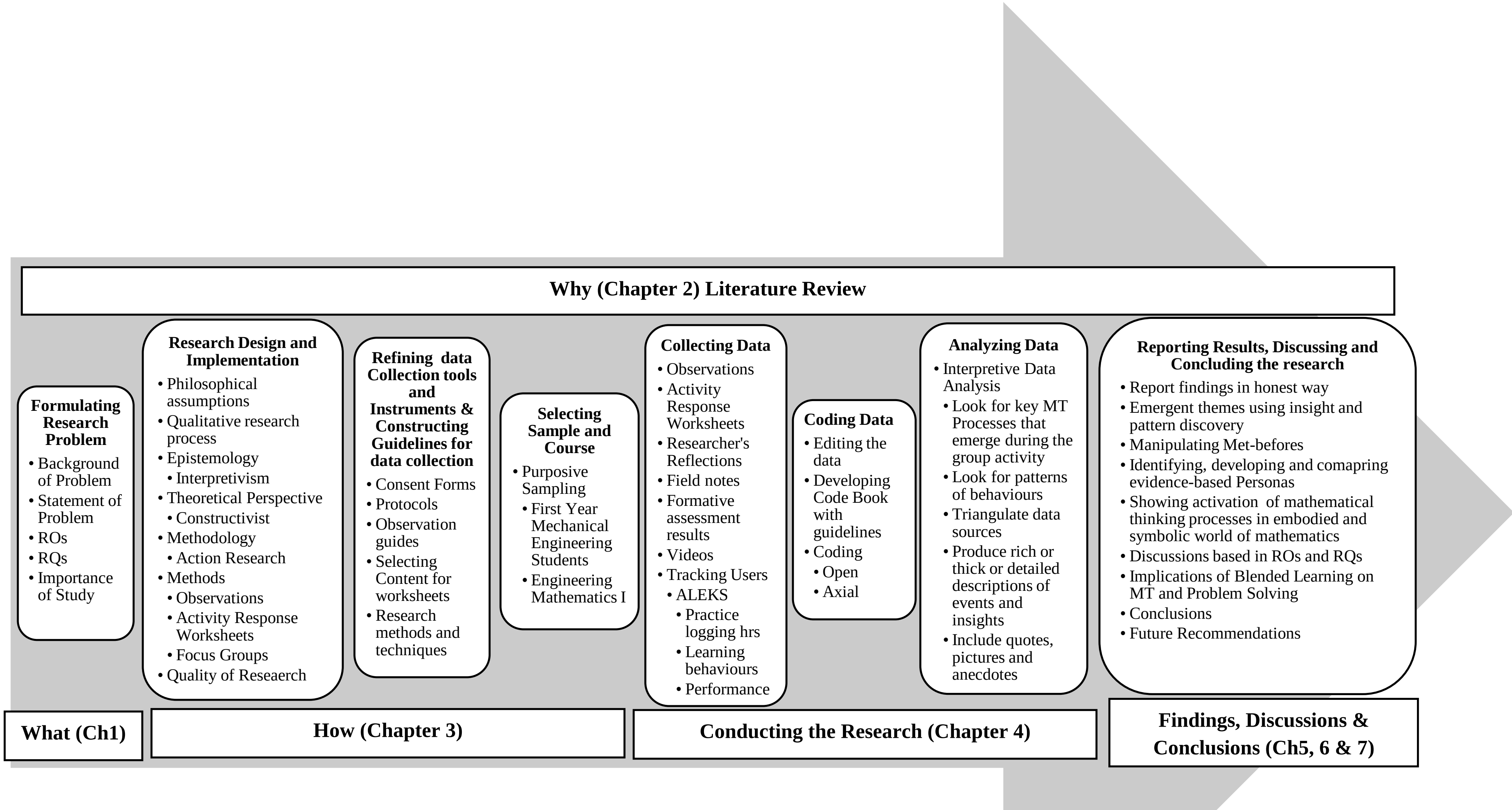


Figure 1.1: Research Process of this research

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

All the descriptions in this thesis are backed by comprehensive literature review as well as the constructive feedback of peer researchers and experts. During the first stage of the literature review (LR), the researcher distinguished the gaps in the literature (Section 1.3) so that the research contribution can be located in relation to those gaps. During the second stage of LR, three main concepts/ideas are explored that is blended learning (Section 2.4), mathematical thinking (Section 2.5), and mathematical thinking as problem solving (Section 2.6) resulting in an adapted theoretical framework for this study. The need and rationale to develop “student personas” during the main study is also briefly discussed in this chapter. During the third stage of LR, different worldviews are reviewed and compared with the researcher’s epistemological, ontological, methodological and axiological assumptions and beliefs followed by a comprehensive review (Appendix H) and the comparison of emerging methodologies in engineering education research with the action research. Throughout the current study, the researcher tried to make sense of the great extension of the engineering education research as well as practices, and demonstrated a creative and critical engagement with the ideas of others as the research progresses (reported throughout the research process in different chapters of this thesis).

In this chapter, the researcher reviewed the relevant literature by focusing on the key concepts of this research. Before reviewing the literature for blended learning, mathematical thinking, and problem solving strategies, the researcher contextualized the current research by explaining the challenges in engineering education and the role of mathematics education for engineering students.

2.2 Challenges in Engineering Education

We are living in the world of challenges, but at the same time we all want the healthier, more dynamic, more creative, more sustainable, and more auspicious lives. For all the above futuristic needs, we depend on brains who are engineers and scientists. From embedded technology of household appliances to high-speed microprocessors and micro-controllers in smart phones, from a minor disease to deadly cancers, we look forward to the technological advancements. Electrification, automobiles, airplanes, water supply and distribution, electronics, radios and televisions, agricultural mechanizations, computers, telephones, refrigeration and air conditioning, internet, highways, imaging, household appliances, spacecraft, health related technologies, petrochemical technologies and petroleum, fiber optics and laser, nuclear based technologies and high-performance materials are selected examples of the greatest engineering achievements of the 20th Century (Aisha Mahmood et al., 2013).

The structural disproportion between today's need of the world, and the production of, engineers and scientists, along with the risks and uncertainties of counting on imported and foreign talent, should be given real attention. There is an utter need to address the problem of producing a workforce locally that consists of capable engineers and scientists who will be equipped to face the global challenges of the new century (Aisha Mahmood et al., 2013). Rover (2008) highlighted the competence-based approach in "Global Engineering Excellence" report and stated global engineer as generally knowledgeable, professionally competent, innovative and entrepreneurial, culturally flexible, market savvy, well versed about world market demands, professionally adjustable, adaptable and mobile. A global competent engineer (Downey et al., 2006) should also be knowledgeable, positively inclined towards professionally different opinions related to problems in hand and ready to learn from local as well as international expertise, equipped with the ability to analyze the way people live and practice around the globe, biased to deal with the diversity of people's perception about problems and their solution (Downey et al., 2006).

Keeping in view the wide-ranging challenges faced by engineers like the ever-increasing rate of technological developments and advancements, the emerging innovations in new fields, the expected population shifts to progressing areas of the worlds and the evolving need of integrated technology in our daily lives, it is vital for engineering education to deal with these challenges effectively. Furthermore, "the importance of technology in health care, public policy, national security, and business will require that engineers have a variety of professional aptitudes in addition to technical proficiency" and "the scope and nature of the challenges facing engineering education make it critical that our directed effort be to characterize and improve the entire system of engineering education" (Fortenberry, 2006). A transformational change in engineering education seems inevitable to prepare prospective engineers so that they can face the challenges of evolving innovation. Thus there is a need to create learning experiences more engaging, more motivating, more diversifying in a variety of setting for example in the laboratories, in the actual classroom, synchronously face to face/online or asynchronously online activities (EERC, 2006). To accomplish that, we require new and better kinds of teaching and curricula, which in turn require engineering faculty to think about teaching and learning in more scholarly ways (Fink et al., 2005) and we also need to "get on with the task of making deep and solid inquiries into learning processes, using the best methods we can bring to bear to advance scientific knowledge and understanding of learning from the variety of research perspectives that are available" (Anderson et al., 2000) and need to focus "to the emergence of a new research trend that attempts to develop better understanding of the nature and processes of teacher change and the factors that affect these processes" (English, 2002).

2.3 Context of the Study: Mathematics education for engineering students

The role and importance of mathematics not only proved to be the core to engineering education but also maintained its distinctive position as a gatekeeper to get into engineering education. Mathematics achievement is the major motivation for future brains to opt for engineering programs at engineering universities. Strength in mathematics is also a gauge of a nation's scientific and technological standing. Therefore, mathematics education is a big concern all over the world and also becomes a hot polemic in the Ministry of Education (MOE) Malaysia (Mahmood et al., 2012). During the transition to engineering universities, it is normal that students bring preconceptions with them from many domains of knowledge as well as misconceptions and numerous other idiosyncrasies based on their previous learning experiences. Research shows that even good students struggled as they encounter prior mathematical ideas and concepts in a new setting and faced various inhibitions in invoking their mathematical thinking powers (Yusof et al., 2014).

Mathematics has been perceived as an efficient tool to investigate the world around, to identify the problems and to equip an engineering professional to systematically and logically solve them (Bajpai, 1985) but "there is a clear distinction between mathematics as a discipline (which is an essential part of engineering education) and the application of that discipline in the various branches of engineering" (Scanlan, 1985). During mid-eighties, Bajpai (1985) raised an issue that mathematics requires redesigning the way we teach it, to improve the quality of engineering education and Scanlan (1985) added that "the effective and useful design of mathematics courses for engineering students must involve a continuous and informed dialogue between engineering and mathematics departments to which each must contribute fully." After two decades, Cardella (2007b) raised the similar issue of responsiveness towards the rapid changes occurring in engineering and the need of constructive contribution of mathematics to improve the quality of engineering education. However, we are still struggling to find an optimal resolution of the above issue.

Mathematical practices (*Common Core State Standards for Mathematics*, 2010) should incorporate the standards like sense making during problem solving activities, logical and abstract reasoning, mathematical modelling, manipulating tools, precise handling and utilizing structures, searching and articulating uniformity in repeated reasoning. According to the conducted studies in mathematics education, the mathematical intelligence of a traditional school goer should have the following nine skills (Sriraman, 2009): to generalize, abstract, and differentiate mathematical structures; to manage the data; to grasp logical thinking and implication; to think heuristically and analogically along with posing related problems; to use mathematical operations in reverse order; to figure out the regularities in mathematical patterns; to take timely decisions while solving the problems; to perceive mathematical relationships; and to differentiate between theoretical and experimental rules. However, the sociocultural basis of mathematical intelligence is developed by measuring, counting, playing, designing, explaining and locating in traditional and cultural activities of a specific community. Kuhn, Lakatos and Polya's inspiring ideas have changed our perception about the history of science, mathematics and technology, that caused a "paradigm shift" promoting innovative teaching and learning practices and materials through many innovative projects world-wide in order to empower our students besides getting educated (Kuhn, 1962; Lakatos, 1976; Polya, 1957).

According to the mathematics Curriculum and Malaysia Education Blueprint, "every student needs to master a range of important cognitive skills like creative thinking and innovation that is the capability to innovate, to produce novel possibilities, and to generate new ideas or knowledge; the problem-solving and reasoning means the ability to anticipate problems and to approach issues logically, inductively, critically, and deductively (in order to find solutions), and to eventually make decisions; and the learning capacity means to independently drive self-learning leading to lifelong learning"(Curriculum Development Centre and Ministry of Education Malaysia, 2004; "Malaysia Education Blueprint 2013 - 2025," 2013). They recommended the use of various teaching and learning strategies included approaches those are suitable with the purpose of instilling mathematical thinking, pedagogy emphasizing on making sense, making connections, representing, reasoning, problem solving, communicating as mathematical processes, integrating Internet Communication Technologies (ICT), developing high order thinking skills (HOTS), facilitating teachers with teaching aids and encouraging constructivist approaches (Zanzali, 2012) in the classroom.

Homer (2012) who is a lecturer in engineering mathematics at University of Bristol described engineering as an art of creating and improving our environment. He further added that engineering involves planning, designing, constructing, assessing and controlling but all these are impossible without mathematics. Every engineering discipline like electrical, mechanical, chemical and aerospace engineering are developed on common mathematical foundations. Industry demands engineers who can apply mathematical thinking skills in real scenarios. Engineering graduates are required to apply their logical skills to formulate a problem, their modelling skills to translate the problem into mathematical notations, their designing skills to develop a prototype and their mathematical thinking skills to question and interpret the outcomes and their engineering thinking skills to materialize the solution. Students with high order mathematical and engineering thinking skills are leading the industry, academia and the related research and the core mathematical skills are transferable to all types of engineering disciplines (Homer, 2012). Goold and Devitt (2013, p. 245) further added that "it is the use of broader mathematical thinking – tacit mathematics, rather than the more explicit 'syllabus' mathematics – that is of most value to all engineers in their workplace."

Engineering expertise of a civilization always maintained its significance for a sustaining modern economy and its progress towards the future advancements, whereas the inclination towards engineering as a career has diminished, in Western as well as in Eastern Countries (Becker, 2010; Elliott, 2009; Forfás, 2009; Goold and Devitt, 2013; King, 2008; McKinsey, 2011; Organisation for Economic Co-Operation and Development, 2010). The expected supply of engineers facing major academic hurdle due to the gate keeping attribute of mathematics (Croft and Grove, 2006; King, 2008). The proficiency in mathematics is the major requirement for engineering programs. However, the role of mathematics in future careers is not clear to many students (Wood et al., 2011). "Some see mathematics as the gateway to engineering, paving the way for sound design; others see mathematics as a gatekeeper, denying entry to otherwise talented would-be engineers" (Winkelman, 2009). It is also highlighted by the research that the experiences of engineering professionals related to mathematics are different from what they have experienced during their academic exposure. (Alpers, 2010; Cardella, 2007a; Gainsburg, 2006; Goold and Devitt, 2013; Trevelyan, 2009). However, it is evident that the major engineering practices depends on engineers' mathematical thinking skills, like contextual and prior experiences, reasoning and justification of inferences, designing new solutions (Gainsburg, 2006). Problem solving, including working collaboratively on complex problems, critical thinking, complex data analysis, numerical reasoning, and appropriate applications of technology are essentially required by employers (English 2002). The role of mathematical thinking in engineering design (Gainsburg, 2006) and its link to design thinking (Cardella and Atman, 2007; Cardella, 2006) strengthens its value for future engineers.

Keeping in view the above discussion, the following sections will review the literature related to developing and implementing blended learning environment, highlighting the conceptual regularities of mathematical thinking and exploring the evidence-based strategies for problem solving.

2.4 Blended Learning

The underpinning human abilities to think mathematically, the students' current knowledge state and their prior experiences related to mathematical thinking (Tall, 2013) are the key factors to be considered before understanding how future engineers learn to think mathematically. By providing a new environment for learning to cultivate mathematical thinking explicitly in the real engineering classroom, we need to make deep inquiries in order to understand the impact of classroom innovation based on new pedagogies (Light and Case, 2011). The traditional face to face and completely online course are the opposite ends of the education spectrum and blended learning is somewhere in the middle. By providing the subjective engagement experience in the classroom, the flexible model of blended learning removes the place and time restrictions using online learning.

Ragan (2013) in his project, Connexions of Rice University for the joint changes and open educational resources, defined Blended Learning as "the planned integration of online and face-to-face instructional approaches in a way that maximizes the positive features of each respective delivery mode." He further added, "Blended

Learning is an approach to course design that brings together the best of both face-to-face and online strategies. This combination aims to build from each approach to create an innovative and effective learning experience for students."

These designed courses employing blended learning are also named as "hybrid", "mixed-mode", and "flexible learning" and are becoming popular day by day (Larson and Murray, 2008). Blended learning has become the state-of-the-art in the history of technology-focused training. During the evolutionary steps, which led us to where we are today, we started with traditional instructor-led training as shown in Figure 2.1. Several variations are available on the blended learning experience from fully integrated model to a minimally integrated model. The Sloan Consortium defines blended learning as any course where 30 to 79 percent of the course content is delivered online (Allen et al., 2007, p. 5).

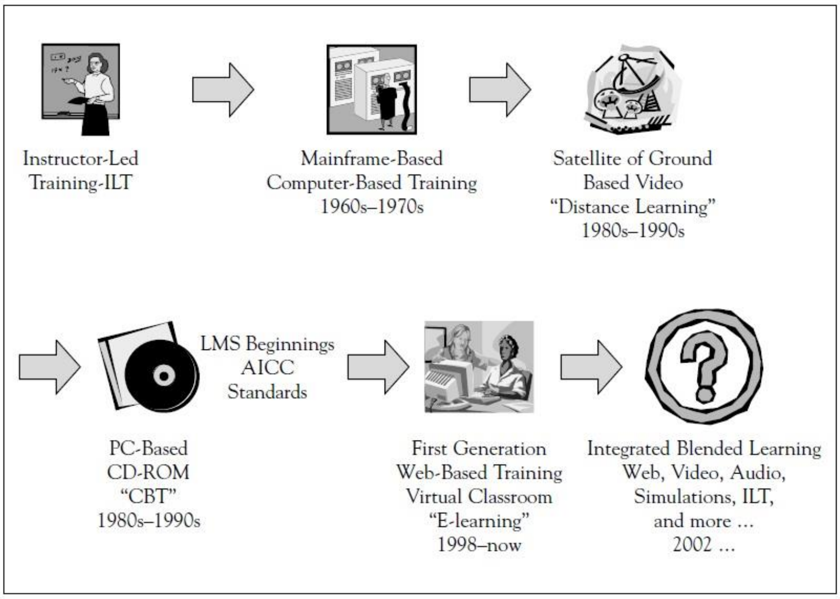


Figure 2.1: Evolution of Technology-Based Training (Thorne, 2003)

Blended learning has emerged as a mean to utilize resources, time and effort in an efficient manner, to help instructors in improving their skills, to maximize learning by involving experts from top class universities who can help to overcome the deficiencies of the traditional classroom and to build up a multicultural classroom community (Bourne et al., 2005). Duet Teaching can be achieved by utilizing remotely recorded video segments of experts in the classroom. The in-class teacher can scaffold students using traditional and non-traditional methods to make sure they master the mathematical concepts along with higher order mathematical thinking skills.

By introducing blended learning, curriculum sharing, and cross-institutional instructional design in engineering education, it is more likely to improve the quality of instruction. However, political, social, technical and supervisory problems can emerge while implementing blended learning (Bourne et al., 2005). It is highly recommended by Bourne et al. (2005) for engineering institutions to design blended learning instructions, to train the instructors around the globe to implement it, and to target the bigger community and to overall improve the quality of engineering education.

As predicted by researchers (Bonk and Charles, 2006; Bourne et al., 2005), the online education and traditional on-campus education are becoming more blended or integrated as shown in the Figure 2.2 to entertain the factors like time, space, attitude, disparity in knowledge, and cognitive demands. Kashefi (2012) also suggested the use of blended learning for engineering mathematics to support the mathematical thinking of new students joining engineering education.

Many instructors and commentators place the poor performance of fresh engineering students in problem solving to a deficit of skills and/or knowledge by assuming the “lack of mathematical and problem-solving skills or correct understanding of the underlying concepts” (Gupta and Elby, 2011). Anticipating the potential of blended learning to better meet the individual needs of students, providing more time for reflection, accommodating different learning styles, designing engaging activities can further be controlled by the dimensions (Picciano, 2007, p. 28) listed in Table 2.1 along with their possible variations.

In this study, “the measure of the impact of blended learning will be the qualitative shift in the learning process and outcomes of learning itself” (Garrison and Vaughan, 2008, p. 148). The selected values of the dimensions of blended learning for this study are also listed in Table 2.1 in the right column.

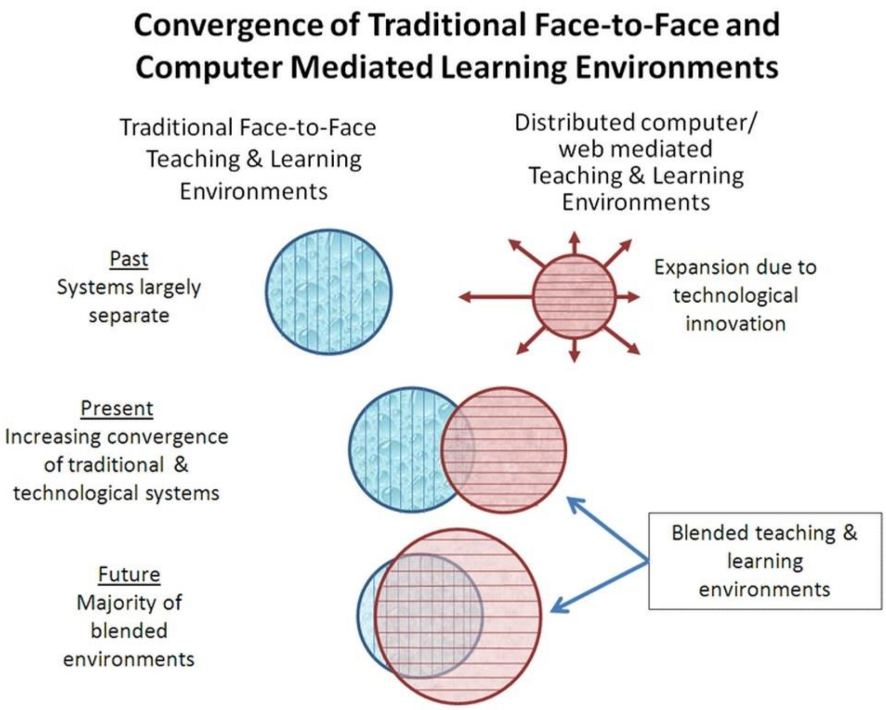


Figure 2.2: Past, Present and Future of Blended Environments (Bonk and Charles, 2006)

Table 2.1: Dimensions of Blended Learning along with the possible variations. The selected values for this research are given in the last column and italicized

Possible Variations	**BL Dimension**	***Selected values in this research***
Synchronous – Asynchronous	Time	*Asynchronous*
Face-to-Face – Online	Place	*Face-to-Face and Online*
Cooperative – Competitive	Pedagogy	*Constructive and Collaborative*
Text – Multimodal	Technologies	*Multimodal (Worksheets, Videos, ALEKS)*
Cohort – Self paced	Format	*Weekly guided lab sessions*
Home-institution's courses – Others	Courses	*Mathematical Thinking Lab as a complementing component of Engineering Mathematics I*
Local – Distant	Participants	*First year engineering students*

2.4.1 Benefits of designing Blended Learning Environments

Bonk and Charles (2006) described the potential benefits of blended learning environments and the researcher evaluated those benefits for this study and described them in Table 2.2.

2.4.2 Challenges in designing Blended Learning Courses

Besides many benefits of blended learning, there are also potential challenges that instructors often face when developing and implementing a blended learning environment. Students and instructors often are required to acquire new skills and spend more time preparing for class because designing and implementing an effective blended learning environment is difficult. Table 2.3 listed some of the specific challenges that have been highlighted by previous studies. There are other logistical challenges in developing blended learning courses that will be discussed during the design process.

2.4.3 Key elements while designing a course based on blended learning model

Designing a course based on blended learning model will work differently for different students, in a particular subject, and in a unique context. Additionally, the aspects of blended learning instruction are dynamic rather than static. The process for designing a blended learning environment is cyclical not linear, and requires instructors to continually adjust their teaching practices to match the needs of their students. This section is organized into five parts; Exploring and Selecting the Resources, Instructional Design Principles, Design for Blended Learning, Course Structure and Class Activities and Community.

Table 2.2: Potential Benefits adapted from (Bonk and Charles, 2006) and MOOC "Blended Learning with edX" (edX, 2015) along with the related responses for this study given in right column and italicized.

Potential Benefit	**Description**	***This research***
Social Benefits of face-to-face interactions	Social benefits of classroom training focused on subjects that gain the most from face-to-face interactions	*We anticipated social benefits in MTL because of collaborative learning during the face-to-face interaction*
Improved Access to Resources	Individualized benefits of self-paced learning, use of new technology and online resources, virtual instruction when campus space and or instructor time is limited, or when students are located remotely. Students have convenient, on-demand access to course materials and lectures	*In MTL, BLOSSOMS modules and problem solving activities will be made available online*
Improved retention and reinforcement	Improved retention and reinforcement through follow-up mechanisms on the Web	*Partial follow-ups mechanisms through ALEKS and social media is planned*
Greater flexibility	Greater flexibility to meet the different learning styles and levels of individuals and customization, to use class time in new and more varied ways, rather than only using class time for lecture delivery.	*Expected in this study during the implementation of blended learning*
Extended communities of practice	The ability to create extended communities of practice where peer-to-peer interactions provide a vital learning environment	*Anticipated in this research*
Student-driven, Learner-centered	Online learning can provide students with more options regarding subject matter and mode of instruction to cater to their individual backgrounds, needs, and interests, the ability to review the materials at their own pace and re-watch the lectures until the content makes sense	*Students will be given an access to ALEKS to review their concepts and to re-build their knowledge base*
Improved metacognitive awareness	The more autonomous nature of blended learning nudges students to improve their metacognitive awareness, which has been shown to be an important element of student success (Bransford et al., 2000). In other words, when educators use blended learning models, they can encourage students to reflect and think about how they learn and change their study habits to match their needs.	*Improved metacognitive awareness is anticipated in this research due to the reflection and extension sub-stages of selected problem solving strategy.*

Table 2.2: Continued

Potential Benefit	**Description**	***This research***
Matches broader trend	Better aligned with the increasing importance of technology, students' engagement with online learning modules enhance their technological literacy, which is quickly becoming a necessary skill for the 21st century, students and faculty become more adept at using technology	*Improved students' engagement with BLOSSOMS modules is expected.*
Additional opportunity to collaborate and interact	Students from around the globe can easily share, discuss, explain, and comment on learning materials. It is also easier for students to connect their lived experiences with technology to classroom materials when discussions can be integrated with social networking tools. By using current, technological tools in classroom experiences, educators can make discussions more interactive, help create a borderless community of learners, and empower both educators and peers to give more immediate feedback to each other (Bonk and Charles, 2006).	*Face-to-face opportunity to collaborate and interact will be provided through in-class problem solving activities*
It matches how the brain works	Ratey (2001) found that brain cells only grow when people are actively engaged. Additionally, the connections in our brains disappear if they are not repeated and reinforced. Bransford et. al (2001) explains that humans do not passively record events in their brains, but rather their brains actively process, recall, categorize and store information in a variety of different regions of the brain. They also highlight that the functional organization of students' brains depends on and benefit from robust, engaging educational experiences.	*Expected to match how the brain works through blended learning in this research*
Preparing students to become life-long learners	The skills listed above are very dynamic. There is no end point for creativity and innovation, students will likely work with many different people from many different cultures throughout their careers, media's primary form of distribution has changed from paper to smartphones and will likely continue to develop, and many future job descriptions have not been written. Students will need to improve these skills throughout their lives, and for the majority of that time, their primary teachers will be themselves.	*Blended Learning environment will help students to become life-long learners though self-awareness and empowerment of their own mathematical thinking powers*
Immediate Feedback	The instructors and learners are able to get more immediate feedback throughout the course on students' understanding. Instructors can use a variety of sources from formative assessments to discussion forums to gather information about their students. Students also have more tools available to them to be able to monitor their own progress, assess what they may need to spend more time on, and ask for assistance from their peers and instructors.	*Immediate feedback will be designed through online resources*

Table 2.3: Potential Challenges extracted from MOOC "Blended Learning with edX" (edX, 2015). The additional discussion specific to this study is given in right column and italicized.

Potential Challenge	**Description**	***This research***
Complex decision-making	Deciding which elements of a blended learning model requires in-person interaction and which ones are best mediated by technology is a complex decision (Aldrich, 2006)	*We will use BLOSSOMS modules as online component whereas problem solving activities will be conducted face to face*
Students need guidance	Dalhstrom, Walker, and Dziuban (2013) found that while students expect technology to be used in the classroom, they still want the instructor to provide some guidance for its use. Therefore, it is important for instructors to use technology purposefully in their courses and make it clear how it is benefiting their students.	*A detailed guidebook will be developed to facilitate the students along with the just-in-time guidance from class instructor*
Privacy Concerns	As technology becomes more integrated with students' academic and personal lives, students become more wary of their privacy (Dahlstrom et al., 2013)	*Students' Privacy will be respected and will be taken care of.*
Resistance to change	Some institutions and regions are resistant to changes in pedagogy that include the use of ICT (Hamuy and Galaz, 2010; Mouyabi, 2011).	*Researcher and the supervisors are willing to introduce new pedagogy*
Lack of teacher's training	Many faculty members have little training and experience using online tools in the classroom, and feel that they will not be able to use them effectively to meet students' needs (Hunt et al., 2014).	*Researcher attended training sessions and online courses on designing and implementing BL environment*
Time consuming and need more effort	The instructors and students both need to develop an understanding of what a blended course is. Most K through 12 systems around the world still use traditional classroom models and students have been trained to be successful in that environment. With a blended approach, instructors are asking students to try a new approach. Therefore, setting expectations and developing understanding of this new mode of learning can take time and effort.	*Researcher expected time-constraint and conflicting met-befores in implementing BL*
Disparity in skills requires extra support	This leads to another challenge, addressing the skills gaps students may have when they first take a blended course. They may have limited experience with using online learning tools, working in groups, and participating in discussions. Students may need extra support to keep them up-to-date with the first few weeks of course material.	*Scaffolding will be used to provide extra support for the students having low mathematical thinking and problem solving skills*
Gradual transition to BL	Instructors, institutions, and students may be resistant to change. Therefore, it is important to implement blended learning models gradually, so that instructors and students can adapt and develop an understanding of a new approach to teaching and learning.	*25% of the total course time will be will be conducted through BL to make change acceptable for the first year engineering students*

Table 2.4: Key element: Exploring and Selecting Resources while designing a blended learning course extracted from MOOC "Blended Learning with edX" (edX, 2015). The additional discussion specific to this study is given in right column and italicized.

Key Elements	**Description**	***This research***
Exploring and Selecting the Resources	***Institutional resources:*** Is your institution ready to run a blended course? Will the administration support your efforts? Does your institution have a media department, a center for teaching, or an office of digital learning that you can leverage to help you develop your courses and course materials? Do you have enough time and money to design, develop, and produce the course? How would you record your findings and experiences, conduct research, and build your institutional knowledge base for blended learning?	*Special permission will be taken to conduct the weekly 1 hour long MTL session with the consent of classroom teacher. Instead of developing blended learning lessons, the researcher selected BLOSSOMS modules for the online component and researcher developed mathematical thinking oriented problem solving activities for the face 2 face component. Data will be collected and recorded manually as well as digitally. Written activity responses and online data will also be collected.*
	Course team personnel and support: Do you have a team that has the necessary experience with tasks like video editing, education technology implementation, and in some cases, computer programming, to be able to design and deliver both the online and classroom components of the course? The best course teams will also be able to offer technical support to their students. Ideally, the course team should also plan to spend time after the initial run to improve and iterate on the course design for future runs.	*Three graduate students as research assistants (RAs) will be engaged for the data collection and stage I Data analysis.*
	Physical and technological infrastructure: Does your institution has the baseline physical and technological infrastructure to run a successful blended learning course?	*Computer Lab will be allocated for collaborative and active learning activities in this study. For a blended learning experience to be successful, students will have access to computers and the internet, in the lab. The researcher will be using Google forms and Facebook polling facility to get instant feedback about the course. The researcher will likely be using a projector in the MTL.*

2.4.3.1 Exploring and Selecting the Resources

Three types of resources should be explored to run a successful blended learning course-- institutional resources, course team personnel and support, and physical and technological infrastructure as described in Table 2.4.

2.4.3.2 Instructional Design Principles

Research has shown that well-designed, effective learning environments are learner centered, knowledge centered, community centered, and assessment centered. Much of the information presented in this sub-section is from Bransford, Brown and Cocking's (2000) book "How People Learn: Mind, Brain, Experience and Schools." They describe in depth how learning environments should be learner-centered, knowledge-centered, assessment-centered, and community-centered. These four key characteristics are summarized in Table 2.5. A learning environment that is both knowledge-centered and learner-centered empowers students to take more ownership of their learning process and supports their metacognitive development. The effectiveness of assessments depends on the extent to which the learning environment is learner-centered, knowledge-centered, and finally, community-centered (Bransford et al., 2000).

2.4.3.3 Design for Blended Learning

It is important to understand that blended learning not only requires educators to reconsider their pedagogical practices, but also requires students to acquire new learning skills. Terrence Doyle (2008) lists eight skills that need to be cultivated in students for them to be successful: Students must learn how to learn on their own; they must develop the communication skills needed to collaborate with others; they must take more control for their own learning; they must learn how to teach others; they must learn how to make presentations; they must develop lifelong learning skills; they must develop their metacognitive skills; they must develop their abilities to evaluate themselves, their peers, and their teachers.

The instructors must manage at least two distinct learning environments: the face-2-face classroom, and the online space. Therefore, it is important to consider the following concepts when designing a blended learning course: Aligning the online and in-class activities, addressing issues of student accessibility regarding both connectivity and the design of the course materials, designing a clear course structure to help students that are less familiar with courses that use blended learning models and promoting a culture of collaboration between online and in-class.

A successful blended learning course uses a mix of the two types of classrooms--online and brick-and-mortar--to improve the pedagogical experiences of both the students and the teacher. One way to design an effective course is to use the backwards design process. This process should be completed in three stages: First, ***establish your learning objectives*** for the course; these will serve as the guidelines throughout the course design process. Second, ***design the assessments***; we should decide how we would measure students' progress against those goals. Third, ***choose your instructional strategies***; we must select the activities that students will complete to achieve the course's learning objectives.

While the course is running, and after it has completed, the practitioner should be actively thinking about his/her learning objectives. The following elements can be used in course to monitor student understanding and engagement: Assessments and coursework, the discussion forums to track student questions and concern, polls and surveys, and student evaluations. It is important to remember that what students learn in the classroom becomes valuable when they begin to apply it to solve problems in their personal and professional lives. Furthermore, becoming self-directed learners is one of the keys to them becoming lifelong learners.

Table 2.5: Key element: Instructional design Principles while designing a blended learning course extracted from (Bransford et al., 2000) and MOOC "Blended Learning with edX" (edX, 2015). The additional discussion specific to this study given in right column and italicized.

Key Elements	Description	*This research*
Instructional Design Principles	***Learner-centered environment:*** Learners have been found to base new knowledge on what they already know, therefore it is important to help students connect what they learn in the course with their experiences outside of the classroom. Instructors should seek to learn about student's unique interests, backgrounds, and competencies, and then design course content that builds upon that foundation. One of the potential benefits of a blended learning environment is that there tends to be more student-teacher interaction, which enables teachers to gain a better understanding of what their students know, and where students may be likely to struggle. Teachers then will be better positioned to respond to their student's needs. Building a learner-centered environment is only one piece of the puzzle in designing effective learning environments.	*Knowing students, knowing what they know and knowing their prior experiences, believes and attitudes will be targeted in this research.*
	Knowledge-centered environment: To design knowledge-centered learning environments, instructors should consider not only what they're teaching, but also why they're teaching it. Courses should be designed to not only help students master the course content, but also to put that content into a broader context relevant to their own lives. Students should be able to solve real world problems based on their knowledge base in the future. Therefore, course activities and curricula should give students opportunities to use what they're learning to complete meaningful tasks. By moving course lectures and other course materials online, instructors can design more challenging learning activities that promote sustained active learning in the classroom.	*Mathematical thinking oriented problem solving activities will be developed to help students manage and apply their knowledge using context-rich problems*
	***Assessment-centered environment*:** Students can become even more self-directed learners when learning environments are also designed to be assessment-centered. Designing an assessment centered learning environment does not necessarily mean that instructors should increase the number of worksheets, quizzes, and exams they require their students to complete. Instead, an assessment-centered approach implies thinking carefully about the formative value of work assigned to learners. Formative assessments are meant to provide both teachers and learners with valuable feedback that can help them alter their approaches to other problems and learning experiences in the future. In other words, assessments should be used to help instructors and students monitor their progress towards the courses learning objectives and improve their teaching and learning practices. This is different from more traditional summative assessments that are primarily focused on measuring what students remember.	*Students should be assessed formatively using worksheets, quizzes and focus groups*

Table 2.5: Continued

Key Elements	**Description**	***This research***
Instructional Design Principles	***Community-centered environment:*** This learning environment treats a classroom like a community. Effective learning communities take into account not only the students and teachers in the classroom, but also the roles they play in their broader school, town, and global communities. Learning environments should be designed to facilitate connections between learners and their larger community, because one of the primary purposes of education is to help students become functional, contributing members of society. If work in the classroom is detached from the norms, cultures, and needs of the real world, its value is less significant. In community centered learning environments, instructors design lessons that nudge students to apply what they learn in their classes to their everyday lives. What might this look like in practice? By integrating **social media** into your courses, you can increase student interaction with their peers (both in-class and online) while the course is running, and also encourage them to share resources and materials with each other after the course has ended. Additionally, by using social media within a course, you are in fact teaching students valuable media literacy, information, and communication technology (ICT) skills. Students learn that social media tools can be used for other purposes like education, and not just for entertainment. Moreover, by providing additional access points to the course, you will be able to reach more potential educators and students from around the world. Yet, like other elements of the course, social media needs to be used with a clear purpose in mind. You can't simply create a facebook page and a hashtag for your course and expect students to use those resources effectively. It is important to monitor these channels, post example articles and resources, comment on students' posts, and, in general, ensure that these resources are being used in a manner that is consistent with your learning objectives.	*In this research, the first year engineering students will be tasking with a collaborative semester long research project to work together in order to solve a real issue in the engineering education*

2.4.3.4 Course Structure

Instructors, who design blended learning courses, emphasized that the experience reinvigorates them as teachers. They enjoyed experimenting with innovative course structures and designing more engaging classroom activities (edX, 2015). However, both instructors and students also highlighted that a blended learning course requires greater investment of time and energy, because it means managing two distinct learning environments-- the in-person classroom and the online space. A well-designed blended learning course comprises more than just putting lectures online. When designing our own blended learning course, there are several factors we should consider ensuring that the online and in-class activities are well aligned and designed with purpose. Factors considered in designing BL course are adapted and described (edX, 2015) in the following paragraphs.

First, be mindful of ***student workload*** when planning the course. It should be avoided to put course materials online simply to allow for further lecture-style presentations in class. Rather, the time in the classroom should be used to engage students in projects and activities that engage active learning strategies.

Another key factor is to ***align online and in-class experiences*** to keep them consistent across environments. For example, establishing the same set of norms and guidelines for student interactions online and in person will facilitate better collaboration and community building. Because students may have never taken a blended learning course before, including a well-defined and articulated core structure is important. A well-designed syllabus should outline how much time students are expected to spend preparing for class and completing assignments; how the course will progress from week to week, including key deadlines; and how they will be graded. Setting clear expectations can alleviate much of the anxiety students might have around transitioning to a new model of learning, like a blended course. It is helpful to provide students with a printer-friendly schedule that includes due dates, discussion and collaboration guidelines, and project descriptions.

The syllabus should also highlight which ***elements of the course*** will be completed ***synchronously and asynchronously***. For example, we may choose to give students access to the online course materials all at once, to allow them to move through the lectures at their own pace, but design in-class group projects that progress in a sequence over the length of the course. What is the best way to make sure that students fully understand the syllabus and structure of the course? Walking through it with them together is the answer. Describing an average week of course-work for the class is a great way to ensure that students understand expectations and definitions of success. If students are curious about the reasons for choosing the blended format for the course, it is recommended to be transparent and explain the rationale behind all the decisions.

Encouraging students to ask questions and demonstrating our willingness to be open with them can actually begin establishing constructive social norms for our class.

Lastly, throughout the course, ***a high level of interaction*** should be maintained with students by communicating often. Sending regular updates to students, monitoring their in-class and out of class interactions, provide them with opportunities to meet with class teacher and gather feedback whenever possible are some of the recommended methods.

Organizing Course Content

Before we begin designing the course content, it should be identified what content we have already have, both digital and physical. This inventory should help the researcher to distinguish which course materials can be used, which ones can be adapted, and which ones need to be created. Later, we will identify what sequence we want this content to be presented in, and decide which content should be delivered online and which content should be delivered in-class. Blended learning is still evolving, and so it is likely that the teacher and the students will hit some bumps in the road in the first attempt. If students get frustrated and find it difficult to navigate through the course, they may lose motivation and become dissatisfied with the course.

We can reduce this risk by not trying to do too much. We should be selective when choosing which technological tools we first integrate into our course. When deciding which tools to use, we should consider the following questions: Do we have experience using the tool? Do we have a teaching assistant or a course team member who can offer technical support if a tool doesn't work? Does the learning tool further the learning objectives of the course? How? Does this tool create extra work for the students? Does the added value of the tool outweigh the extra work? It is recommended to experiment with new tools and to be innovative with pedagogical practices in a strategic and gradual manner.

Assessment

While there are no universal rules for the use of online assessments in blended courses, it is important to consider how online assessments can complement activities in the classroom, and improve specific learning skills. With this in mind, there are two key suggestions to consider:

1. Use assessments in a formative rather than a summative manner whenever possible. Using assessments to simply measure what students remember, and to assign them grades, does not help them apply or deepen their understanding of the material. In fact, it may encourage students to only focus on "what is on the test." However, using not-for-credit, multiple choice questions to help students review what they have learned and to give them instant feedback is quite beneficial.

2. Assessments are more than just knowledge checks. They should be designed in a way that encourages students to apply what they have learned to other situations, engage in deeper analytical thinking, and extend their knowledge beyond the course content.

Grading Policy

The grading policy for a class needs to be consistent with the learning objectives. For instance, if we think it is important for students to be able to make presentations, to learn how to collaborate with each other, and to be able to write academic papers, then the breakdown of grades should reflect those objectives. Also, if the goal of the course is to teach students to reflect on their learning experiences, learn from their mistakes, and seek to make sense of their learning experiences, then it is important to design assessments that are formative in nature.

2.4.3.5 Class Activities and Community

Blended learning models allow us to rethink our classroom activities. This is an exciting opportunity to develop robust learning exercises that encourage students to become more active learners and engage in higher level cognitive activities. A few types of in class exercises are explored here, but there are many other types that exist and more that have yet to be designed, so teachers need to be creative.

Group Work

Blended learning models free up more time for students to interact with each other in class. Students should be given ample opportunities to participate in group discussions, collaborate to solve problems and complete projects, and give each other constructive feedback. As always, clear guidelines should be provided and a clear set of norms to scaffold students' interactions should be established as they learn how to collaborate and communicate effectively. If successful, group activities can yield deeper learning outcomes and help students develop many 21st century skills, which include communication, collaboration, and critical thinking. Collaborative projects are one type of group activity we can implement in our blended classroom and are often designed to be either problem based or project based. Problem-based learning tasks challenge groups of students to explore case studies and other real world problems and work together to find a solution. These types of activities are convergent in nature in which students explore a variety of different solutions to problems but end up choosing one. Project-based learning activities, on the other hand, are more interested in students applying a diverse set of skills to complete a project. Project-based learning activities tend to be more divergent and open ended. While completing both problem and project-based learning activities, students may engage in similar tasks, employ similar skills, and produce similar results. In fact, activities can be both problem based and project based. For example, in a computer programming class, the instructor can design an activity that asks students to produce a computer program that completes a specific task. Whereas in a physics course, students could be asked to solve a specific problem and then build a model or extend the concept based on the results. Yet, in practice, there tend to be some noteworthy differences between problem based and project-based learning. For example, problem-based learning tends to focus on one subject while project-based learning is often interdisciplinary. Project-based learning activities often require students to get direct, real world experience while completing their projects, which is not necessarily a key component of problem-based activities. In addition, project-based activities often take more time to complete and can extend over the span of a course. The outputs for project based and problem-based learning activities can take many forms. Students can conduct case study analyses, prepare for in-class debates, make presentations, engage in hands-on labs, create documentary films, design future project proposals, and so on.

When choosing an activity for a course, it is always important to ask how will that activity help the students achieve the learning objectives of the course. Activities must be designed and chosen with purpose. The best course activities will use active learning strategies to get students to engage more fully with their learning experiences.

Constructive Feedback Sessions

Learning how to give and receive feedback is an essential skill for everyone to learn. Bob Dignen (2014) highlighted five reasons why he believed feedback may be the most important skill for people to learn: Feedback is omnipresent; another word for effective listening; an opportunity to motivate; essential to improve performance; and a way to keep learning.

Therefore, we should provide students a variety of opportunities for students to exchange feedback with both the teacher and their peers. To ensure that feedback sessions are constructive, we should design them with specific goals in mind, and we should learn effective feedback strategies so that we can serve as a model of excellence for our students. Additionally, like with the discussion groups, we should set clear norms and expectations for these sessions.

Presentations

Student presentations encourage development of oral presentation skills and allow students to synthesize knowledge and develop expertise in specific areas of a topic. In order to encourage active listening by other students in the classroom during presentations, we can integrate a peer evaluation assignment. We can provide students with a rubric by which to evaluate their fellow student's presentation, thereby encouraging critical thinking and engagement.

Debates

Debates are an excellent form of in-class activity that provides students with an opportunity to develop a number of 21st century skills (e.g. critical thinking, communication, collaboration, information literacy, critical thinking, communication, self-direction, presentation). There are a number of different types of debates that we can use in our classrooms, for which guidelines are readily available online from a number of sources.

Hands-On Labs

Many learning theories are based on the idea that people learn most effectively when they are interacting with their learning experiences. Intuitively, this makes sense especially when we think of subjects like chemistry, biology, medicine, and others that prepare students for obvious hands-on professions, but "hands-on" can also apply to any activity in which students apply what they are learning to produce something meaningful. Some obvious examples are statistics classes in which students spend a substantial amount of time actually manipulating and analyzing data, or computer programming classes in which students code functional programs.

Case study analysis

Case studies give students an opportunity to analyze in depth a person, event, organization, policy, law, political movement, etc. Case study analyses often require students to look at the subject from a variety of perspectives using both quantitative and qualitative research techniques. The relevant case studies should be clearly connected to the course content. Depending on the learning objectives for the case study, we can have students work individually or in groups, and have them report their findings in different forms (e.g. presentations, written reports, poster sessions).

2.5 Mathematical Thinking

In this section, the researcher will try to conceptualize the way mathematical thinking is defined and perceived followed by the mathematical thinking and learning from multiple perspectives and then describe the mathematical thinking processes.

2.5.1 Definition and Perception

> "It is one thing to assert that what we really want is for learners to develop their mathematical thinking; it is quite another to be precise about just what that means. We may feel certain that learners who actively think about their mathematics, who develop mathematical 'habits of mind', who use their natural powers, will also do better on examinations and tests than those who are trained in a few techniques and typical problem types."
>
> (Mason and Johnston-Wilder, 2004, p. 184)

Working at different levels of mathematical thinking empowers a brain with valuable skills of critical analysis, timely identification of fallacies and biases, risk assessment and creative attempts to solve problems, thus helps a human to perceive and decode the information embedded in the environment around us (Mathematical Sciences Education Board and the Board on Mathematical Sciences National Research Council and National Research Council, 2000). Mathematical ability involves effective thinking with conceptual learning; students need to be taught to think logically along with practicing the numerical problems, but on the contrary, they do practice a problem, and then repeatedly do the same kind of problems until that is hard-wired in their brains (Pearse & Walton, 2011).

> "Thinking mathematically is not an end in itself. It is a process by which we increase our understanding of the world and extend our choices. Because it is a way of proceeding, it has widespread application, not only to attacking problems which are mathematical or scientific, but more generally. However, sustaining mathematical thinking requires more than just getting answers to questions, no matter how elegant the solution or how difficult the question".
>
> (Mason et al., 2010, p. 140)

According to Sfard, (2008) mathematical thinking is a communication tool and is used in languages unintentionally whereas Leron (2003) synthesizes the social, cognitive and biological roots of the latest research in mathematical thinking. He separated three levels of mathematical thinking as hard-wired rudimentary arithmetic in the brain supporting the existence of innate abilities, the informal mathematics based on daily cognitive mechanisms easily learned by experiencing the world around and the formal mathematics learnt through formal learning processes using formal mathematical language, abstraction, de-contextualization and proof (Leron, 2003). After conducting the meta-analysis as summarized in Figure 2.3, Argyle (2012) was able to realize an abundant treaty about mathematical thinking:

> "First is that "mathematical thinking" is a specialized function distinctive from generalized thinking. However, it remains difficult to

delineate exactly where mathematical thinking ends and where scientific thinking, for example, begins. Second is that "mathematical thinking" is best seen as a ***continuous, cyclical process of cognition*** in which a person strives ***to make sense*** of a vast sea of sensory data, map the mathematical world, attend to social convention all while coping with the individual differences in belief of every mathematical thinker. Ultimately, there is much yet to be learned about mathematical thinking, but now, perhaps, there is a potential basis for a strong paradigm to emerge from the Kuhnian pre-science in which such research has been mired." (Argyle, 2012)

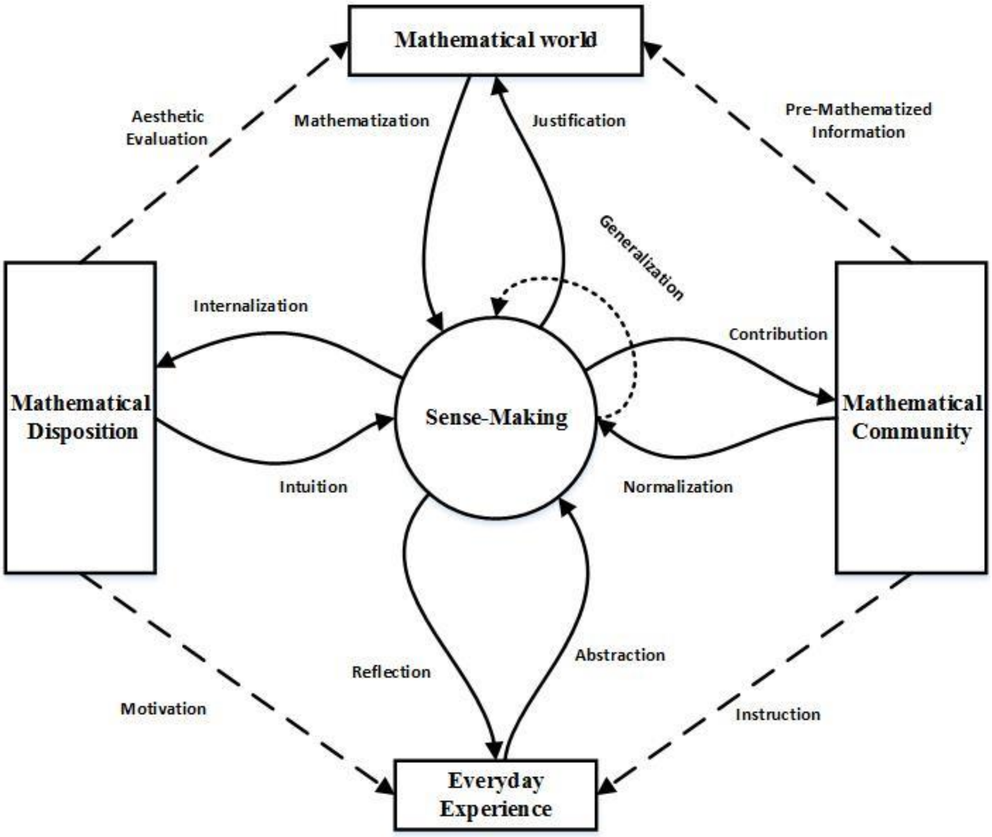


Figure 2.3: A Meta-analytic Model of Mathematical Thinking (Argyle, 2012)

The outcome of Argyle's research is a meta-analytic model of mathematical thinking as shown in Figure 2.3. He further explained:

"The model consists of five nodes and their various interconnections. The five nodes can be distinguished into three groups: the cognitive nodes – the mathematical world and everyday experience; the socio-

> cultural nodes – mathematical community and mathematical disposition; and, most importantly, the central node of sense-making. The basic premise of the model is that sense-making is the active process of mathematical thinking and the other four nodes provide the data which sense-making digests, as it were. The solid-line arrows are the flow of information through sense-making. In contrast, the dotted-line arrows represent processes which are not inherent to mathematical thinking but which are, in some sense, inseparable from the process."
>
> (Argyle, 2012)

2.5.2 Mathematical Thinking and Learning

Corte and Verschaffel (2007) reported four interlinked components of mathematical thinking and learning as competence, learning, intervention and assessment as depicted in Figure 2.4 along with their working regularities (Mahmood et al., 2012).

Cultural variations in perception and thinking also give rise to variations in mathematical thinking by showing the link between culture and cognition. Directive and self-consciousness of thoughts, feelings and actions need to be explored and neural activity patterns are required to be recorded to know the myth of mathematical thinking (Ames & Fiske, 2010). Cognition of problems, perception of objects and scenes, and emotions of cultural practices within a community and/or society, are considered the basic components in understanding the cognitive processes of mathematical thinking. The above mentioned components combined together help to answer how variation in mathematical thinking is guided by learning culture (Rule, Freeman, & Ambady, 2011). Gutchess et al., (2011) reported the influence of culture on memory by studying the cognitive processes across cultures. Pros and cons in information processing due to cultural norms are identified and culture based schemes are suggested to eliminate mathematical thinking discrepancies. Distinct neural activities in brain in a distinct sequential manner are recorded to identify different information processing mechanisms and thus propose ways to implement the improved cognition required for different human brains (Gutchess et al., 2011). The latest research findings in the field of cultural psychology provided sufficient evidence that different ethnic groups possess contrasting thinking and processing styles. A comparison between people of Western and Asian cultures are carried out and results showed that the former possess an analytical thinking style that is familiar to important central objects, but less sensitive to the contexts, whereas the latter are trained in a holistic thinking style that is accustomed to background and contextual information (Han, 2010; Mahmood et al., 2012).

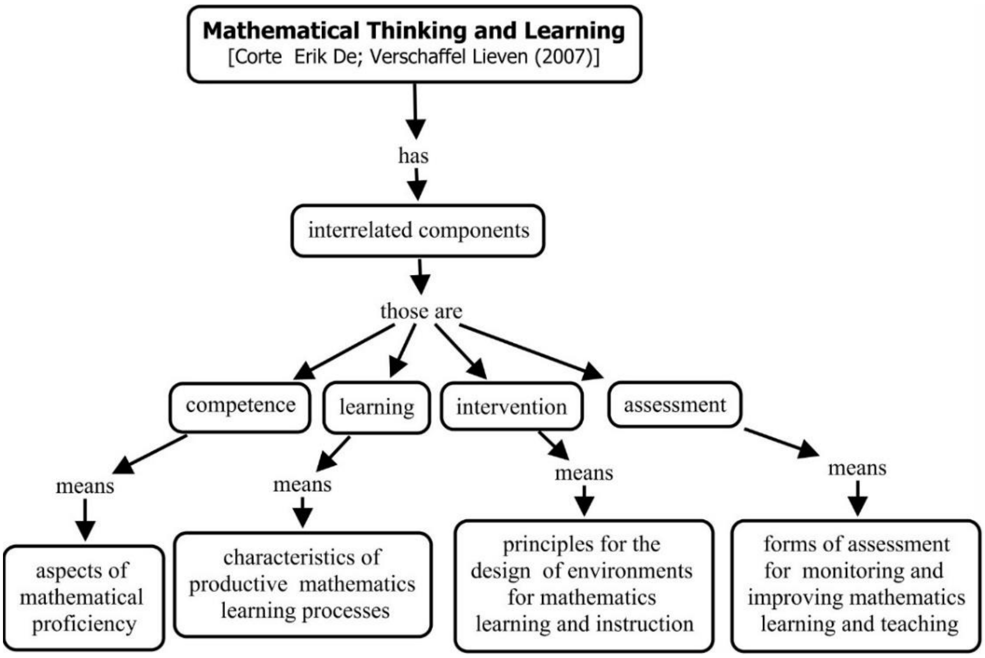


Figure 2.4: Interrelated components of mathematical thinking and learning

Cultural practices influencing multi-level thinking processes and neural correlates of primary thinking processes like perceptual or attention-based processing is also reported by the latest brain imaging research. Cultural specific thinking styles, thus emerged through understanding of thinking processes and their related neural activity patterns (Han, 2010). The bidirectional relationship between cultural neuroscience and mathematical thinking is shown in Figure 2.5. The self-explanatory concept map developed during the initial conceptualization of this study, shows the three basic contributors that is innate mathematical abilities, informal educational culture and formal educational culture developing mathematical thinking and recorded by cultural neuroscience (Mahmood et al., 2012).

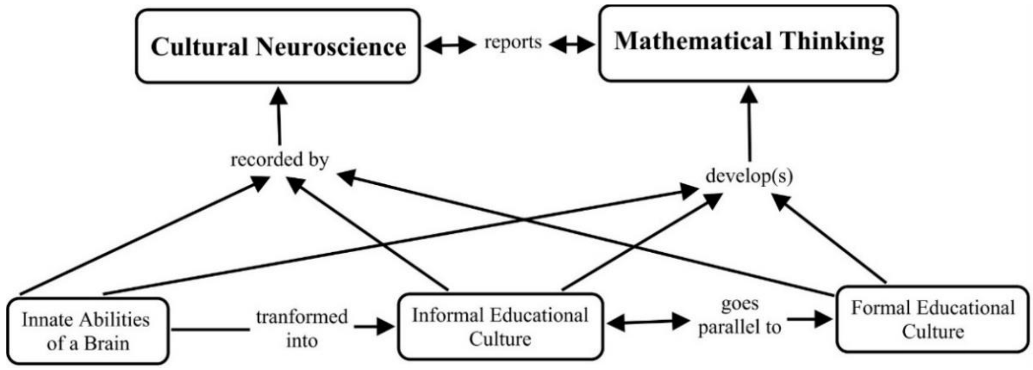


Figure 2.5: Bidirectional relationship of cultural Neuroscience and mathematical thinking

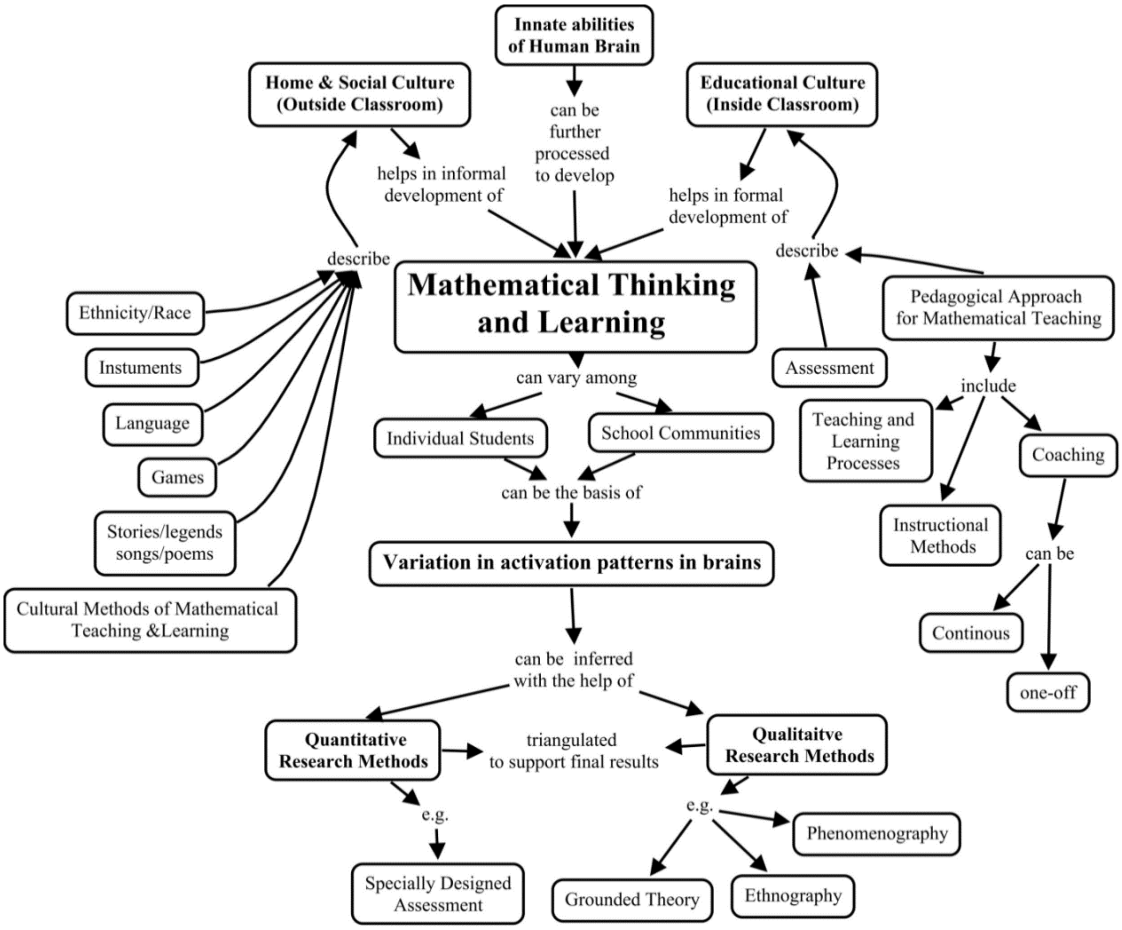


Figure 2.6: Initial conceptualization of Mathematical Thinking and Learning during this study

Three active input streams, namely the innate ability to learn mathematics by the young brains, home and social culture contributing as informal development and educational culture as formal development to activate mathematical thinking processes are shown in Figure 2.6. The variation in mathematical thinking among individuals; among social and cultural communities would result in variations of activation patterns of brain and the final results can be interpreted through qualitative as well as quantitative research methods. The contributing variables for home and social culture are language, ethnicity, instruments, games, stories, legends, songs, poems and cultural methods of mathematics teaching and learning and all these participate as informal development of mathematical thinking whereas the first contributing factor in formal educational culture is the pedagogical approach of mathematical teaching that includes teaching and learning processes e.g. representation, visualization, mental images, abstraction, reasoning etc. in the classroom, instructional methods, and coaching styles; second contributing factor is assessment method (Mahmood et al., 2012).

Ginsburg et al., (2008) further described several contributing factors in mathematical thinking and learning such as instinct learning abilities, biological basis for prime concepts , physical environment, the contribution of the social environment in multiple ways and concluded that all young learners have the ability, chance and drive to learn basic mathematical thinking skills. They also reported that most of the research has been focused on numbers than other mathematical features and concluded that number-based thinking is extremely different among different communities. Young brains have inbuilt ability to rote learn, use counting rules, work with concrete as well as abstract concepts, employ different strategies for addition, work with complex procedures. An urgent need to understand the divergence in mathematical learning trajectories is required (Daniel, 2008) and will help to recognize the cultural influence on these abilities.

According to Tall (2002), there are several ways of mathematical thinking, the various facets of thinking are highly affected by cultural variations and their contextualization. He also reveals that the secret to understand the mathematical thinking is not only hidden in knowing how individuals construct their mathematical knowledge but also in studying how they think mathematically in groups of people. Therefore, the goal of inquiry about the mathematical thinking would be well served by sociological rather than psychological research. In the following section, the researcher will summarize the inductive teaching and learning methods and its foundational roots in constructivism.

2.5.3 Inductive Teaching and Learning Methods for Mathematical Thinking

Teaching mathematics in an old-fashioned deductive way is getting obsolete day by day and there are more demands to innovate mathematics education by introducing new methods of teaching and learning. The growing popularity of inductive teaching and learning methods in the engineering education predicts these methods to be as effective in secondary education as in higher education.

Problem based, project based, inquiry based, discovery based, case based and just-in-time are some of the popular ITL methods (Prince and Felder, 2006). The authors described the methods, their similarities, dissimilarities as well as the efficiencies of these methods. They conclude the inductive methods to be more effective than the deductive methods and thus focusing on a broader range of outcomes. In inductive teaching and learning, the instructor introduces specific examples, observations, case studies, real world problems, or experimental data and asks the students to interpret, analyze or solve the problem. Teacher's role as a facilitator is to scaffold the learning process and help them to be independent learners. The inductive teaching and learning methods also fall under constructivist, collaborative, cooperative and active learning methods (Prince and Felder, 2006). Table 2.6 summarizes the similarities of the inductive method and guide the researcher to select the ones more suited to the respective research objectives. It can be clearly seen from Table 2.6 that all the inductive methods have built-in attribute of active learning and collaborative learning and can also be adapted at the any required levels. Table 2.7 even shows the dissimilarities in terms of the end products, but those are of negligible value once the methods are implemented.

Inductive learning here does not mean that we altogether eliminate the deductive learning, but the inductive learning in an effective teaching strategy should follow the deductive learning. In the next sub-section, the researcher will discuss the foundations of inductive teaching and learning described by Prince & Felder (2006).

Table 2.6: Feature similarities of the inductive methods adapted from Prince & Felder (2006)

Features/Methods ↓ →	IBL	PBL	PrBL	CBL	DBL	JITL
Questions and Problems	by def	always	always	always	always	always
Complex, ill-structured, open-ended real problems	possibly	by def	usually	always	possibly	possibly
Projects	possibly	possibly	by def	usually	possibly	possibly
Case Studies	possibly	possibly	possibly	by def	possibly	possibly
Concepts/Content Discovery	always	always	always	usually	by def	always
Instruction modification based on students' responses	possibly	possibly	possibly	possibly	possibly	by def
Self-Directed Learning	possibly	usually	usually	usually	always	possibly
Active Learning	always	always	always	always	always	always
Collaborative/ Cooperative (team-based) Learning	possibly	usually	usually	possibly	possibly	possibly

IBL(Inquiry Based Learning); PBL(Problem Based Learning); PrBL(Project Based Learning); CBL(Case Based Learning); DBL(Discovery Based Learning); JITL(Just-In-Time Learning)

Table 2.7: Dissimilarities of the inductive methods in terms of end product adapted from Prince & Felder (2006)

Methods	End Product
Inquiry Based Learning **(IBL)**	an answer to an interesting question
Problem Based Learning **(PBL)**	extensive analyses of real or hypothetical scenarios
Project Based Learning **(PrBL)**	a formal written and/or oral report
Case Based Learning **(CBL)**	extensive analyses of real or hypothetical scenarios
Discovery Based Learning **(DBL)**	discover theories, facts, patterns
Just-In-Time Learning **(JITL)**	Questioning prior to lecturing based on provided material

2.5.3.1 Foundations of Inductive Teaching and Learning

I: Constructivism

According to the worldview or paradigm of constructivism, learning is considered as an active and constructive process that helps us to construct knowledge about the social world and ourselves. The learners not only construct the knowledge based on their prior knowledge, but also create their own subjective representations of objective realities. "Knowledge does not straightforwardly reflect an external reality, but is contingent on convention, human perception, and social experience" (Elliott, 2005). Originators and important contributors of constructivism are, Piaget, Dewey, Bruner, Vygotsky, and others (Bruner, 1961; Dewey, 1997; Piaget, 1972; Vygotsky, 1978) and their research lead us to use this theoretical foundation for developing mathematical thinking skills.

> "If the constructivist model of learning is accepted—and compelling research evidence supports it— then to be effective instruction must set up experiences that induce students to construct knowledge for themselves, when necessary adjusting or rejecting their prior beliefs and misconceptions in light of the evidence provided by the experiences. This description might serve as a definition of inductive learning."
>
> (Prince and Felder, 2006)

The researcher will thoroughly explain the constructivism as theoretical perspective basis of this study under Section 3.3.2 of qualitative research process.

II: Research Cognition

A strong support for constructivism and inductive teaching and learning is provided by the psychological and neurological research results surveyed by Bransford et al. (2000), reported by Prince & Felder (2006) and replicated here:

> "All new learning involves transfer based on previous learning."
>
> (Bransford et al., 2000, p. 53)

"Motivation to learn affects the amount of time that people are willing to devote to learning. Humans are motivated to develop competence and to solve problems."

(Bransford et al., 2000, p. 60)

"Learners of all ages are more motivated when they can see the usefulness of what they are learning and when they can use that information to do something that has an impact on others."

(Bransford et al., 2000, p. 61)

"Helping students transfer what they have learned in school to everyday settings of home, community, and workplace is a function of the similarity by transfer tasks and learning experiences"

(Bransford et al., 2000, p. 73)

"Transfer can be improved by helping students become more aware of themselves as learners who actively monitor their learning strategies and resources and assess their readiness for particular tests and performances."

(Bransford et al., 2000, p. 67)

III: Intellectual Development and Approaches to Learning

There is a tendency to approach learning in one of the following three ways by students:

"Some students take a *surface approach*, relying on rote memorization and mechanical formula substitution, making little or no effort to understand the material being taught. Others may adopt a *deep approach*, probing, questioning, and exploring the limits of applicability of new material. Still others use a *strategic approach*, doing whatever is necessary to get the highest grade they can, taking a surface approach if that suffices, and a deep approach when necessary. Another goal of instruction should be to induce students to adopt a deep approach to subjects that are important for their professional or personal development."

(Prince and Felder, 2006)

Inductive teaching promoting deep learning approach can also support intellectual growth:

> "Several conditions of instruction have been shown to promote a deep approach, including interest and background knowledge of the subject, use of teaching methods that foster active and long-term engagement with learning tasks, and assessment that emphasizes conceptual understanding as opposed to recall or the application of routine procedural knowledge."
>
> (Prince and Felder, 2006)

The other precursors to intellectual growth are posing "serious challenges to students' low-level beliefs in the certainty of knowledge and the role of instructors as providers of knowledge" by introducing "open-ended problems that do not have unique well-defined solutions" (Prince and Felder, 2006).

IV: Learning Cycle-Based Instruction

Learning cycles to work through the sequences of thinking and problem solving activities are found effective in well-known instructional models (Prince and Felder, 2006).

2.6 Mathematical Thinking as Problem Solving

Focusing mathematics through the lens of "problem solving" (Carlson and Bloom, 2005; Polya, 1957) does not mean solving well-structured problems rather it means "wrestling with ill-structured problems" (Schoenfeld, 1992, 1985), solving "open-ended" problems (McGinn and Boote, 2003) and "modelling of complex problem situations (Lesh and Harel, 2003). "Problem solving is the activity that occurs when the individual (or individuals) concerned is (or are) faced with a problem situation for which the precise nature of the problem and its solution are not initially evident" (Tall, 2013, p. 176). Based on the literature review focusing on different aspects of mathematical thinking that could be included in the education of engineers as described in Table 2.8, the researcher drew her first inspiration from George Polya's work. Therefore, the researcher first described the problem-solving model by George Polya in the next sub-section followed by Mason's problem solving strategy and Schoenfeld's framework of problem solving analysis.

2.6.1 Polya's Problem Solving Model

George Polya was a famous mathematician who contributed to the development of new mathematical ideas. His books on mathematical thinking and courses related to teachers' education are full of intuition and wisdom. He migrated to the USA and influenced mathematics teacher there (Mason and Johnston-Wilder, 2004). He reflects on the forms and types of mathematical thinking as quoted below:

> "Getting food is usually no problem in modern life. If I get hungry at home, I grab something in the refrigerator, and I go to a coffee shop or some other shop if I am in town. It is a different matter, however, if the refrigerator is empty or I happen to be in town without money; in such a case, getting food becomes a problem. In general, a desire may or may not lead to a problem. If the desire brings to my mind immediately, without any difficulty, some obvious action that is likely to attain the desired object, there is no problem. If, however, no such action occurs to me, there is a problem. Thus, to have a problem means: to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable, aim. To solve the problem means to find such action. A problem is a 'great' problem if it is very difficult; it is just a 'little' problem if it is just a little difficult. Yet some degree of difficulty belongs to the very notion of a problem: where there is no difficulty, there is no problem. … the solution of any problem appears to us somehow as finding a way: a way out of a difficulty, a way around an obstacle." (Polya, 1962, p. 117)

Table 2.8: Aspects of Mathematical Thinking that could be included in the education of engineers adapted and modified from (Cardella, 2007b). The additions related to this research are italicized.

Aspect	**Definition/Description**	**Main Theoretical Source**	***This Research***
Knowledge Base	Cognitive Resources: Mathematical Content Knowledge	Schoenfeld (1992)	*ALEKS Reports*
Skill Base	Mathematical Thinking and Problem Solving skills		*Written Activity Response Analysis*
Beliefs, Attitudes and Affects	Beliefs about mathematics and one's mathematical ability, Feelings towards mathematics, attitudes towards mathematics, Emotions or feelings experienced	Schoenfeld (1992)	*Strengths and Weaknesses, Good and bad Experiences, Study and Exam Habits, Expectations, fears, need and difficulties (Google Forms) using inductive coding and 2-staged analysis*
Met-befores	Current structure that students have in their brains as a result of experiences they have met before related to mathematical thinking, learning and problem solving	Tall (2013)	*Classroom Observations, Math autobiography (Google Forms) and 2-staged analysis*
Problem Solving Strategies	Global or local strategies learned from mathematics courses	Pólya (1946), Schoenfeld (1992), Mason (2010)	*Mason's Problem Solving Strategy using worksheets and F2F student-teacher interaction and Written Activity Response Analysis*
Use of Resources	Social Resource: Peers, Experts	McGinn and Boote (2003), Schoenfeld (1992)	*Social Resource: Peers, MTL Teacher, BLOSSOMS Expert*
	Material Resources: textbooks, time, computers		*Material Resources: time (1hr/week), ALEKS, BLOSSOMS Videos*
	Use of Resources: metacognitive processes such as planning and monitoring		*Variation in the use of resources among students*
Mathematical Practices	Activities or actions that engineers or mathematicians engage in, or activities that involve mathematics.	Schoenfeld (1992) Cardella (2006)	*Context-Rich Problem Solving Activity based worksheets are developed , Using Prompts, Questions and Themes in the F2F*

	UNDERSTANDING THE PROBLEM
First. You have to *understand* the problem.	*What is the unknown? What are the data? What is the condition?* Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? Draw a figure. Introduce suitable notation. Separate the various parts of the condition. Can you write them down?
	DEVISING A PLAN
Second. Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.	Have you seen it before? Or have you seen the same problem in a slightly different form? *Do you know a related problem?* Do you know a theorem that could be useful? *Look at the unknown!* And try to think of a familiar problem having the same or a similar unknown. *Here is a problem related to yours and solved before. Could you use it?* Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible? Could you restate the problem? Could you restate it still differently? Go back to definitions. If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other? Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?
	CARRYING OUT THE PLAN
Third. *Carry out* your plan.	Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?
	LOOKING BACK
Fourth. *Examine* the solution obtained.	Can you *check the result?* Can you check the argument? Can you derive the result differently? Can you see it at a glance? Can you use the result, or the method, for some other problem?

Figure 2.7: Detailed stages of Polya's Problem Solving Strategy (Polya, 1957)

Mathematical thinking and the use of natural powers of sense-making are related to each other in a circular manner. "Mathematical thinking is applying powers to mathematical topics; mathematical topics arise as the result of thinking mathematically!" (Mason and Johnston-Wilder, 2004, p. 187). Although it is hard to specify what makes a problem mathematical, or what it means to solve a problem mathematically. Polya promoted the methodical problem solving as:

> "What is know-how in mathematics? The ability to solve problems–not merely routine problems but problems requiring some degree of independence, judgment, originality, creativity. Therefore, the first and

> foremost duty of the high school in teaching mathematics is to emphasize methodical problem solving."
>
> (Polya, 1962, pp. xi–xii)

The problem-solving approach by Polya has four stages. First stage is to comprehend the problem, the second is to formulate a strategic plan, third is to execute it and the final stage is to evaluate the solution (Polya, 1957). The details of all the stages are shown in Figure 2.7. Polya's approach served as a foundation for future research in problem solving. Mason et al. (1982) and Schoenfeld (1985) further added components to understand the process of problem solving explicitly (Tall, 2002).

2.6.2 Mason's problem solving strategy

> "John Mason (b. Ontario, 1944–) was profoundly influenced in his teaching by seeing a film in 1967 of George Polya teaching some undergraduates, by year-long contact with the mathematician and teacher J. G. Bennett (1897– 1974) and by contact with Gattegno"
>
> (Mason and Johnston-Wilder, 2004, p. 24)

In a classic text, Mason et al. (2010) explain how to think mathematically through engaging readers in the act of thinking for themselves.

> "Three kinds of involvement are required [to think mathematically]: physical, emotional and intellectual. Probably the single most important lesson to be learned is that being stuck is an honourable state and is an essential part of improving thinking. However, to get the most out of being stuck, it is not enough to think for a few minutes and then read on. Take time to ponder the question, and continue reading only when you are convinced that you have tried all possible alleys. Time taken to ponder the question and to try several approaches is time well spent." (Mason et al. 2010:p ix-x)

Mason presents a more empathetic framework for problem solving that has three stages. The “entry” stage is the first that corresponds to the first two stages of Polya’s framework whereas his second and third stages are named as “attack” and “review” equivalent to the third and fourth stage of Polya’s framework. In the entry phase, the person solving the problem tries to understand the context of the problem by manipulating given information and then summarizing the given and required information along with recalling the learnt strategies at the end of this phase. The attack phase is all about trying different strategies and ends with success or a failure. If it is a success then he can move on to next stage, otherwise he will have to revisit the first phase to select another way of tackling the problem. The last phase is all about reviewing the procedural steps of the problem solving. The transition in mood from quantitative manipulation to qualitative attacking and then reviewing helps the learner to keep on iterating the process to solve new problems (Mason et al., 2010). A summary of the book “Thinking Mathematically” is illustrated in the form of questions and answers compiled in Table 2.9.

2.6.3 Schoenfeld’s framework for Problem Solving Analysis

Schoenfeld (1985) not only presented a framework for problem solving, but also guided for the analysis of successful and unsuccessful attempts to solve difficult mathematical problems. He discussed four necessary and sufficient factors to fully understand the problem solving failure or success process. The knowledge base is the first factor that is related to prior knowledge of the learner; problem-solving strategies are termed as a second factor that helps to understand the selection of tools and techniques to tackle an unseen problem. Monitoring and self-regulation as a third contributing factor describes the resource management capabilities of individuals. The last factor is beliefs and affect that helps to understand the individual’s way of relating mathematics with them as well as with their environment and society to solve the problems around them. Since the above stated factors are necessary and sufficient, if we miss any of the above factors of the analysis of somebody’s problem solving attempt, then there is a risk of missing major information about the overall process of success or failure. There can be situations when mathematical knowledge base is not sufficient to make the problem solving successful. Sometimes the use of heuristic problem solving strategies speedup or slowdown the process to reach final goal. In the same way, the effective and ineffective management of available resources can help to make the problem solving process as final success or failure. Individuals' beliefs about his capabilities and the other three factors are also important to analyze the problem solving and may lead to success or failure (Schoenfeld, 1985).

2.7 How People Learn Meta-Framework for Blended Learning

How People Learn (HPL) model known as meta-framework for instructional design is defined by Bransford et al., (1999) and its four main components are shown in Figure 2.8. The underlying theories related to student-centered, knowledge-centered, assessment-centered and community-centered frameworks (Marilla, 2008) are shown in Figure 2.9 to 2.11 (Mahmood et al., 2012) with the help of self-explanatory concept maps. The selection of related theories is based on the research objectives.

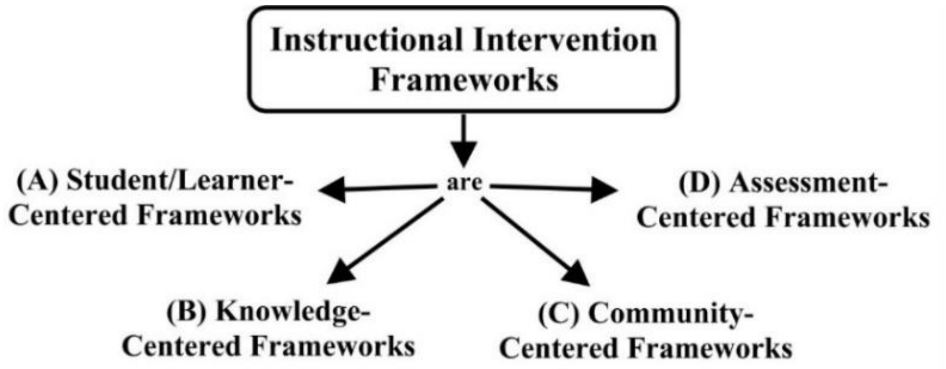


Figure 2.8: Meta Framework of HPL showing all types of instruction intervention frameworks

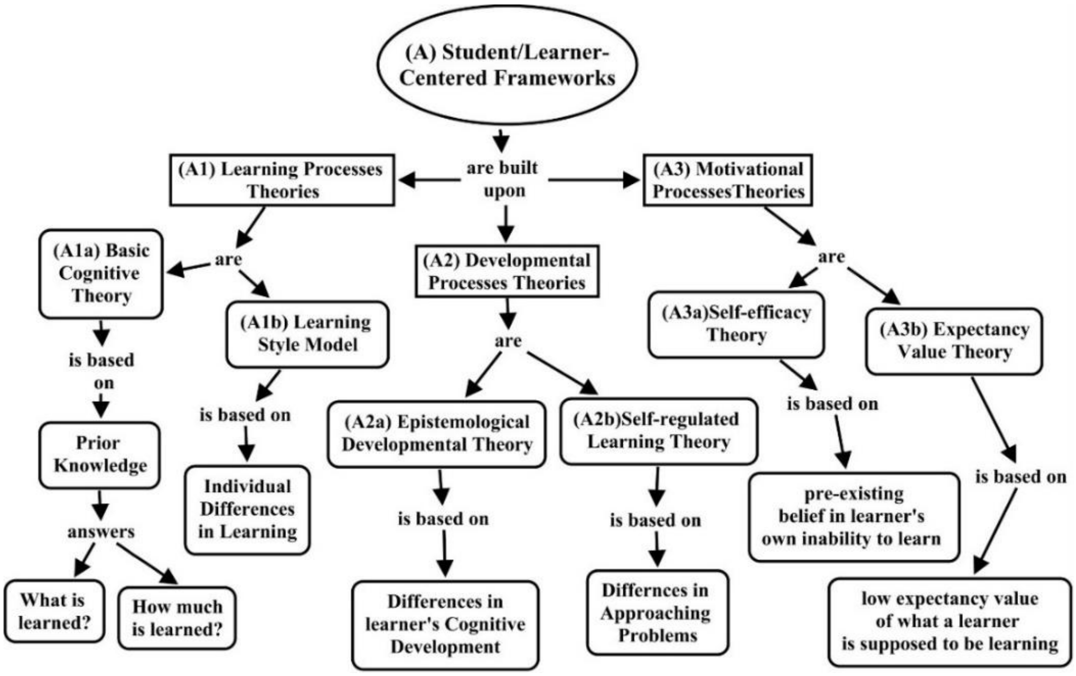


Figure 2.9: Student/Learner- Centered Frameworks and related theories

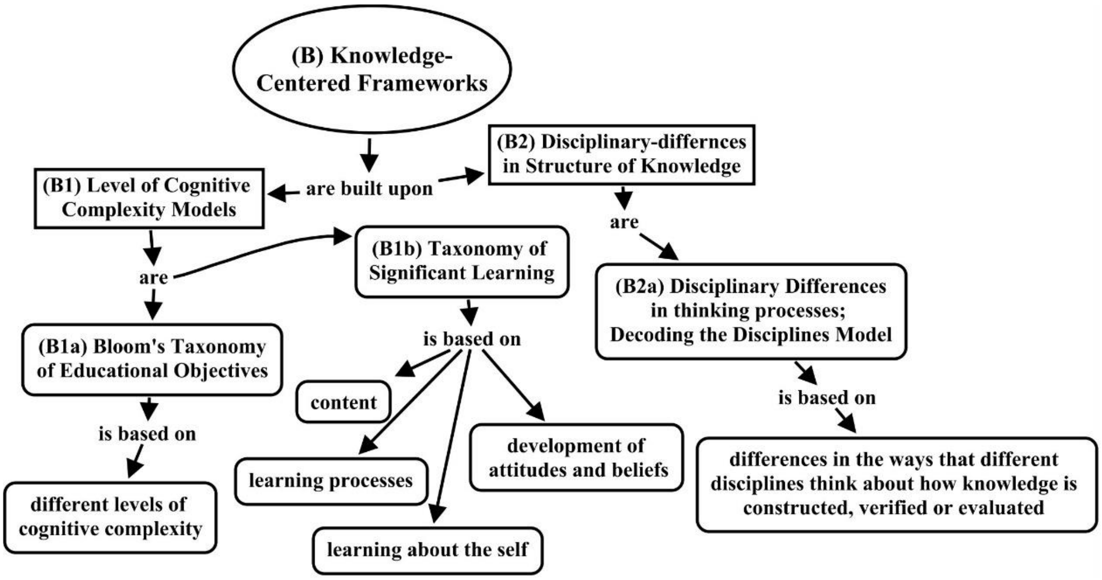


Figure 2.10: Knowledge-Centered Frameworks and related theories

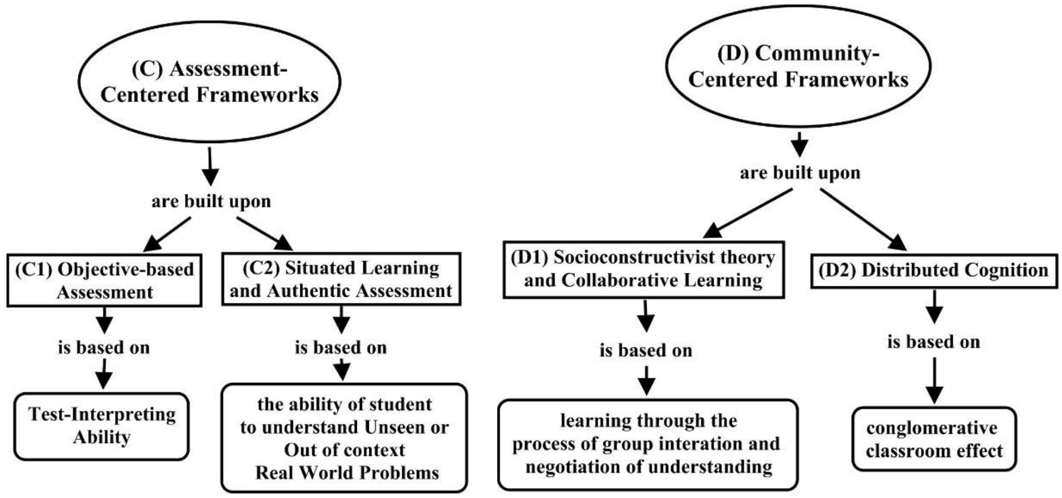


Figure 2.11: Assessment Centered and Community Centered Frameworks and related theories

The HPL meta-framework recommends that classroom environments should simultaneously be learner-centered, knowledge-centered, assessment-centered, and community-centered.

2.8 Three Worlds of Mathematical Thinking Theoretical Framework

Research in mathematical thinking showed that long term human learning is based on our shared genetic structures and individualized knowledge base as a product of our personal experiences in life. According to Tall (2008), we all are born with some fundamental human characteristics embedded in our genetic structures that enable us to think mathematically and our personal learning skills grow as we construct knowledge by going through different experiences and situations in our lives (Tall, 2008). Tall used the term "set-befores" to refer to the innate abilities of human brains like number sense, level sense, weight sense, and attention sense to help us socialize with others around. He then identified three fundamental set-befores to support us to think mathematically. *Recognition* of patterns, their similarities and dis-similarities, *repetitions* of schemas until they become hardwired in our brains, and *language* to describe our knowledge, feelings and attitudes about what is happening around us. The development of mathematical thinking of humans depends profoundly on these three set-befores. He also coined a term called “met-befores” that refers to “a current mental facility based on specific prior experiences of an individual." The met-befores can be either consistent or inconsistent with a newly met situation. A consistent met-before leads to advantage and an inconsistent met-before causes mental confusion that hinders the learning process of new concepts (Tall, 2008).

According to Tall, the **embodied**, **symbolic,** and **formal** modes of mathematical thinking involve pattern variants to make sense of whatever is happening around us mathematically. *Perceptions* provide foundation for the embodied world around us. We see thing as we perceive to make sense of them, leading to Van Hiele type of development based on recognizing of objects along with perceiving and describing their different properties (Pegg and Tall, 2005). *The symbolic manipulations* (calculations) provide the reliance for the "proceptual" world. We can symbolically check an arithmetic or algebraic statement to be true or false. It supports the transition of a human brain from embodied concepts to mathematical thinking processes in the symbolic world (Pegg and Tall, 2005; Tall, 2004). The *proof* supported by conceptual but formal definitions provide base for formal world of mathematics (Tall, 2008). By combining the embodied concepts with symbolic manipulations, we can learn to experience their bidirectional interaction with the formal world (Tall, 2004). Tall presented the above three different ways of representation for the conceived and constructed concepts as three interrelated worlds of mathematical thinking shown in Figure 2.12 and painted them as constructions from objects as one's own perception, actions on those objects and the properties related to them. By exploring foundational issues and their relationship with the long-term development of mathematical thinking, concepts can be achieved through Tall's framework with general expression. It covers from the ways of making sense of the world that surrounds us and by using our perceptions and actions to foster more sophisticated ideas using the ability to utilize language and manipulate symbols in succession (Tall, 2013).

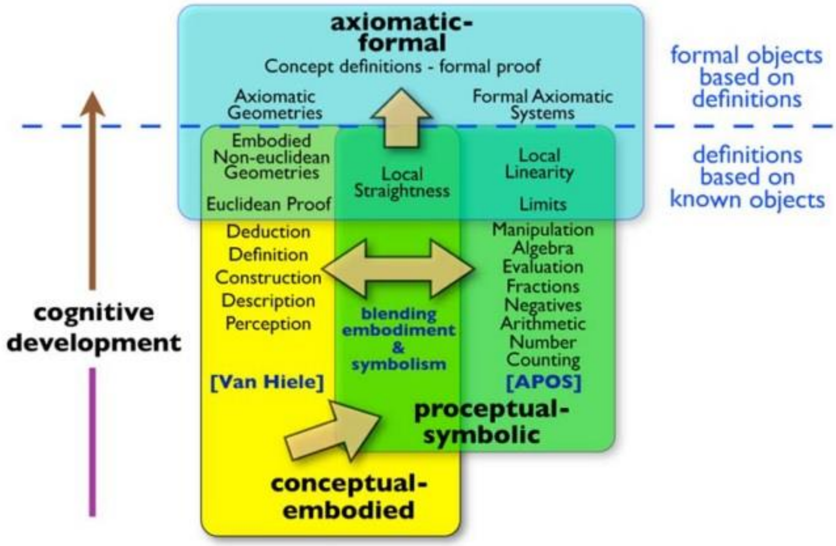


Figure 2.12: The Three Worlds of Mathematics illustrated by selected aspects. (Tall, 2008)

Guided by Tall's framework of three worlds of mathematics, our work will focus on students' met-befores related to mathematical thinking, learning, and problem solving. To make sense mathematically, learners construct new knowledge on the top of the already met-before experiences in their lives. Sometimes, these met-before experiences support learners to learn new concepts and sometimes they feel inapt when a new context is encountered. Knowing the set-befores of humans and exploring the met-befores of first year engineering students within three distinctive worlds of mathematics can help us to interpret the insights during their historical journey of mathematical thinking and learning.

Since axiomatic formalism was not emphasized in previous curriculum as *"it builds formal knowledge in axiomatic systems specified by set-theoretic definition, whose properties are deduced by mathematical proof,"* mentioned by Tall so the researcher did not include that in the theoretical framework of this research. Conceptual and Symbolic worlds of mathematical thinking are adapted as a theoretical basis of this study as shown in Figure 2.13.

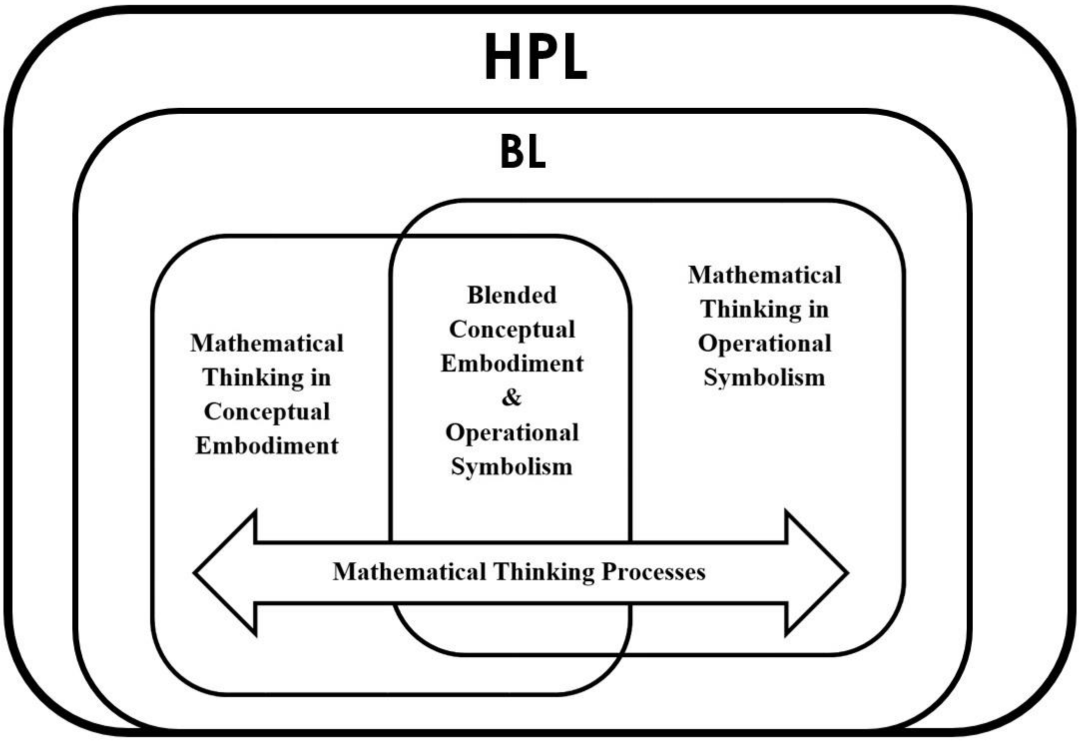


Figure 2.13: Theoretical framework for this study: Conceptual and Symbolic activation of Mathematical Thinking processes (Tall, 2008) in a blended learning environment based on HPL meta-framework

In this study, the researcher intended to investigate the key met-befores related to the fundamental concepts like "functions" for first year engineering students before extending it to the class of advanced concepts like "transcendental functions" and before introducing the mathematical thinking lab (MTL). Some met-befores will be expected to be supportive whereas others might cause hindrance in learning the extended concepts. The researcher also planned to address the emergent need of investigating student's educational background, their prior experiences with mathematics, their current knowledge state in mathematics, their submitted individual and group works and their expectations and fears in future mathematics learning. Knowing how our students think will help to understand their mathematical thinking patterns, also making sense of their errors will help to improve the instruction. The researcher will explore the diagnostic insights of the underlying issues at the start of the main study and draw guidance to redesign the mathematical thinking experience for first year engineering students.

2.9 Student Personas

Jennifer Turns and her colleagues shared some insights about student personas through a poster presentation. They highlighted:

> "The engineering education community needs to accelerate the rate at which research on engineering students influences educational practice. Personas are a technique that is garnering attention as a productive way to "personify" user research and get it into the hands of designers as they practice design. Personas are composites based on user research data that capture key elements of the dataset to represent an archetypal user. Personas are labeled with a name to provide easy reference to the persona (and therefore to the underlying research results). Narratives are easier to remember than abstract or detailed research results and therefore personas include a narrative that illustrates the data represented." (Turns et al., 2013)

They also investigated the use of students' personas as "a tool to help the engineering education community communicate implications of research on engineering students to engineering faculty, staff, and students." They developed four personas based on existing research on the engineering students' experiences at the University of Washington (UW), shared those personas with professional educational developers, engineering professors, and engineering administrators through a series of workshops. They found that workshop attendees empathized openly with the problems the students (personas) were facing, spent time speculating about the details of the student experiences, and discussed a variety of potential approaches (and roadblocks) for addressing similar student challenges in their own complex contexts (Turns et al., 2013).

Coinciding with the stage II analysis of this research, the researcher got a chance to attend one of the above mentioned online workshop "Bridging the Gap between Research Findings and the Design of Educational Experiences" on the 25th of January 2014 facilitated by Jim Borgford-Parnell, Jennifer Turns and Toni Ferro from the University of Washington (UW), and learned "how educators can utilize the personas to improve their practices." The purpose of the workshop was to use personas as a method of communicating education research to Community of Practice (CoP) that is "a group of people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis" (Wenger et al., 2002). The above experience gave birth to the idea of developing personas for this study. However, instead of creating fictitious personas, there is a need emerged to develop perosnas (Faily and Flechais, 2011; Goodwin, 2008) based on evidences collected through a qualitative research unfolding their potential as pedagogical tools in engineering education. The researcher adapted the persona development process from design (Nielsen, 2013), explained in the Chapter 3 (Section 3.10) and described the narratives of student personas in the Chapter 5 (Section 5.3) from the lens of an action researcher. The researcher used pseudonyms for the personas to make the description interesting and to address the need of anonymity.

2.10 Research Paradigm Considerations

There are a multitude of 'accepted' research paradigms in engineering education (Chism et al., 2008). To avoid the confusions, "it is therefore important to explicitly state the chosen paradigm option" (Baillie and Douglas, 2014). This section will provide a foundation for the paradigm selection (Section 3.2) process of this study. Generally, engineering education researchers distinguish among four paradigmatic alternatives: post-positivist, interpretive, critical and postmodern/post structural (Chism et al., 2008; Koro-Ljungberg and Douglas, 2008). This differentiation among post-positivist, interpretive, critical paradigms and postmodern/post structural as proposed by Creswell, Chua and other researchers has been studied and employed by many other researchers (Chua, 1986; Creswell, 2007; Guba and Lincoln, 2005; Koro-Ljungberg and Douglas, 2008). It is equally applicable to the field of engineering education research. Comparison among four epistemological paradigms in Tables 2.9a, 2.9b and 2.9c will provide a summary of issues that would help the researcher to determine her own philosophical assumptions (Chism et al., 2008; Chua, 1986;

Creswell, 2007; Guba and Lincoln, 2005; Heinze, 2008; Koro-Ljungberg and Douglas, 2008). Ontological, epistemological, methodological, and axiological philosophical assumptions are discussed for all the four paradigms in tabular form. The choice of methodology is dictated by the assumptions on epistemological beliefs. Epistemological and methodological assumptions are therefore interrelated and constitute beliefs about knowledge (Chua, 1986). In Table 2.10, other assumptions related to methods, purpose, role of the researcher and research outcome are compared for all four paradigms.

When researchers decide to conduct a qualitative study, they embrace their own philosophical assumptions and try to find the closest agreements through exploring the available worldviews that lead to "how to further proceed" in their research. Creswell (2007, pp. 16–18) described five philosophical assumptions but the researcher's freedom in this study was constrained for Rhetorical (what is the language of research?) assumptions, so excluding Rhetorical assumptions, the remaining four are described below:

> ***"Ontological*** (What is the nature of reality): Relates to the nature of reality and its characteristics. Researchers embrace the idea of multiple realities and report on these multiple realities by exploring multiple forms of evidence from different individuals' perspectives and experiences. ***Epistemological*** (How researchers know what they know): Researchers try to get as close as possible to the participants being studied. Subjective evidence is assembled based on individual views from research conducted in the field. ***Methodological*** (Comparison of methods used in the process of research): inductive, emerging, and shaped by the researcher's experience in collecting and analyzing the data. ***Axiological*** (The role of values in research): Researchers make their values known in the study and actively reports their values and biases as well as the value-laden nature of information gathered from the field."

Table 2.9a: Comparison among epistemological perspectives. Adapted and modified from Koro-Ljungberg & Douglas (2008), Creswell (2007) and Chua (1998) Researcher's modifications are in the last column and italicized.

↓Philosophical Assumptions →Paradigm	**Post-positivist**	**Interpretivist (constructivism, social constructionism, hermeneutics, phenomenology, symbolic interactionism)**	**Critical/ Emancipatory**	**Postmodern/ Post-Structural**	***Researcher's beliefs and point of view***
Epistemological Assumptions • How researchers know what they know? • What is considered to be true' and 'how it can be proven? • What is the nature of the relationship between the research and the researcher?	• Modified Objectivist. The goal is objectivity, but pure objectivity is impossible. • Results are "probably" true. • Theory is separate from observation that may be used to verify or falsify a theory. • Hypothetico-deductive account of scientific explanation accepted.	• Researcher and participants are linked, constructing knowledge together. • Scientific explanations of human intention sought. Their adequacy is assessed via the criteria of logical consistency, subjective interpretations, and agreement with the actors' common sense interpretation.	• Criteria for judging theories are temporal and context-bound. • Knowledge is mediated reflectively through the perspective of the researcher.	• Grand narratives • Grand narratives are the theoretical frameworks that describe social behavior. • In place of grand narratives, the "post" perspectives emphasize differences, multiple perspectives on the same phenomenon, and lack of absolutes. • "Deconstruction" is an important element.	• *The researcher believes in subjective interpretations and agreed with the actors' common sense interpretations*. • *The researcher believes in closely observing and interacting with the participants of the study.* • *Subjective evidences should be collected to support the findings*

Table 2.9b: Comparison among epistemological perspectives. Adapted and modified from Koro-Ljungberg & Douglas (2008), Creswell (2007) and Chua (1998) Researcher's modifications are in the last column and italicized.

↓Philosophical Assumptions →Paradigm	Post-positivist	Interpretivist (constructivism, social constructionism, hermeneutics, phenomenology, symbolic interactionism)	Critical/ Emancipatory	Postmodern/ Post-Structural	*Researcher's beliefs and point of view*
Ontological Assumptions • What is the researcher's view of the nature of reality? • How does a researcher relate the research to the nature of reality and its features?	• Critical Realism. • Single falsifiable and "real" objective reality. • Empirical reality is objective and external to the subject. • Human beings are also characterized as passive objects; not seen as makers of social reality and cannot know it for sure	• Relativist • Multiple subjective realities. • Social reality is emergent, subjectively created, and objectified through human interaction • All truth is "constructed" by humans and situated within a historical moment and social context. Multiple meanings exist of perhaps the same data	• Historical Realism. • Reality can be understood, but only as historically constructed and connected to power. • Multiple subjective and political realities. • Human beings have inner potentialities, which are alienated (prevented from full emergence), through restrictive mechanisms.	• Multiple fragmented realities	• *Researcher embraces the notion of multiple subjective realities* • *The researcher intends to explore and report on various forms of evidence from different individuals' experiences, perspectives, and written activity responses.* • *The researcher also believes that human interaction is essential to comprehend the subjective realities.*

Table 2.9c: Comparison among epistemological perspectives. Adapted and modified from Koro-Ljungberg & Douglas (2008), Creswell (2007), Guba & Lincoln (2005) and Chua (1998) Researcher's modifications are in the last column and italicized.

↓Philosophical Assumptions →Paradigm	Post-positivist	Interpretivist (constructivism, social constructionism, hermeneutics, phenomenology, symbolic interactionism)	Critical/ Emancipatory	Postmodern/ Post-Structural	*Researcher's beliefs and point of view*
Methodological Assumptions • How can enquirer find out? • How to justify the selected methods that are used in the research process?	• Includes both quantitative and qualitative methods of data collection and analysis. • Seeks reduction of bias through qualitative validity techniques (e.g. triangulation) and quantitative allow for generalization favored.	• Generally qualitative, research through dialogue. • Variants are Narrative Analysis, Action Research, Case Study, Ethnography, Grounded Theory, Discourse Analysis Phenomenology, Phenomenography	• Focused on investigator/ participant dialogue, uncovering subjugated knowledge and linking it to social critique • Historical, ethnographic research and case studies more commonly used	• Grand Narratives	• *The researcher believes in real life interaction with informants, so action research is employed in this reseaerch.* • *The research focuses on improving practice by understanding her students, their met-befores, and investigating the challenges related to their learning about how to think mathematically during problem solving activities*
Axiological Assumptions • What is the role of values and biases in the research?	• Excluded • Influence denied	• Included • Formative	• Included • Formative	• Included • Formative	• *Researcher's values and biases are known and actively reported formatively*

Table 2.10: Comparison among epistemological perspectives. Adapted and modified from Koro-Ljungberg & Douglas (2008), Creswell (2007), Guba & Lincoln (2005), and Chua (1998) Researcher's modifications are in the last column and italicized).

↓Items →Paradigm	Post-positivist	Interpretivist	Critical/ Emancipatory	Postmodern/ Post-Structural	*Researcher's beliefs and point of view*
Methods • How to select research methods and approaches?	• Methods and variables defined in advance, hypothesis driven	• Methods and approaches emerge and are to be adjusted during the study	• Methods and approaches designed to capture inequities	• Methods and approaches generated during the study • When data are deconstructed, they are broken down to reveal hidden or latent meanings, revealing the assumptions inherent in the data and their interpretation.	• *Researcher used multiple methods and techniques those were deductive, inductive, emerging, and were shaped by the researcher's experience in collecting and analysing the data.*
Purpose • What is the purpose of the study?	• To find relationships among variables, to define cause and effect	• To describe a situation, experience, or phenomenon	• To produce a socio-political critique	• To deconstruct existing 'grand narratives'	• *To investigate and describe the conflicting met-befores of students while thinking mathematically* • *To design, implement ,explore and communicate the pragmatic implications of Blended learning on Mathematical Thinking during Problem Solving*
The role of researcher • What is the role of the researcher?	• The researcher is detached from participants and distanced from data	• Researcher and participants are partners	• Researcher and participants are activists	• Researchers and participants have various changing roles	• *The researcher is an instrument and works in collaboration with other teachers and participants*
Outcome or research product • What are the outcomes of the research?	• Context-free generalizations	• Situated descriptions	• Critical essays, policy changes	• Re-conceptualized descriptions of the phenomenon	• *Situated descriptions of the met-befores related to students' mathematical thinking and a commentary on the activation of mathematical thinking processes and student personas as potential pedagogical tools.*

2.11 Methodology Considerations

"Research methodology consists of the assumptions, postulates, rules, and methods (the blueprint or roadmap) that researchers employ to render their work open to analysis, critique, replication, repetition, and/or adaptation and to choose research methods" (Given, 2008, p. 516). Research methodology and research methods are often used interchangeably in literature, here research methods are considered as "the tools or techniques with which researchers collect their data." The overall research methodology of a research means selecting tools or techniques wisely based on the "larger set of assumptions and procedures." The constituent components of research methodology are overarching paradigms, facets of research design, methods of data collection, and data analysis and knowledge dissemination (Given, 2008). While discussing "a position on methodology" in mathematics education research, Burton (2002, p. 4) provided a few guiding questions: "why the researcher chose that focus; why the study was designed by the researcher in that way; why alternatives were rejected; what were the questions the researcher was asking; and how the researcher ensured that confidence could be felt in the data gathered and in their analysis" of those data. The researcher utilized these guiding queries to describe the research methodology of this study. Cousin (2009) helped to understand the association between method and methodology being not straightforward because "different people might use the same methods with quite different values and aims in mind" (Cousin, 2009, p. 5). Light and Case (2011) also helped to grapple with "methodology as a critical area" to maintain the quality of this research and to cover the scope of this research and that helped the researcher to develop herself as an engineering educator.

2.12 Summary

In this chapter, the key concepts, ideas, and rationales were discussed followed by the research paradigm and methodology considerations.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

The previous chapter outlined the review of literature related to the key areas of this research followed by the rationale of using Mason's problem solving strategy and BLOSSOMS modules to design and implement a blended learning environment conducive to mathematical thinking and developing student personas. In this chapter, the research design process is discussed as an important part of the overall research process as shown in Figure 1.1 of Chapter 1. The research design process is replicated from (Heinze, 2008) based on the philosophical assumptions of the researcher, the research settings, and the research questions under investigation.

Before discussing the philosophical assumptions of this study, it is essential to understand that there is no "ultimate" or "ideal" set of these assumptions and the dominance of one philosophical paradigm to another is not claimed. The researcher compared the available philosophical paradigms while keeping in view the research context of activating mathematical thinking processes during problem solving activities through blended learning. The three interwoven research sub-domains, those of blended learning, mathematical thinking, and problem solving have a common implication on engineering mathematics, making the discoveries of this research significant to prospective mathematics and engineering educators.

Firstly, the underlying philosophical assumptions are formulated, which focus on classifications based on Chism (2008), Chua (1986), Creswell (2007), Guba & Lincoln (2005), Heinze (2008), and Koro-Ljungberg & Douglas (2008) and their widely accepted classifications in scientific and social research. The classifications are used in the process of research design to examine the beliefs about reality, knowledge, and comparison of methods. The researcher discussed four paradigms – post-positivist, interpretive, critical, and postmodern/post-structural, and their applications in the current research state. The nature of this research demands to adopt interpretive paradigm but with sound justifications. This led the researcher to select an action research as the research methodology. The rationale for the selection of the action research will be given in Section 3.3.3 followed by the cycle and process of action research along with its limitations and characteristics. Finally, the data collection methods, data analysis, and employed processes are documented.

3.2 Research Paradigm Selection

The researcher selected the interpretivist paradigm or worldview for this entire research because of its similarity with the researcher's philosophical assumptions presented in the last column of Tables 2.12 (a), (b), and (c). This research paradigm will enable the researcher to view multiple subjective realities based on the students' verbal and written activity responses and to describe a situation, experience, or phenomenon dealing with mathematical thinking processes within a blended learning environment. Due to diversity in the processing styles of the human brain, methods and approaches of the above analysis would emerge and have to be adjusted during the study. This requires the researcher to work on a partnership basis with the respondents to explore their mathematical thinking processes based on a validated stimulus in the form of problem solving activities. The findings will lead the researcher to record situated descriptions presenting the activation of mathematical thinking processes during those activities through blended learning. In this research, the researcher bracketed the contextual boundaries around the geographical region of Malaysia, specific mathematical thinking processes, first year engineering students in a local research university (RU) and constructivist teaching that is "based on the assumption that learning is a process whereby a student is actively engaging with constructing their knowledge. The role of the teacher is that of "guide on the side" and as a facilitator of that learning" (Heinze, 2008) and blended learning environment.

3.3 Qualitative Research Process

Qualitative research is all about knowing what people think, and how they feel, what they say they think and how they say they feel in "their natural settings, attempting to make sense of, or interpret, and phenomena in terms of the meanings people bring to them." (Given, 2008, p. 312) It involves feelings and impressions, rather than numbers. Its focus is multi-method involving interpretative, subjective, and naturalistic approach (Nelson et al., 1992, p. 4).

The qualitative research process has four key elements as described by Crotty:

> "***Methods:*** the techniques or procedures used to gather and analyze data related to some research question or hypothesis. ***Methodology:*** the strategy, plan of action, process, or design lying behind the choice and use of particular methods and linking the choice and use of methods to the desired outcomes. ***Theoretical perspective:*** the philosophical stance informing the methodology and thus providing a context for the process and grounding its logic and criteria. ***Epistemology:*** the theory of knowledge embedded in the theoretical perspective and thereby in the methodology." (Crotty, 1998, p. 3)

The research process of this study is shown in Figure 3.1 and the adoption process and rationalization of research components will be discussed in the following sub-sections (Section 3.3.1-3.3.8) by debating the selection of respective epistemology (Interpretivism), theoretical perspective (Constructivism), methodology (Action Research), rationalizing the choices of research components, action research cycle and process, and research methods.

3.3.1 Interpretivism

The overarching perspective of interpretivism i.e. The theory of knowledge in this research is likely to consider the situation, participants and researcher's interpretation of contextual truth. This research will try to reveal the individual stories of the participants learning in new environments and to identify the underlying truth (Chism et al., 2008) embedded in their mathematical thinking elicitied by problem solving activity responses.

The interpretivist epistemological worldview has the following attributes (Chism et al., 2008; Oates, 2006): Increased understanding of people's subjective experiences of multiple realities; descriptions of the situation; researcher reflexivity; dynamic socially constructed meaning; methods and themes that emerge during the study; researcher and participants are partners; multiple interpretations; context-bound descriptors and study of people in their natural settings.

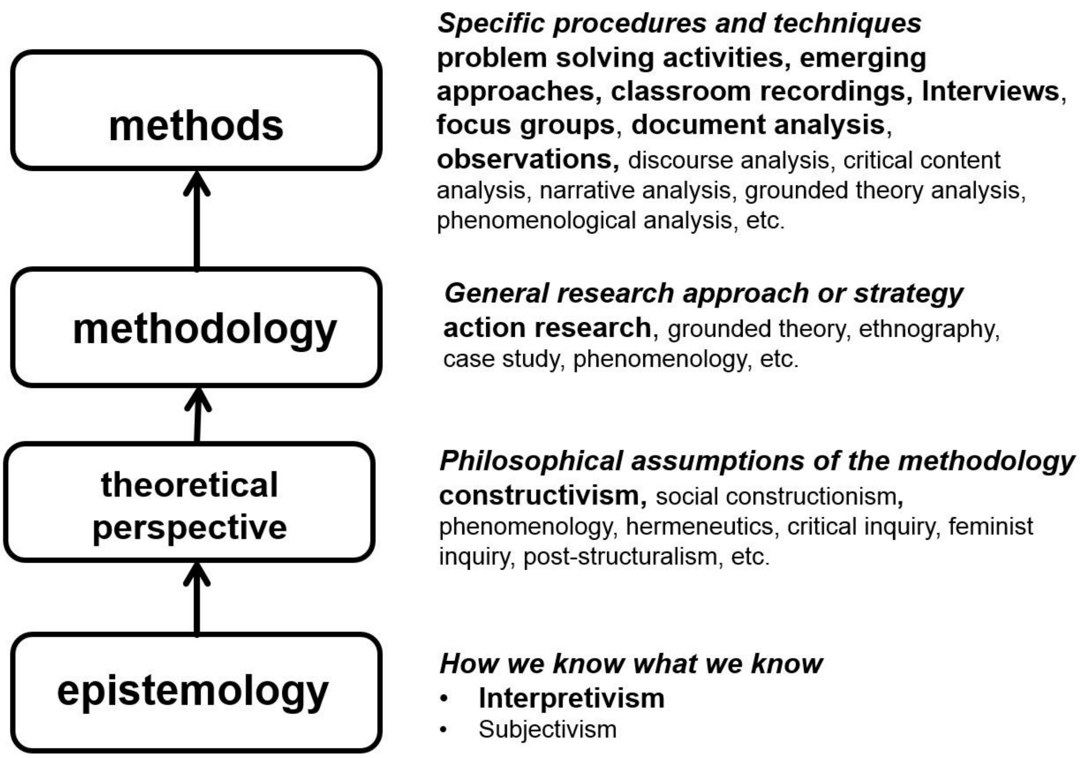


Figure 3.1: Qualitative Research Process adapted (Crotty, 1998) and modified by the Researcher for this study

Subjective realities are related to the fact that people perceive reality, it implies that different people might perceive the same situation differently. Thick context bound descriptions of multiple subjective realities are used to make inferences and decisions. Dynamically and socially constructed meaning highlights explanation of a certain incident and is transferred through social structural patterns like textual descriptions, which differ between groups and evolve slowly. Interpretations due to the reflexive work are fouled by subjective beliefs and perceptions of researcher, which should be stated in detail to understand their impact on the research process and its interpretations. Processes should be studied in natural settings instead of research laboratory so that real life complexity can be gained to its maximum level. Deviating from the positivist approach of having one conclusion, interpretivists scrutinize multiple interpretations and try to make inferences on the basis of most meaningful and convincing ones (Heinze, 2008). In quantitative research, the researchers usually hide their motivations or choices because of their positivist/post positivist beliefs that they are not "significant variables in their research; thus in testing a hypothesis, they expect other researchers handling similar data to come to the same conclusion that they find" (Bassey, 1995). By contrast, ".. to the interpretive researcher the purpose of research is to describe and interpret the phenomena of the world in attempts to get shared meanings with others. Interpretation is a search for deep perspectives on particular events and for theoretical insights. It may offer possibilities, but no certainties, as to the outcome of future events" (Bassey, 1995, p. 12).

3.3.2 Constructivism

Under the overarching epistemology of interpretivism, there are various theoretical perspectives that can be used to have additional focus to explain "how reality is defined." Out of multiple choices like constructivism, social constructionism, hermeneutics, phenomenology and symbolic interactionism, constructivism is selected because of its compatibility with the research purpose, problem, and questions.

"A study based on the constructivist perspective seeks to identify a particular meaning and how that meaning was created by individuals"(Chism et al., 2008, p. 6). The self-explanatory concept map shown in Figure 3.2 tries to provide a quick understanding of the concept of constructivism. It depicts that constructivism is an approach to learning in which students create and discover new knowledge, develop metacognitive skills and are responsible for their own learning, whereas the teacher is ranked as a coach, process experts, behaviour/skill model and follows the principle of "guide on the side" rather than "sage on the stage". The constructivist-learning environment is not only positive, interactive, cross-disciplinary and project based but also allows the instructional designer to plan learning activities that are interesting, engaging, socially cognitive, collaborative, authentic and based on real-world. These activities have not predefined content and outcomes; rather allow lateral, tangential, parallel, nonlinear, and linear threads of inquiry. The resulting meta-cognitive skills are comprised of elements like i) knowledge connectivity and construction, ii) self-efficacy, control and awareness, iii) monitoring, planning and evaluation.

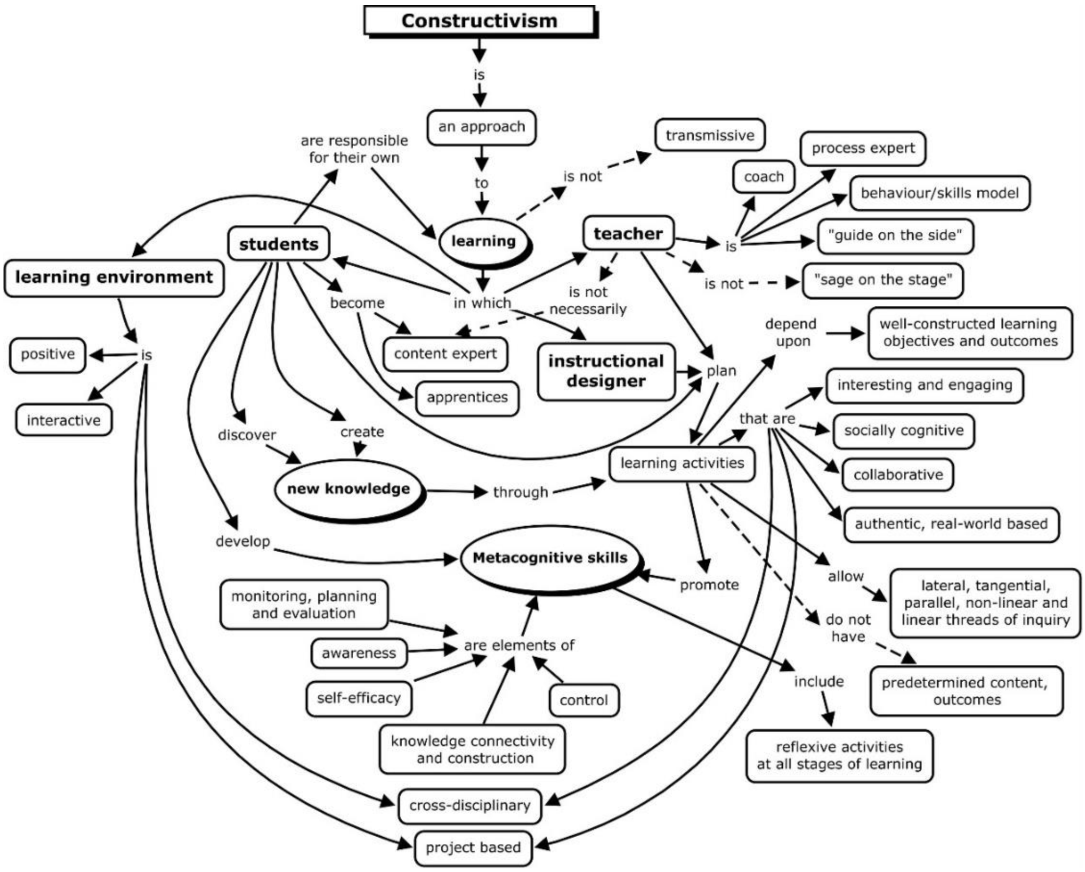


Figure 3.2: Concept Map of Constructivism adapted and modified from (Macdonald, 2008)

According to the theoretical perspective of constructivism shown in Figure 3.3 as a concept map, learning is considered as an active and constructive process in this study. The learners not only construct the information based on their prior knowledge, but also create their own subjective representations of objective realities. Originators and important contributors of constructivism are, Jean Piaget, John Dewey, Bruner, Lev Vygotsky, and others (Bruner, 1961; Dewey, 1997; Piaget, 1972; Vygotsky, 1978) and their research lead the researcher to use this theoretical foundation for developing mathematical thinking skills among future engineers. The concept map also shows different models based on constructivism as a constructivist instructional model, conceptual change model, and predict-explain-observe model along with their operational components. The constructivist instructional model is employed in this research. Constructivism is also the foundation of inductive teaching and learning methods as shown in the same concept map.

In this research, the construstivist perspective is further guided based on the following sense:

> “Constructivism is used to advocate movement away from lecturing towards investigations and discussion in order that learners would be supported in ‘making sense’, in ‘constructing meaning’. But the constructivist notion that learners have to make their own sense, or to be enculturated into the practices of the community applies equally well to learners sitting in rows in lectures and learners exploring for themselves or engaging in scientific debate. What matters is whether the learners are in a position to be able to see, hear and make sense of what they are told and shown.”
>
> (Mason and Johnston-Wilder, 2004, pp. 229–230).

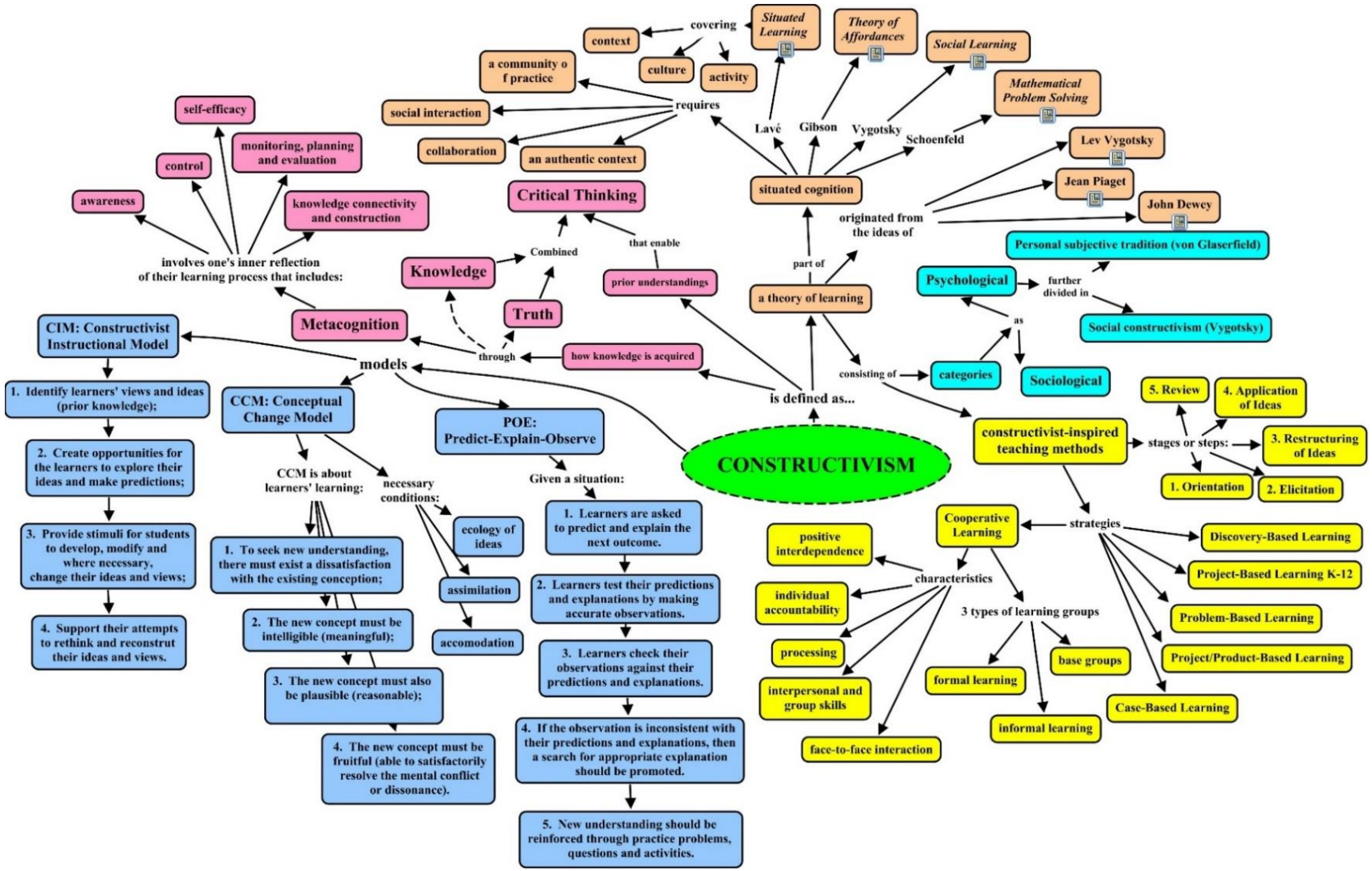


Figure 3.3: Another concept Map of Constructivism adapted and modified from (Maydonik, 2010)

3.3.3 Rationalizing the Choice of Action Research for this Study

Koro-Ljungberg and Douglas (2008) highlighted "the limited explicit discussion" about methodology "in the engineering education literature" in general and "in mathematics education research (Burton, 2002)" in particular. The dominance of qualitative research design in engineering education for the last 15 years (Jayarajah et al., 2012) and the emerging methodologies in engineering education (Light and Case, 2011) helped to rationalize the choice of action research out of the pool of ethnography, case study, phenomenography, grounded theory, narrative analysis, discourse analysis, and action research. All the emerging methodologies are concisely discussed (Light and Case, 2011) based on their defining features like definitions, brief history, study type, the techniques employed in leading a research and prospective challenges to justify their use (Creswell, 2007) in Appendix H [Tables H(a) to H(g)]. A summarized comparison of emerging methodologies with action research is compiled based on the discussion by Light and Case (2011), Creswell (2012, 2007) and (Whitehead, 2009) in Appendix R that helped the researcher to decide which approach is best to use and rationalized the choice of action research as a living educational theory for this study.

The second rationale for the choice of action research is based on the context of this study. The previous studies highlighted the lacking mathematical thinking and problem solving skills among future engineers whereas there is still an absence of a broader picture of what actually happens when we use innovative pedagogies to improve the above-required skills among the prospective engineers. Most of the research efforts were mainly limited to the cognitive aspect of mathematical thinking using theoretical opinion (34%) and more on the learners and learning trajectories mostly using teaching experiment (18%) and then by Glaser and Strauss (1967) constant comparison method (11%) (Argyle, 2012). Whereas only 9% qualitative work has been done by exploring the above issues (Argyle, 2012) and there is minimal research reporting on teaching and teacher's trajectories and giving a holistic picture of the whole ecosystem while implementing new pedagogies in the actual engineering classroom.

The third rationale for the choice of action research was based on “the kind of enquiries” and those were “How do I improve the way I learn about my students, their learning trajectories in a new environment and the way I tackle the on stage issues?” “How the transformation occurs during my epistemological transition from a positivist practitioner to a reflective teacher, with the intent of improving my practice and constructing knowledge in my own living educational theory (Whitehead, 2014)?” This study was a form of self-reflective problem solving, which enabled the researcher to better understand and solve pressing problems in educational and social settings. The action (what I do) aspect of action research is about improving practice. The research (how I learn about and explain what I do) aspect is about creating knowledge about my practice (McNiff and Whitehead, 2010).

After critically comparing all the emerging methodologies based on their defining features and tenets, and the methodological considerations of the current research, the researcher decided to use action research because of its suitability with the context, purpose, problem, and research questions of this research. The permission to use active intervention is also favoured in selecting the action research as a valid methodology for this research while rejecting other methodologies due to the “restriction of influence on a situation by the researcher” (Heinze, 2008).

The current research is about considering the following factors: social complexity of the given research setting, dealing with social phenomena during problem solving, group dynamics, language and culture, informants specific profiles, space and sitting arrangements. All these factors are challenging in real life. It is very hard to articulate the cause and effect of different variables. Replicating the exact situations is difficult for different students, teachers, venues and the learning environments (Heinze, 2008).

Mann and Chang (2010) further mentioned three distinct but interwoven strands of action, knowledge, and learning to form the practice presented by Cheery (1999, p. 4) that helped the researcher to feel confident of her choice of action research methodology. Action research, unlike other emerging methodologies in engineering education, “allows for the creation of change whereas concurrently studying the effects" (Heinze, 2008). An interpretive worldview allows the action research to be carried out in an iterative manner incorporating the multiple subjective realities in the classroom, implementing new practices through blended learning, identifying issues and improvements during the intervention. Another advantage of action research as compared to other research methodologies is that "it acknowledges the existence of knowledge embedded in the practice rather than in vacuum" (Heinze, 2008). In conclusion, action research seems to be the most suitable methodology for this research. It is compatible to the interpretive philosophical beliefs, researcher's assumptions and research area which gives the researcher the freedom to intervene actively in the research. Additionally, action research offers a chance for action researchers to have insights into the big systems and allows them to work as researchers and practitioners simultaneously. Action Research cannot be used to draw comparisons, show statistical correlations, or demonstrate a cause and effect relationship (McNiff and Whitehead, 2006). It can be used when you want to evaluate whether what you are doing is influencing your own or other people's learning, or whether you need to do something different to ensure that it is working (McNiff and Whitehead, 2006). According to Carr & Kemmis (1986),

> "Action research activity has two essential aims, both to improve and to involve. The focus of this improvement lies in three key areas: improving a practice; improving the understanding of a practice by practitioners and improving the situation in which the practice takes place." (Carr and Kemmis, 1986, p. 165)

In this particular research, the researcher addressed a local, practical problem, i.e. first year engineering students' lacking skills in mathematical thinking, their pertinent characteristics and associated challenges in different scenarios. The researcher also tried to empower, transform, and emancipate first year engineering students from situations that constrain their mathematical thinking powers/skills.

3.3.4 Rationalizing the Choice of BLOSSOMS Modules as Pedagogical Tools for this Study

During the phase of exploring and selecting resources for this study, the researcher searched for effective solutions and got amazed to see the development of Open Educational Resources (OERs). The researcher was overwhelmed by the online resources intended to promote mathematical thinking and satisfying most of the requirements for effective problem solving tasks. One of such initiatives was Massachusetts Institute of Technology (MIT)'s programme, identified as BLOSSOMS (Blended Learning Open Source Science or Mathematics Studies). The researcher's interest in using BLOSSOMS increased when "Dr. Zaini Ujang, Vice Chancellor of Universiti Teknologi Malaysia (UTM), and Professor Richard C. Larson, Director of the BLOSSOMS Project at MIT in Cambridge, Massachusetts USA, announced the kick-off of the BLOSSOMS Project in Kuala Lumpur starting on January 7, 2013. BLOSSOMS is an educational project of MIT, aimed at high school classes in (STEM) Science, Technology, Engineering and Mathematics" (UTM, 2013). The main goals of MIT-UTM Blossoms Project (2013) are: "to excite young people about math, science & engineering, and STEM careers; show the relevance of science and mathematics to their real world; develop critical and creative thinking skills, moving away from rote memorization and teaching to a test; introduce the in-class teacher to technology-enabled education in a supportive way that allows her/him to stay in charge of the class; encourage teachers away from a lecture format to a more active, problem-based style of teaching; develop an awareness, understanding and appreciation for other cultures."

Reviewing the above goals and undertaking the opportunity of MIT-UTM BLOSSONS Project, the researcher decided to use selected BLOSSOM modules as pedagogical tool in designing a blended learning environment conducive to mathematical thinking. By using ready to use BLOSSOMS modules as online component , the researcher intended to save time for the in-depth investigations during the implementation phase of the blended learning environment.

3.3.5 Rationalizing the Choice of Masons' Problem Solving Strategy for this study

Drawing the inspiration from Polya's work, evaluating based on the criteria to select a universally acceptable problem solving strategy (Woods, 2000), knowing the popularity among the local researchers (Abdul Rahman, 2008; Kashefi, 2012; Mohammad Yusof and Abdul Rahman, 2004; Zeynivandnezhad, 2014), predicting the popularity gain in western world of mathematical practice (Boaler, 2013), and feeling the need of an empathetic framework (Personal Communication, Jennifer Turns, April 4, 2014), ***Mason's Problem Solving Strategy*** is selected for this study.

3.3.6 Rationalizing the Choice of developing Students' Personas

The development of the engineering students' personas in this study is guided by the idea that engineering educators need to speed up the rate at which research informs educational practice. The design world has a similar problem, "how to get user research to influence the design of objects." The researcher adapted the concept of persona from design and brought it into the engineering education design. The researcher presented personas as a way to help the engineering education community communicate implications of research on engineering students to engineering faculty, staff, and students. In the world of design, personas are tools, used to help designers keep user research in mind while designing. Fictional personas of a person pertaining important characteristics within a research context are usually described (Grudin & Pruitt, 2002) as narratives. Literature from design highlighted its impact on the improvement of design process. (Adlin and Pruitt, 2010; Anvari and Tran, 2013; Faily and Flechais, 2011; Goodwin, 2008; Grudin and Pruitt, 2003; Madsen et al., 2014; Nielsen, 2013; Ward, 2010). The use of personas is quite new in engineering education and there are a few attempts developing fictitious student personas (Turns et al., 2013) to improve design of teaching and learning practices and to bridge the research-practice gap. However, evidence-based personas have not been developed in engineering education yet. To explore the value of personas in the area of engineering education, the researcher developed eight evidence-based personas from this research project, with an intention to share those personas with professional educational designers, engineering professors, and engineering students through a series of workshops (in near future). During the validation process, the researcher conducted a similar kind of workshop with six engineering educators and found that workshop attendees not only empathized with the challenges the personas were facing, but also discussed a variety of potential approaches for addressing similar students' challenges in their own complex contexts. Hence, in this research the researcher developed Personas as potential pedagogical tool to bridge the gap between research and the engineering education practice. Since, personas are popular in design but new in engineering education, most of the literature and development process is adapted from design (Nielsen, 2013) fulfilling the need to develop pedagogical tools to communicate the research findings to the community of practice (CoP).

3.3.7 Action Research Cycle and Process

In this section, the tenet, key characteristics and action research cyclic process will be discussed.

There are many other terms available to describe the educational practitioner's research. Some of them are: 'educational action research' (Carr and Kemmis, 1986); 'self-reflective enquiry' (Kemmis, 1982); 'exploratory teaching and learning' (Allwright and Bailey, 1991); and 'classroom research' (Hopkins, 1985). Several other variants of action research, mentioned by Power and Naysmith (2005) and Heinze (2008), were Dialogical Action Research (Mårtensson and Lee, 2004), Canonical Action Research (Lindgren et al., 2004), Educational Action Research (Carr and Kemmis, 1986), Collaborative Practice Research (Iversen et al., 2004), Classroom Research (Hopkins, 1985), Participatory Action Research (Street and Meister, 2004; Whyte et al., 1991), Action Science (Argyris et al., 1985), and Self-Reflective Enquiry (Kemmis, 1982). The purpose of this work is not to explore the different interpretations of action research given by different authors mentioned above, instead to shed light on the action research interpretation implemented for this work. The researcher felt more gravitated towards the idea of "The Living Theory" developed by Jack Whitehead in

1976 (Whitehead and McNiff, 2006, p. 32) and to introduce an interpretive living theory of her own practice.

The simple action research process is "cyclical with four inter-related stages: plan, act, observe, reflect (see Figure 3.4 below)", "collaborative in two senses: that many action research activities are best carried out with colleagues (practitioner-researcher); that action research always involves the participants, at least in knowing what is being explored and why", "qualitative rather than quantitative with the emphasis on descriptions rather than numbers" and "reflective, involving critical reflection on both the process and the outcomes" (Power and Naysmith, 2005, p. 5). Kemmis (1982, p. 7) further explained the simple action research process as follows:

Research Question (Outcome of Literature Review): The process of action research is usually initiated by a question about students' learning and teaching methods and moves forward through four phases of action research. ***Plan*** **(**to develop an action plan in order to improve what is happening in the classroom): During the plan stage, the researcher ponders and asks herself "how I might investigate the question I have identified and what possible changes I might make", then s(he) consults appropriate practitioners as well as researchers to discuss and explain his/her intention to do this research and why. "In the first stage, the teacher-researcher finally completes an action plan. This plan includes a summary of literature findings, recommended actions, and the identification of individuals responsible for action and those who need to be consulted and informed. The plan also indicates who will monitor and collect the data, the time line for data collection, and the resources needed to carry out the action" (Creswell, 2012, p. 581). ***Act*** **(**to act to implement the plan): In this stage, the researcher takes the planned actions by initiating an enquiry or changing the current practice and making sure that his/her students understand why he/she is doing this. ***Observe*** **(**to observe the effects of employed action contextually): During this stage, the researcher observes and records the responses to his/her action. He/she also notes if there are any changes in the behavior of the students. The student's feedback is also collected and recorded. ***Reflect*** **(**to reflect on outcomes and effects to facilitate further planning, consequent actions and repeat in a series of cycles): This stage is important in reviewing what the researcher has done and critically reflecting on the outcomes;

he/she may find that his/her action and observations have raised further questions. The process is then repeated in a cyclic manner (as illustrated in Figure 3.4).

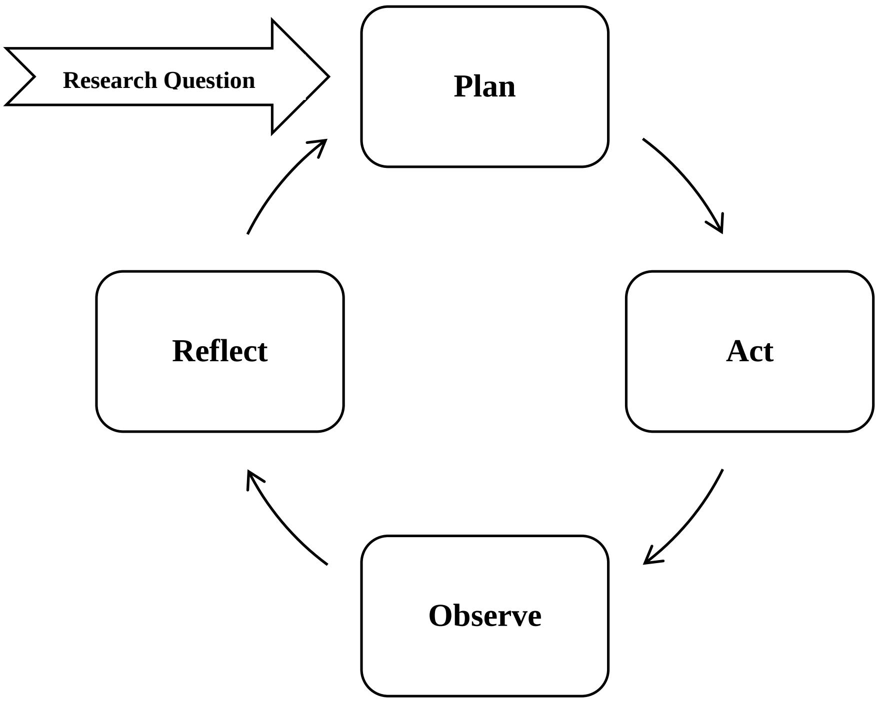


Figure 3.4: Simple Action Research Process (Kemmis et al., 2014)

The Tenets, characteristics, and stages of Lewin's cycle for action research are summarized in Table 3.1.

Table 3.1: Tenets, Characteristics and stages of Lewin's cycle for action research (Heinze, 2008)

Tenets (Baskerville and Myers, 2004)	Characteristics (Oates, 2006)	Stages of Lewin's cycle (Burns, 2000)
I: "concepts are defined by their consequences" II: "truth is embodied in practical outcome" III: "logic of controlled inquiry" & "rational thought is interspersed with action" IV: "human action is contextualized socially"	• An emphasis on change • Several data generation methods • Iterative cycle of plan – act – observe – reflect • Concentration on practical issues • Collaboration with practitioners • The research outcomes are practical and theoretical	(1) Initial idea (2) Reconnaissance (3) Plan (4) Implementation (Action) (5) Monitoring (Observe) (6) Evaluation (assessing written, verbal and attitudinal responses, reflections from students and researcher) (7) Review of the plan

In Table 3.1, the first column listed the tenets based on pragmatic philosophical ground which support the action research and influence its characteristics as listed in the second column. These tenets are reported by (Heinze, 2008) and stated as follows:

> "The first premise is Peirce's tenet that all human concepts are defined by their consequences. The second is James' tenet that truth is embodied in practical outcomes. The third is Dewey's logic of controlled inquiry, in which rational thought is interspersed with action. The fourth premise is Mead's tenet that human action is contextualized socially, and human conceptualization is also a social reflection."
>
> (Baskerville and Myers, 2004)

The third column presents the action research process of Lewin's cycle as further shown in Figure 3.5.

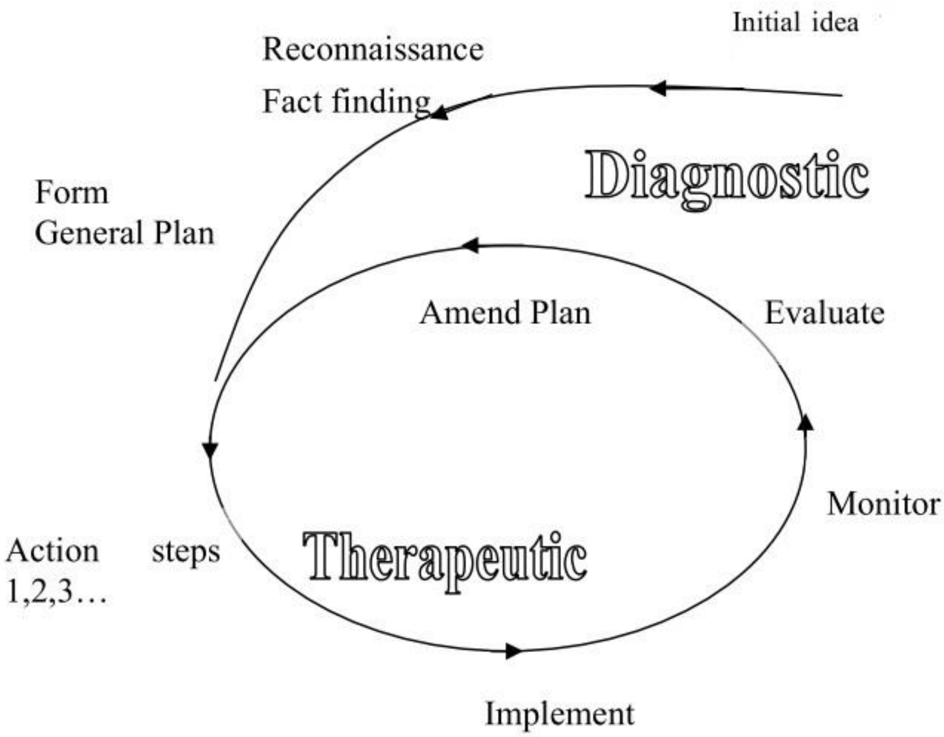


Figure 3.5: Lewin's cyclic model for action research, adapted from (Burns, 2000)

As shown in the Lewin's cyclic model and discussed by Heinze (2008), "the action research process can be divided into two stages: the diagnostic and the therapeutic. The former is concerned with the general 'problem' identification and the latter with active improvement by carrying out the iterative fact finding, planning, implementing and evaluating. It enables the formation of a local theory which informs practice. Although it is difficult to achieve the contribution to both theory and practice and is the 'double challenge' for action researchers, the action research can still be rated as successful when either a theoretical or a practical contribution is achieved (Oates 2006)."

By combining the stages of Lewin's cycle with Kemmis and McTaggart's cycle, the researcher developed a modified cycle as presented in Figure 3.6 for this research.

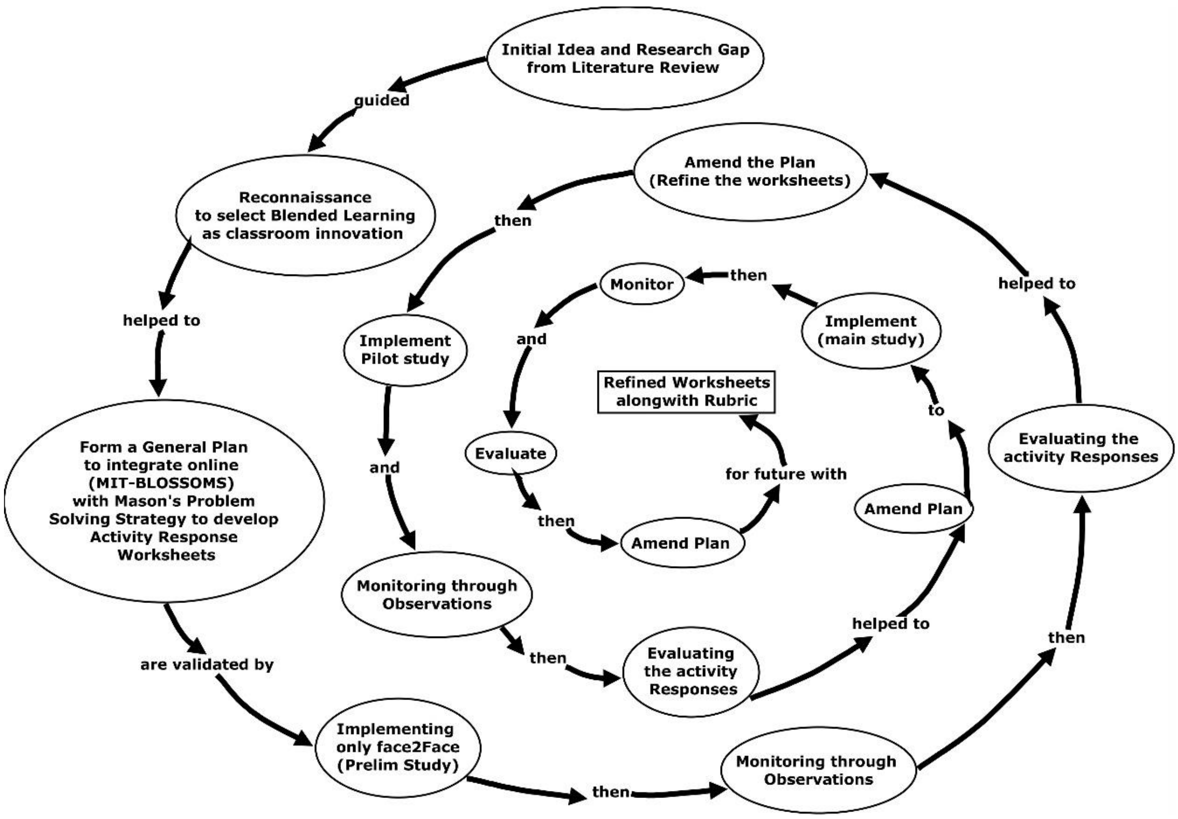


Figure 3.6: Lewin's cyclic model (Burns, 2000) combined with Kemmis and McTaggart (2014), adapted and modified for this research

The explanation is extracted, combined, and modified in the form of a self-explanatory concept map as shown in Figure 3.7 adapted from Kemmis and McTaggart (2014), Power, R., & Naysmith, J. (2005) and Burns (2000) to explain what needs to be done during different stages to actuate the modified cycles shown in Figure 3.6. While conducting the action research in real life settings, the action researcher may face the following issues and criticism: involvement of multiple variables which are beyond the researchers' control; the resulting learning process may not necessarily be the same as illustrated in the modified version; the non-sequential research process due to exploratory, opportunistic, and emergent nature of the action research and every stage of the research cycle is communicating with every other stage; infinite iterative nature of research might raise a question of, when to end the iteration?

To avoid the biased research interpretations, it is advisable to conduct action research in collaboration with other researchers:

> "Action is socially relative, and this makes the action researcher a participant observer. Further, it explains why collaborative teams are essential. In order for action to be formulated in the social setting, the formulators must be socially situated in that setting. A collaborative team is necessary to provide the "others" who will invoke the responses in the reflective self. Otherwise, action is not rationalized or operationalized in the reality of the social world. (Baskerville and Myers, 2004)" (Heinze, 2008)

Based on the emergent needs because of the tenets, characteristics and stages of action research cycle, the implications for this work are listed in Table 3.2.

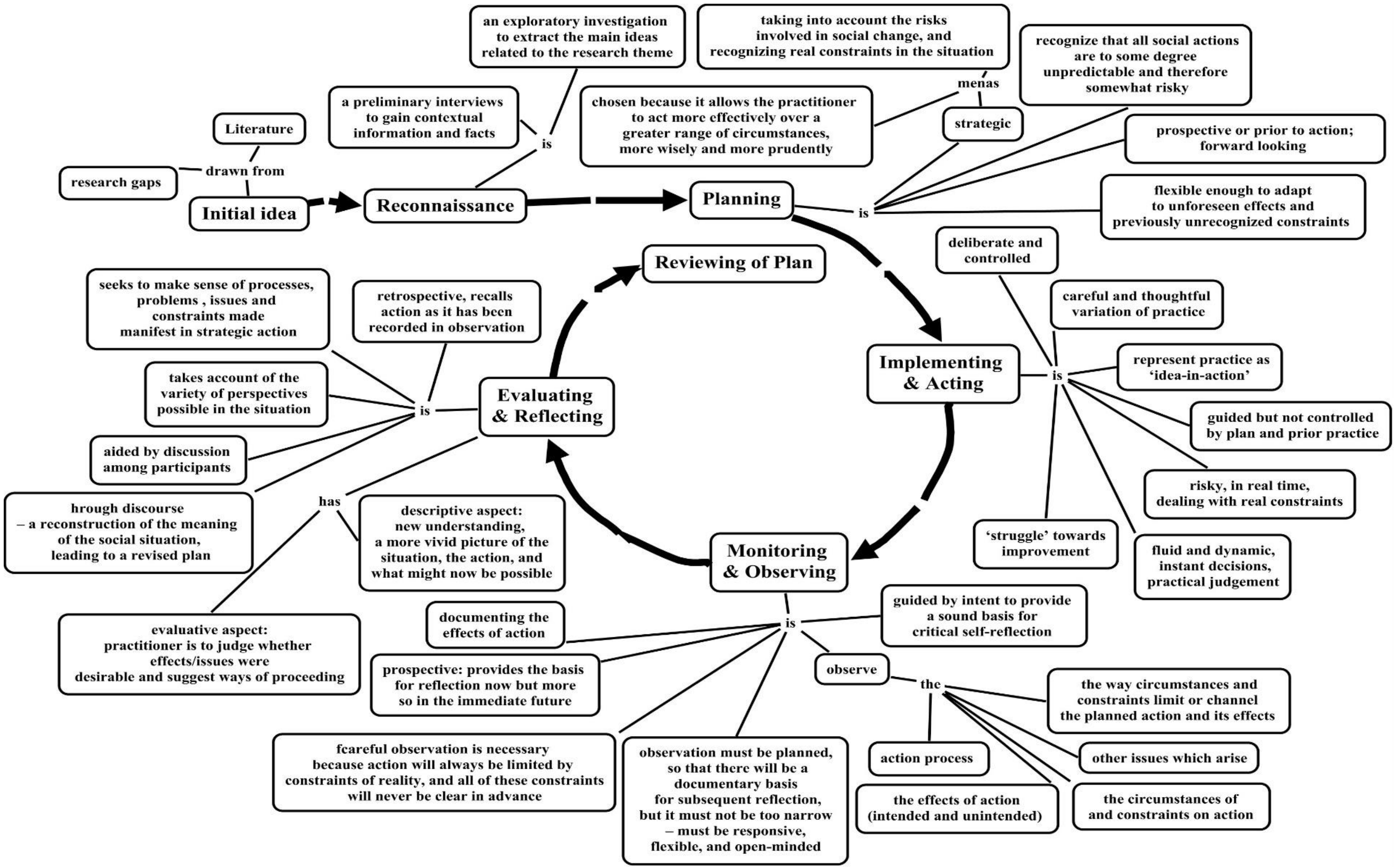


Figure 3.7: Explanatory Concept Map showing the different stages of an action research cycle

Table 3.2: Needs alignment of action research and given research problem adapted and modified from (Heinze, 2008), modifications are italicised

Action research implications	***Is the need met in given research setting?***
Need for improvement in practice	*Yes, improvement of a course in terms of implementing its intended objectives to improve mathematical thinking skills of first year engineering students*
Need for "open mindedness" in relation to data sources	*Yes, data from multiple sources will be collected*
Need for a plan of actions and iteration	*Yes, the plan is conducted and iterated during the preliminary, pilot and main study*
Need for real-world research settings	*Yes, the MTL is conducted in a local Research University (RU) of Malaysia in the real classroom as a complementing part of Engineering Mathematics I for first year engineering students.*
Need for participant observation and collaborative reflection	*Yes, the data collection team including research assistants, researcher and staff are willing to work in collaboration with each other. They collaboratively observe and reflect on the learning process*
Need for contribution to theory and practice	*Yes, the class-teacher is also a supervisor of this research and is keen towards change in teaching and learning environment. Looking back to literature fulfilled the need for theory's contribution . This research tried to bridge the theory- practice and practice-research gaps.*

3.3.8 Selected Research Methods

According to Creswell,

"Action research has an applied focus. Similar to mixed methods research, action research uses data collection based on either quantitative or qualitative methods or both. However, it differs in that action research addresses a specific, practical issue and seeks to obtain solutions to a problem. Thus, action research designs are systematic procedures done by teachers (or other individuals in an educational setting) to gather information about, and subsequently improve, the

> ways their particular educational setting operates, their teaching, and their student learning. Educators aim to improve the practice of education by studying issues or problems they face. Educators reflect about these problems, collect and analyze data, and implement changes based on their findings." (Creswell, 2012, p. 577)

Oates further added that the qualitative data, "include all non-numeric data – words, images, sounds and so on – found in such things as interview tapes, researcher's diaries, company documents, websites, and developers' models" (Oates, 2006, p. 266).

The current research has the characteristics of ad hoc opportunistic method design (Heinze, 2008), using an opportunistic or emergent purposive sampling technique (Patton, 2002, p. 244) i.e. commonly used for action research (Tomal, 2010, p. 30), which allow the researcher to follow new leads during fieldworks, take advantage of unexpected and deploy flexibility to facilitate various interpretations offering 'how' and 'why' explanations. Data type considerations for this research are described in Appendix P. Multiple data collection methods and techniques are used in this research. Different data sources, why and how the researcher used these methods are listed in Table 3.3.

3.4 Research paradigm implementation

Knowledge creation in this research is subjective due to the accepted interpretive beliefs, means that the research context, research participants and the researcher's background will influence the knowledge produced through this process. The detailed descriptions related to research settings, research participants and researcher's background would help the reader to comprehend and then make sense of the think descriptions of processes and interpretations throughout the research. The research setting will be described in detail whereas the participants' consents are collected but their identity will be protected by anonymity. In this section, the researcher will first discuss the research setting, then the description of the participants will be given and finally the researcher's background will be explained.

Table 3.3: Overview of data sources and rationale behind those adapted and modified (Heinze, 2008)

Data source	Why?	How?
Literature (Journal Papers, Conference Papers, Books, Reports)	To get a better appreciation of general issues, surrounding cultivation of mathematical thinking and problem solving and implementation of blended learning in higher education using action research.	Literature search is conducted in the researcher's own university, via the electronic databases offered through the library. The researcher used keywords, Blended learning, mathematical thinking, problem solving, action research, personas, etc. for the literature search.
Students' Profile (Demographics written and online)	To know your students, their mind-sets, attitudes, perceptions and expectations from the course	Math autobiographies were collected in class and online Google forms were created to record the students' profiles
Observations	In order to capture the personal interpretation of events. Addressing the second pragmatism tenet: "truth is embodied in practical outcome"	One research assistant observed face-to-face interactions of teacher with students in the classroom and the researcher updated her own research log and reflective journal. Student-student and teacher-student interactions are also observed through videos recorded by another research assistant.
Activity Responses (Written responses on the worksheets)	To explore the activation of Mathematical thinking processes in the embodied and symbolic world of mathematics.	The written activity responses are collected after every module conducted in the mathematical thinking lab sessions.
Focus groups	In order to get an appreciation of different interpretations of reality. Addressing the fourth pragmatism tenet: "human action is contextualized socially"	Student focus groups are used to serve data triangulation focusing on the experiences and feedback of practice
Informal Interviews	To achieve a deeper understanding of individuals' views and observations of commonly experienced events.	Informal interviews and conversations helped t the issues emerged during the intervention
Documents & Formative Assessment current knowledge state, MTL participation and class participation	To gain an understanding of programme relevant events, students' relative positioning in the class and group, and decisions that took place even though the researcher was not able to be part of them	Course related documents; results of formative assessment in the course are collected from the class teacher. Researcher extracted results from ALEKS to gauge students' current knowledge state and recorded students' formative assessment based on their participation in the MTL.

3.4.1 Research setting description

An innovation-led and graduate-focused Research University (RU) is selected for this study. The international campus is located in Kuala Lumpur, the capital city of Malaysia whereas the main campus is located in Johor Bahru, the southern city in Iskandar Malaysia, which is a vibrant economic corridor in the south of Peninsular Malaysia. The preliminary and main study is conducted in the main campus of the RU whereas the pilot study is conducted in its international campus. After implementing the Outcome Based Education (OBE), the RU welcomed the implementation of the elements like active learning, think-pair-share and problem solving, discovery learning, blended learning and peer questioning in the teaching and learning practices (Harun, 2012).

The current action research practically started in mid-2013, which coincided with the new intake in engineering program. Guided by preliminary and pilot study, a batch of 52 students from the first semester, first year mechanical engineering were selected for the main study to let them introduce mathematical thinking lab (MTL) as an intervention for one hour per week for the whole semester. The course of Engineering Mathematics I was restructured while keeping in view that these first year engineering students need to attend class for three mornings per week and a hour of mathematical thinking lab (MTL) session per week. This research was more focused on mathematical thinking lab (MTL) sessions whereas the researcher also observed the same batch in their routine lectures during the same course throughout the semester. During the MTL sessions, an intended objective of engineering mathematics is targeted to make the engineering students able to think mathematically. The mathematical thinking based problem solving activities were designed and implemented through blended learning during those MTL sessions.

Participants of this study are described next.

3.4.2 Description of participants

The MTL sessions were designed and implemented through blended learning. The online component was implemented using the selected BLOSSOMS modules, the face-to-face sessions were conducted through constructivist teaching using Mason's approach of prompts, questions and themes, and students were instructed to record their written activity responses on the designed worksheets. Students, lecturing staff, research assistants, and the researcher's background is briefly introduced as follows:

3.4.2.1 Students as active participants and informants

We started a preliminary study with nine voluntary respondents from different faculties at the main campus of the RU followed by a pilot study with 32 respondents from first year engineering diploma students at the international campus of the same university. The researcher used the preliminary study to validate the first version of the worksheets whereas the pilot study was the full test run comprising of two pilot cycles of the action research (before the main study). For the main study and two main cycles of the action research, the researcher selected a batch (52 students; 47 males and 5 females) from the first year Mechanical Engineering program at the beginning of their Engineering Mathematics I course. All the participants joined the engineering program from three mainstreams, Type I students came in with high school certificate (15 out of 52), Type II students completed their matriculation studies (35 out of 52) and Type III (only 2 out of 52) students joined after achieving their Diploma in Mechanical Engineering along with some work experience. A sample of demographic data is given in Table 3.4 Actual names are replaced by their group identities for anonymity purposes.

Table 3.4: Sample demographic data for research participants

Group ID	Type	What is your major or field of interest?	What pre-university mathematics classes have you taken?	Where was it taken? (High school, community college, matriculation center, etc.)	GPA
Alpha1	I	Mechanical Engineering	Engineering Mathematics	Foundation Engineering	3.9
Gamma2	II	Mechanical Engineering	Mathematics	Matriculation	4
Sigma3	III	Mechanical Engineering	Calculus, Algebra, statistics	Diploma in Mechanical Engineering	4

During the main study, all the MTL sessions were videotaped supporting classroom observations and the written activity responses were collected through activity response worksheets. The researcher did not differentiate participants (except student personas) during the research discussions in this thesis.

3.4.2.2 Lecturing Staffs

One senior lecturer and one associate professor, involved in teaching engineering mathematics, were from the Mathematical Sciences Department and have a wealth of lecturing experiences. The senior lecturer followed the traditional way of teaching whereas the associate professor used student-centered teaching and learning techniques and the latest technology in her classes. The former participated in the interview whereas the latter facilitated with the required information and documents for the main study. Other lecturing staffs were also involved in providing the critical feedback during peer reviewing and debriefing sessions throughout the research process.

3.4.2.3 Research Assistants

There were two research assistants (RAs) who were officially involved during the pilot study and three research assistants (RAs) who were actively involved during the main study. During the pilot study, one RA was assigned to observe the classroom activities and the other helped in video recording the conducted sessions. In the main study, one RA recorded all the blended learning sessions and the other two RAs alternately recorded the classroom observations. All the research assistants were postgraduate students. The researcher conducted all the MTL sessions through blended learning and her background is described next.

3.4.2.4 Researcher's background

To deal with the subjectivity of interpretive research, researcher's background is outlined in this sub-section. The researcher spent 38 years of her life in Pakistan, completed her B.Sc. in Electrical Engineering from the University of Engineering and Technology at its Lahore campus and Masters degree in Computer Engineering from Lahore University of Management Sciences. She has more than a decade of teaching experience with undergraduate engineering and computer science students, worked in industry as an Enterprise Resource Planning Consultant before pursuing the PhD program in engineering education at the Universiti Teknologi Malaysia. She joined the engineering education program as a practitioner but never been a researcher while teaching in her class. During her exposure to qualitative research, she developed a deep interest in the interpretivist paradigm and concurrently got a chance to meet the pioneers in engineering education who were actively involved in qualitative research. She attended several workshops on qualitative research and data analysis conducted by local as well as international researchers. She was interested in research related to blended learning whereas she was funded through a project to work on mathematical thinking so she combined both her interest and the project demand and came up with an idea backed by the trend of using blended learning for mathematical thinking. To further develop the deep foundation, she joined and completed Massive Open Online Courses (MOOCs) on mathematical Thinking (Stanford University), How to learn

Maths (Stanford University), Design Thinking (Stanford University) and Thinking effectively through Mathematics (EdX). She attended online workshop on students' personas and their effectiveness in research to improve instruction from the University of Washington. The Center for Engineering Learning and Teaching (CELT) at the University of Washington started an NSF-funded project on the early leaders or "pioneers" in engineering education. As a member of the research team, the researcher was trained through online workshops to interview one of those pioneers to document and analyze their "impact trajectories" (contributions, challenges, successes). The stories of the pioneers were critical in guiding, inspiring, and even sustaining the careers of new generations of change agents in engineering education. When the community is still relatively small, the experiences of individual pioneers were shared more informally, with less intentional effort. The growth and transition of the community meant that the new pioneers were less likely to interact directly with those who have paved the way. Moreover, some early pioneers have retired and become harder to contact. In such a system, without support, only a small portion of people's experiences will be remembered. The above research project informed and promoted continuing transformation in engineering education by bridging across and within generations of pioneers and other members of a rapidly growing community while facing challenges similar to those faced by its early leaders. The researcher also worked on an online research activity to revise engineering education research taxonomy in collaboration with other international researchers. The researcher is currently involved in a National Science Foundation (NSF) funded "Engineering Education Research Leadership NetWorkshop (2013 to date) as a mid-career faculty to get support to move into leadership roles, build her communication skills in difficult situations, and develop strategies to raise awareness of and appreciation for the field of EER in the broader engineering community.

The three years of her PhD journey have facilitated her with a gradual and logical shift from positivist to interpretivist worldview. This research has given her a chance to conduct action research and to have the very first experience of being a practitioner and a researcher at the same time. Hence, the findings and discussions throughout this thesis will be presented through her interpretive and engineering education focused lens. The researcher intended to identify and address her subjectivity by regularly recording and utilizing her reflections serving as a backbone of the research process. Reflections in this research are meant to give meaning to experiences; learning from those experiences; bridging the gap between practice and theory; analysing what is similar and what is different; and making decisions of change in the following iterations of action research cycle.

Table 3.5 outlines the preliminary cycle, two pilot cycles, two main cycles of the action research and the participants involved in the respective cycles of the research. This shows that the researcher conducted the preliminary, pilot, and main studies with three different groups of students. The variations of experience throughout the research process provided deep insights while implementing the blended learning to foster mathematical thinking in this research.

Table 3.5: BLOSSOMS modules Conducted by the Practitioner

	Preliminary Cycle	Pilot Cycle 1	Pilot Cycle 2	Main Cycle 1	Main Cycle 2
Academic year, semester and discipline of engineering	2013, Year 1 9 volunteers from Mixed faculties	2013 Semester 1, Year 1 Engineering Diploma students	2013 Semester 1, Year 1 Engineering Diploma students	2013 Semester 1, Year 1 Mechanical students	2013 Semester 1, Year 1 Mechanical students
Conducted Modules	Flaws of Average	The exponentials, Big and Small	Flaws of Average	The exponentials, Big and Small	Flaws of Average

3.5 Research method implementation

The research method implementation will focus on the limitations and ethical issues of action research. Finally, the outline of the four action research cycles is presented in a summarized form.

3.5.1 Action research limitations management and ethics

Action research risk management and mitigation (Heinze, 2008) are kept in consideration during the research design and tabulated in Appendix Q.

Addressing research ethics

> "Ethics is the part of human philosophy concerned with appropriate conduct and virtuous living. Research in general and qualitative research in particular is viewed by most qualitative scholars as moral ethical endeavors because they are human endeavors. The study of ethics can be divided into two areas. First, the common area most influential in research practice is called normative ethics or moral theories. These are frameworks used to decide what is preferable to do among the choices available. The second area is called meta-ethics. These are the assumptions and values underlying normative ethics and moral theories. Meta-ethics, as discussed later, is often associated with research epistemologies and ontologies or the assumptions about what constitutes knowledge and reality and how knowledge is best developed…. research ethics also address the integrity of the research activity. Honesty, openness, and candid revelation of a study's strengths and limitations according to commonly held standards of practice are typical indicators of the integrity of the scholarship."
>
> (Given, 2008, pp. 273–276)

This research intrinsically showed ethics embedded in the action research. Both the normative ethics and meta-ethics were entertained in their possible extend and were discussed in this chapter. The researcher felt full responsibility to follow the academic ethical requirements as practitioners while implementing the action research cycles. She was well aware of the importance of gaining the written consent from the participants of this research, therefore all the informants were requested to fill in and sign their consents at the start of the preliminary, pilot, and main studies. All the consent forms provided brief summary of the research, the commitment to participants for confidentiality, anonymity, with the choice to participate or quit at any stage of the research.

All the written consents were collected and kept in a safe custody at the start of every stage. However, it was difficult to get consent before every single interaction with the participants throughout the research process. Therefore detailed observations, field notes, and researcher's reflective journal will not be published. The zest and main ideas of these observations and reflections are documented in this thesis in edited forms. The researcher described the research process and findings while keeping anonymity of all the participants and assigned them with group identifications (IDs).

3.5.2 Action research cycles' outline

One preliminary, two pilot and two main action research cycles were conducted in this research. A summary of all activities is presented in Table 3.6. The first row shows the academic year and semester of the students during preliminary, pilot, and main studies, the second row shows the modules conducted and the third and fourth rows show the utilization of BLOSSOMS videos and activity worksheets during the respective cycles.

Table 3.6: Summary of action research activities

	Preliminary Cycle	Pilot Cycle 1	Pilot Cycle 2	Main Cycle 1	Main Cycle 2
Academic year, semester, and discipline of engineering	2013 Volunteers from Mixed faculties	2013 Semester 1, Year 1 Engineering Diploma students	2013 Semester 1, Year 1 Engineering Diploma students	2013 Semester 1, Year 1 Mechanical Engineering students	2013 Semester 1, Year 1 Mechanical Engineering students
Conducted BLOSSOMS Modules	Flaws of Average (without the video)	The exponentials, Big and Small (with video)	Flaws of Average	The exponentials, Big and Small	Flaws of Average
Video intervention	No	Yes	Yes	Yes	Yes
Activity Worksheets	Yes	Yes	Yes	Yes	Yes

The data collection activities that took place during the above research cycles are outlined next. Emphasis is given on the BLOSSOM module, participants and activity.

3.6 Data collection

Observations and activity response worksheets were collected during the preliminary and pilot action research cycles. The following two main cycles were then undertaken and based on observations, activity response worksheets, and other emergent data collection techniques like focus groups, informal interviews, documents related to course and results of current knowledge state, the MTL participation-based assessment and the formative assessment for the main course as described in Table 3.3. A detailed description of the data collection process along with a typical sample (example) that of observations, activity response worksheets, and focus groups respectively are recorded in the next sub-sections. A summary of the main and emergent data collection activities during the main study are also described. A data collection and activity log file is maintained throughout the research and provided in Appendix X.

3.6.1 Observations

Given (2008, p. 573) described the important features of observations as:

- Observation is one of the oldest and most fundamental research method approaches.
- It involves collecting impressions of the world using all of one's senses, especially looking and listening, in a systematic and purposeful way to learn about a phenomenon of interest.
- It more typically takes place in natural settings to capture behavior as it occurs in the real world.

- It usually involves direct contact between the researcher and participants though indirect data collection methods such as audio or video recording may also be used.
- It is exploratory.
- It seeks to uncover unanticipated phenomena.
- It uses inductive reasoning with the conceptual constructs used to account for observations being developed during and after data collection from the observed behavior itself.
- Qualitative observational research uses idiographic rather than nomothetic causal explanation.
- It is constructivist in approach, emphasizing meanings that the participants attach to activities and events.
- It recognizes the subjective role of the researcher.
- It acknowledges reactivity to be inevitable on the part of both the observed and the observer and seeks to address and understand this through researcher reflexivity.
- Qualitative observational research is associated with a number of theoretical traditions and broad research methodologies including action research.

One research assistant helped during the pilot study and another during the main study to record the observations according to the observation guide provided by the researcher. The observations were semi-structured and the researcher was looking for 'unusual' or 'interesting' events, cases and participants. The experience of 'practitioner- researcher' was also captured through memos, field notes, and reflective journals.

The ways the researcher recorded the observations were different during the preliminary, pilot, and main study. In preliminary, the session was video recorded and after the session, the reflective notes were documented in the researcher's reflective journal. In the pilot study, one research assistant observed the session, facilitated by the researcher. In the main study, the researcher conducted the MTL sessions; an experienced research assistant recorded the observations whereas another research assistant video recorded the sessions.

An excerpt from observations described in Table 3.7 depicted that the students were fully engaged with the first BLOSSOMS module that was based on the concept of exponentials. They wanted to learn how to think mathematically. They have been informed that they will be graded based on their participation in the MTL. The practitioner also explained what was there on the worksheets and how the responses need to be recorded on the specific areas. She also encouraged them to consider their mistakes as learning opportunities. It was also noted that the introduction part of the module was very engaging because of the story telling nature of the problems being posed.

Table 3.7: An excerpt from observations recorded on the 24 September 2013

Before showing the BLOSSOMS video, the class instructor explains what is there in the worksheets. She explains different phases of mathematical problem solving by making students reflect what they have been doing in the past. She is moving forward and backward in the middle (comparatively the widest space in the middle of the computer lab). She further asks the students, "who has the habit of checking the solution after solving a problem?" Some students (about ten) raised their hands. She explained why checking the solution is important for them and why they also need to develop a habit of reflecting back on their resolutions and as the last and most important step, they need to create new examples replicating the concept learned after the problem solving activity. She encourages the students to ask questions whenever they feel stuck during an activity or otherwise. The students started communicating with each other in their assigned groups. The Instructor allocated 5 minutes to let them review the content on the worksheet. Most of the students are engaged with positive attitudes in this new learning environment whereas a few are looking at their computer terminals.

Humans have the tendency to overlook some of the details during classroom observational exercises due to their selective memory recall or perceptions. To get a complete "in-action" process, the MTL sessions were also video taped. In order to overcome the limitations, the observations were backed by video recordings and then triangulated with the instructor's reflections to extract the students' attitudinal behaviours. This allowed the researcher to gain insights from detailed observations based on handwritten observations and video recordings.

A detailed logbook was maintained in the form of EXCEL worksheets to quickly search the intervention sessions by the researcher, time, venue, document collected and list of activities in that respective session. The log file also informed which data were collected during each session, who collected it and what were the other important things to inform us. Appendix I outlined the tasks undertaken by the research team during the main study. Due to sensitive information, the logbook was not published and was only used confidentially by the researcher. References to the MTL observations were extensively quoted during the description of the action research cycles. However, the researcher presented the logbook entries that she maintained.

3.6.2 Written Activity Responses

Considering the mathematical problem solving nature of the research to activate mathematical thinking processes, data were also collected through specifically designed activity worksheets (Section 2.6). Students' evident behaviours (Rittle-Johnson and Koedinger, 2005), such as their written activity responses were examined to show the activation of mathematical thinking processes in the embodied and symbolic world of mathematical thinking, while considering the students' understanding, misconceptions, and representations of the concepts, the possibility of any change and/or improvement in the students' mathematical thinking skills after the intervention was of interest.

3.6.3 Focus groups

> "Focus groups are a form of qualitative interviewing that uses a researcher-led group discussion to generate data. Since their reintroduction to social science research in the mid-1980s, focus groups

> have become a popular method because, like individual interviews, they can be modified in a wide variety of ways to suit an equally wide range of purposes. They can thus be used for exploratory research, where the participants are relatively free to discuss the topic as they see fit, or they can be used in a more structured fashion, where the interviewer or moderator takes a more active role in controlling the issues to be discussed." (Given, 2008, p. 352)

During the last week of the semester, the students were instructed to attend an interactive evaluation session with their respective groups. The purpose of the focus group was twofold, one to listen to their experiences related to MTL, and the other to collect evidences that they were aware of and were able to think mathematically whilst problem solving. This resulted in 13 focus groups at the end of the action research and Table 3.8 shows their activity summary. In order to encourage students' attendance at the focus groups, marks were allocated for their active participation in the discussion. Feedbacks were also collected at the end of each session to allow them to give comments on their experience of the MTL. The focus groups took place at the selected times and dates by different groups in the study rooms near the students' hostels to make the sessions informal. The duration of each focus group was about two hours. The sessions are also video recorded to provide evidence of the students' engagement and mathematical thinking based verbal communication.

As the first step, every focus group was given an opportunity to retrospectively check whatever they have done in their classes and during the MTL sessions. The researcher then provided them with study guides and asked them to discuss among their groups before explaining the concept of mathematical thinking and its constituting processes. Later, they were asked to record their written responses reflecting their current knowledge related to mathematical thinking. At the last part of the activity, the researcher asked them to describe simple designs (selected by the researcher) from real life and further suggested the change in the design based on some basic requirements. Although the last part was not anlayzed in depth due to the lack of time, was used to provide the evidences of students using their mathematical thinking skills. Discussions were probed wherever perceived as necessary. The initial focus groups were much more open ended in order to decide what to focus on. Some of the themes under consideration were mathematical thinking and problem-solving skills, mathematical thinking processes, students' reflection on the overall process and their feedback related to their learning experiences in the blended learning environment. The written responses were collected every time and used during the stage II data analysis.

Table 3.8: Focus groups activity Summary

Date	Day	Time Slot	Focus Group
23th December, 2013	Monday	10:00AM -12:00PM	Epsilon
23th December, 2014	Monday	2:00PM-4:00PM	Omega
24th December, 2014	Tuesday	10:00AM -12:00PM	Beta
24th December, 2014	Tuesday	2:00PM-4:00PM	Gamma
24th December, 2014	Tuesday	4:30PM-6:30PM	Delta
25th December, 2015	Wednesday	10:00AM -12:00PM	Pi
25th December, 2015	Wednesday	2:00PM-4:00PM	Rho
26th December, 2015	Thursday	10:00AM -12:00PM	Chi
26th December, 2015	Thursday	2:00PM-4:00PM	Alpha
26th December, 2015	Thursday	4:30PM-6:30PM	Phi
27th December, 2015	Friday	10:00AM -12:00PM	Kappa
27th December, 2015	Friday	2:00PM-4:00PM	Lambda
28th December, 2015	Saturday	4:30PM-6:30PM	Sigma

3.6.4 Data collection activities summary during the main study

Overall, data collection was based on observations, focus groups, and written activity responses on the designed worksheets and secondary emergent data collection techniques of math autobiography filled manually by all the 52 students, electronic profiles, classroom video recordings, informal interviews, ALEKS report for prior knowledge and results of formative assessment throughout the class and the MTL sessions. The transcripts were produced for the classroom video recordings to triangulate with classroom observations for quality requirements. The activity response worksheets were collected and scanned to convert them into electronic format.

In short, the following multiple ways were used to gather the information to answer the under-investigated research questions:

1. Students' math autobiography during the introductory session to know their educational backgrounds, previously learned math contents, their met-befores in mathematics in general.
2. Electronic profiles of the students by asking them open-ended questions (online using Google Forms) related to their perceptions, mindsets and future expectations from the course.
3. Students' evident behaviors, such as their written activity response worksheets to categorize their conceptual understanding into different levels (Rittle-Johnson and Koedinger, 2005) and to highlight the activation of different mathematical processes.
4. Classroom observations and video recorded lectures to interpret students' behaviors and attitudes (Given, 2008).
5. Emerging questions being asked informally during the first few lectures to investigate their specific attitudes in the class (Moyer and Milewicz, 2002).
6. Focus groups and students' interviews to find the trajectories of their problem solving practices while doing the assigned activities (Ginsburg et al., 1998; Moyer and Milewicz, 2002).
7. Knowing the current knowledge state for the concept of function using **A**ssessment and **LE**arning in **K**nowledge **S**paces (ALEKS). It is a Web-based, artificially intelligent assessment and learning system that uses adaptive questioning to quickly and accurately determine exactly what a student knows and doesn't know related to a concept (ALEKS Corporation, 2014).
8. Results of formative assessment during the main course and the MTL participation to gauge the relative positioning of the students and to provide a rationale to select the emergent profiles to report in findings and discussions.

During the preliminary cycle of the action research, only recorded video and activity response worksheets were collected. During the action research cycles of the pilot study, classroom observations, video recordings, and activity response worksheets were collected. The volume of the collected data increased during the main cycles due to the emergent need after the preliminary and pilot studies to know more about the informants, their current knowledge states, their prior experiences related to mathematical thinking and their reflections after the activities.

The data analysis process is described next.

3.7 Data Analysis

After outlining the research settings, introducing the action research cycles, providing a detailed account of the data collection techniques and describing the data collection processes via observations, focus groups and written activity responses, this section focused on the two staged data analysis process; stage I during the action research cycles and the post hoc stage II, once the implementation of the cycles were over. There are dedicated sub-sections for each analysis process. The researcher replicated the data analysis from (Heinze, 2008) and adopted Miles and Huberman’s (1994) operational definition of data analysis in this research :

> “We define [qualitative data] analysis as consisting of three concurrent flows of activity: data reduction, data display, and conclusion drawing/verification.” (Miles and Huberman, 1994, p. 10)

Figure 3.8 shows the individual flows of analysis employed in this research. The brief description of each of these three flows is outlined in the following paragraphs.

“The process of selecting, focusing, simplifying, abstracting, and transforming the data that appear in written-up field notes or transcriptions” (Miles and Huberman, 1994, p. 10) is referred to as data reduction/condensation. It started during the formalization of research questions and initial ideas before the data collection phase. At the time of data collection, it started with open coding followed by categorizing, clustering, and identifying themes. The higher order themes then aided in conclusion drawing once the data collection period was over. The data reduction flow of analysis was refined in the subsequent cycles depending on the requirements.

Data display was the second flow of analysis concerning a rational and concise information delivery. The researcher used tables, graphs, concept maps, and diagrams as different display formats in this study. The main purpose data display was to replace it with something easier to understand, quicker to comprehend and to avoid the use of extended text (Heinze, 2008).

Interpreting data and answering research questions helped the researcher to draw conclusions and verify them. The identified themes, emerging patterns and explanations were prompted from the data and helped in the interpretation. Their plausibility was tested during the verification stage of conclusions. In this research, the researcher used triangulation at data collection level, expert audit, and team discussions during this stage.

The constantly iterative data reduction, display and conclusion drawing/ verifications (Miles and Huberman, 1994) made data analysis an interactive process. This happened throughout the data collection through classroom observations triangulated with video recordings, and written activity responses in the form of worksheets. Then the analysis during data collection was carried out to further evaluate the responses and reflections of the participants and researcher. After every cycle of the action research, the plan was reviewed for the next cycle.

3.7.1 Data Analysis -Stage 1: Analysis during data collection

Data analysis in this research was conducted during the data collection (Stage 1) and post data collection (Stage 2). "The analysis during data collection (Miles and Huberman, 1994) stage was conducted in the real life settings of action research. The post data collection analysis was conducted once the researcher was 'detached' from the research settings" (Heinze, 2008). During the first stage, our focus was on classroom observations, students' interactions in the MTL participants and their problem solving activities, whereas stage II was related to researcher's worldview and legitimizing the results by connecting them back to theoretical basis.

As mentioned earlier, real life classroom settings and our limited control over the situation put limitations on this research. Other limitations were the extensive reading on relevant issues, the insufficient time to convert written observations into electronic form and the timely transcription of visual data so the researcher placed greater emphasis on the her field notes and reflections rather than extensively analysing the other forms of data. Field notes were recorded during the action research cycles whereas reflections and voice memos were recorded right after the MTL sessions. At the time of observations, the key themes were perceived (Data reduction) and the information was utilized during the focus group discussions (Data display) and initial conclusions were drawn (Conclusion drawing/ verifications). The Flow Model of data collection is presented in Figure 3.8.

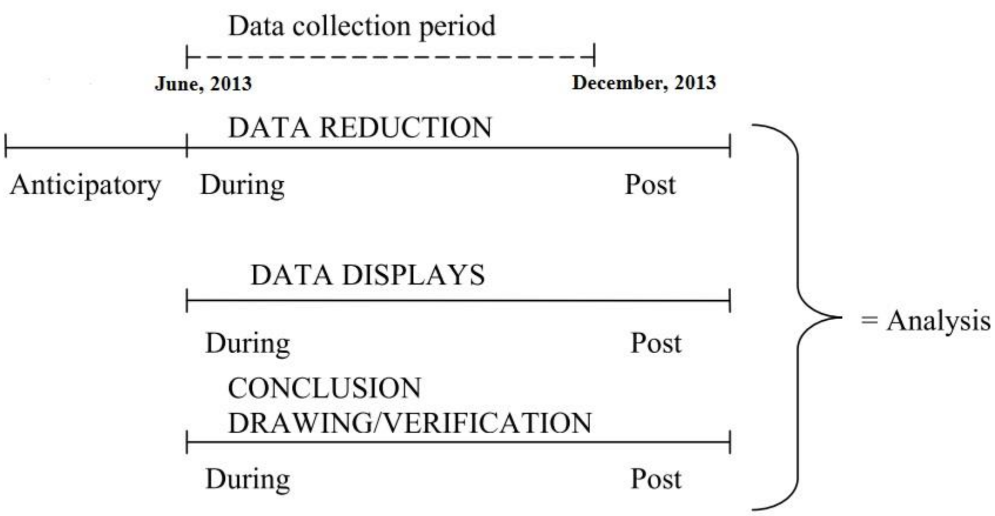


Figure 3.8: Flow Model of data analysis adapted from (Miles and Huberman, 1994)

The hand-written observations of the research assistants and researcher's reflections were used as the initial data collection techniques to refine the focus of this study. For example, the issue of excessive textual information was challenging as mentioned below:

> "I have noticed that excess of text is challenging for the students and detract them to understand the context and conceptual depth of the content. I need to maintain the balance between the printed and verbal textual information transferred during the next action research cycle." (Excerpt from Reflective Journal recorded on 14.06.2013 right after the session).

The data reduction flow was discussed amongst the research team as:

> "The issue related to excess of textual data can be resolved once we implement the MIT-BLOSSOMS modules through blended learning and we can easily eliminate the text related to the visual information in the module and hopefully we will implement the full test version during the pilot study. We should not be using videos in this session in order to validate the worksheets produced by you." (Researcher-Co Supervisor discussion June 17, 2013)

This resulted in reviewing the textual information provided on the worksheets followed by the review of the material posted online with BLOSSOMS modules. The handbook was developed based on the book "thinking mathematically" by Mason (2010) and used to implement themes along with prompts and questions during the teaching practice. The researcher also did the data reduction, display, and conclusion drawing when participants shared their experiences and concerns with instructor informally. Participants shared why they were engaged during the MTL sessions and what made them unhappy. The results of data reduction were shared with the research team (Data display) and discussed before reviewing the worksheets and the action plan (Conclusion) for the following cycles.

The development of worksheets, their content validity, and the analysis of the written activity responses were the main concerns during the first analytical stage. The preliminary study was conducted to serve as the content validity where as a rubric is adapted and developed based on Mason's work to improve the acitivites and to carry out the activity response analysis (Section 3.9) in depth. The first stage of analysis was carried out in a messy situation with limited data and time pressure whereas there was no time pressure during the second stage of analysis. The next sub-section will describe that stage.

3.7.2 Data Analysis -Stage II: Analysis post data collection

Once the data collection phase was completed, the researcher distant herself from the initial processes and started exploring the data in more depth during the post-hoc analytical stage. The researcher also dealt with higher level of abstraction and theoretical underpinning during the second stage. Reflexivity is provided by splitting the entire data analysis process into two stages (Oates, 2006).

All the hand-written observations, researcher's reflections and field notes were digitized using Microsoft Word 2013. The video recordings during the classroom sessions, field notes, focus groups and informal interviews were transcribed, formatted and imported in the qualitative data analysis software QSR NVivo 10. The researcher started coding the textual and visual data into open nodes. Sometimes, the researcher coded the same text sentence/passage or video clip to several nodes contributing to different themes. For example one sentence is coded to the node "Lack of attention', and is also coded to 'Student at Risk' node. Memos were written at the time of coding to make sense of the process.

The researcher first used ***pre-identified deductive coding*** based on Tall's framework (How humans learn to think mathematically) and Mason's Rubric of promoting and assessing mathematical thinking. There was a valid reason for ***pre-identified deductive coding*** especially when the researcher was examining the students' mathematical problem solving activities in the MTL and due to the fact the worksheets were produced using Mason's problem solving strategy having phases (Entry, Attack, Review) and processes (Specializing, Generalizing, Conjecturing, Justifying, Convincing). That covered the cognitive impact of this action research. For affective impact, the researcher used inductive coding to see any logically interconnected sequences or patterns emerging from the data. The concepts of free nodes, tree nodes, parent nodes and child nodes are described below:

> "Free nodes are an unstructured collection of nodes. Use them for ideas which you aren't ready to categorize. Tree nodes are organized into hierarchies, moving from a general category at the top (the parent node) to more specific categories (child nodes). Use them to organize nodes for easy access, like a library catalogue." (QSR 2002)

The inductive coding helped the researcher to bring out emerging categories/themes of students' met-befores, met-afters, challenges whilst problem solving and implications of blended learning. Hence, the coding were a combination of deductive (pre-identified theory-driven) as well as inductive (data-driven). At the end, the deductive coding is reviewed and modifications (addition/deletion) were suggested as needed in that particular part.

During every action research cycle, open nodes are created and grouped together under logical themes/categories. A total of 123 child nodes were created and then logically connected with 13 'parent' nodes during the whole process of coding. One parent node labelled as problem solving activity analysis having eleven child nodes represented the mathematical thinking processes activated in blended learning. Another parent node having three child nodes represented the world of mathematical thinking, thirteen child nodes represented students' met-befores, ten child nodes represented challenges whilst problem solving, eight child nodes represented personas, 13 nodes represented students' met-afters, 6 nodes represented diligence of the students during problem solving, and eight represented teacher–student relationship. Several iterations of reduction, display, and conclusions were undertaken and reported formatively in this thesis.

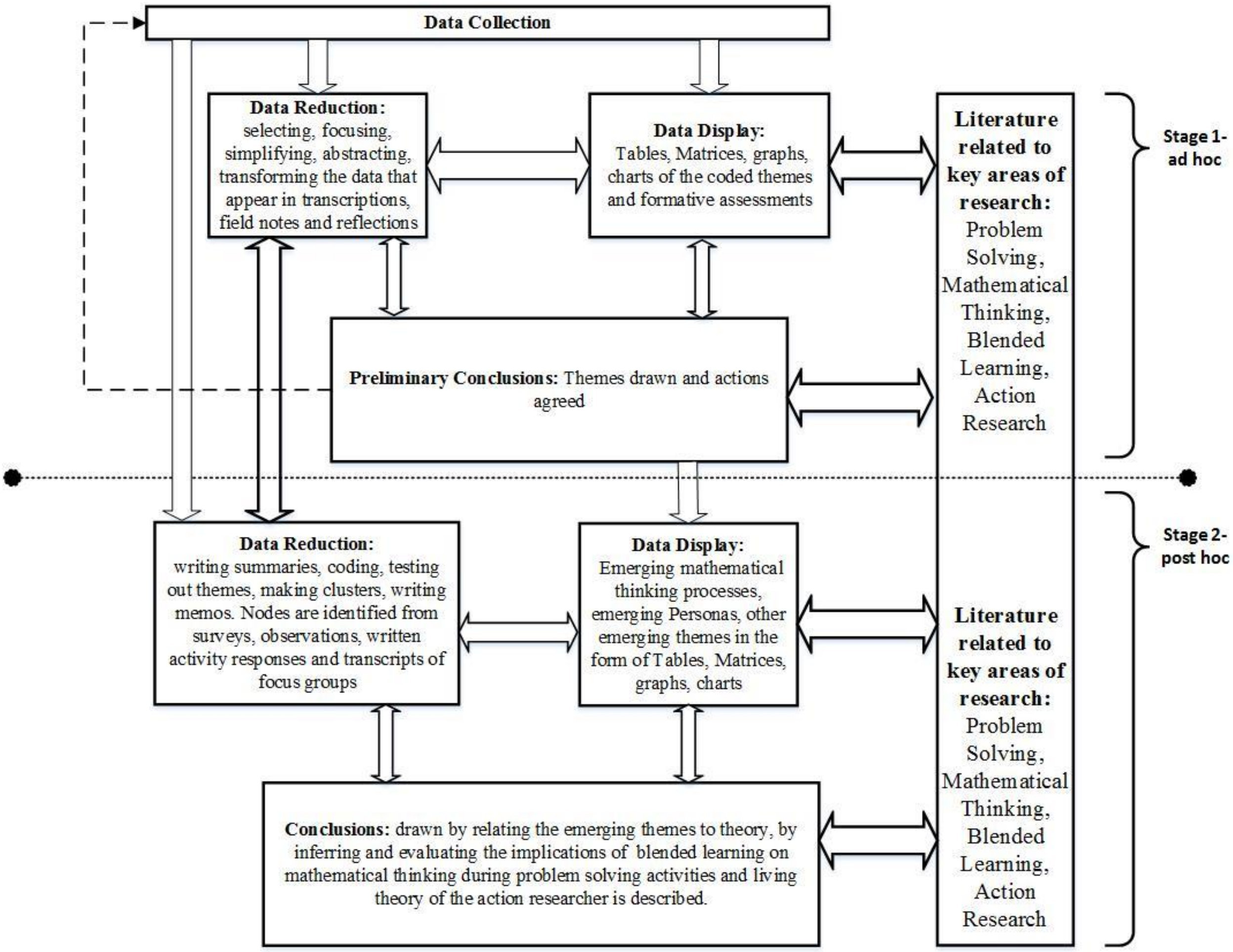


Figure 3.9: Two-staged data analysis in this study

Figure 3.9 shows a two-staged data analysis process. Five iterations were carried out for the reduction, display, and drawing throughout the data analysis of this research.

The next section will present the process to integrate problem-solving strategy with BLOSSOMS modules to create a blended learning environment.

3.8 The conceptual (process) framework to integrate Mason's Problem Solving Strategy with BLOSSOMS modules to create Blended Learning conducive to Mathematical Thinking

The process of integration for blended learning is first devised for this study and then the step-wise process is illustrated to integrate selected BLOSSOMS modules with Mason's problem solving strategy. The researcher anticipated the activation of mathematical thinking processes through blended learning during this study. The process of integration is shown in Figure 3.10 and the steps udertaken are listed below:

Step 1. Select appropriate BLOSSOMS modules (based on complexity level) for the first year engineering students.

Step 2. Extract the content and context from the BLOSSOMS module and use Mason's Problem Solving Strategy to transform them in the form of activity based worksheets.

Step 3. Validate the worksheets without the videos with a small group of first year engineering students during the preliminary research. Get the initial written activity responses, their analysis and refinement of the worksheets based on the activity response analysis.

Step 4. Run a pilot study using BLOSSOMS Videos as a pedagogical tool and Mason's themes, prompts and questions as a scaffolding technique alongwith the already validated worksheets.

Step 5. Analyze the written activity responses during the pilot study and refine the worksheets and strategy for the main study.

Step 6. Run the main study throughout a semester with first year engineering students utilizing a quarter of their time during their very first engineering mathematics course.

Step 7. Analyze the written activity responses with the help of a rubric adapted and developed using Mason's rubric for thinking mathematically.

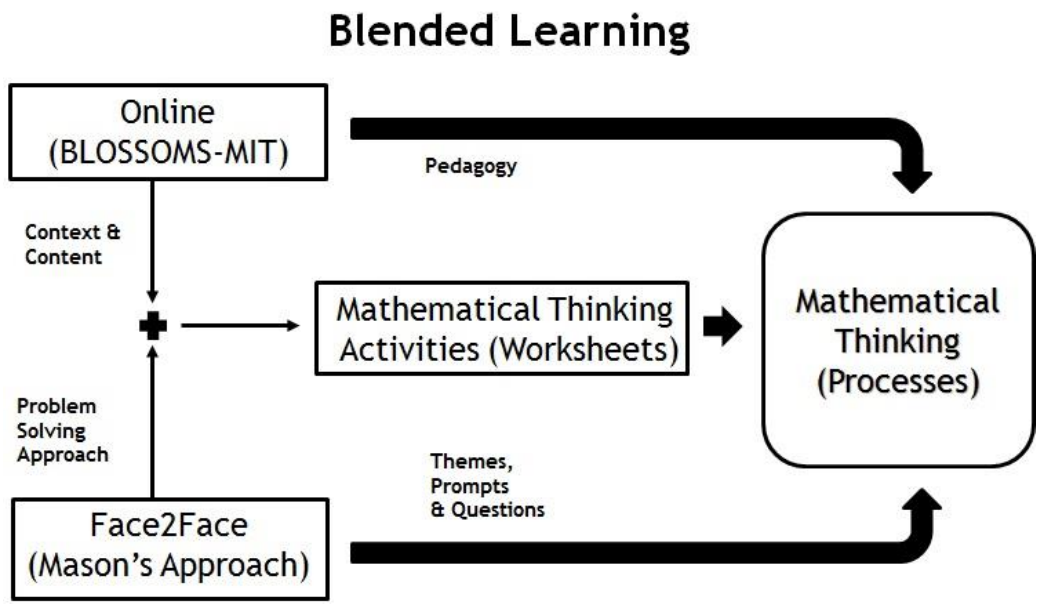


Figure 3.10: Conceptual (process) framework to integrate MIT-BLOSSOMS with Mason's Problem Solving Strategy to implement BL conducive to mathematical thinking

3.9 Problem Solving Activity Response Analysis

In depth, data analysis is carried out utilizing pre-identified deductive coding scheme (Fereday and Muir-cochrane, 2006) for written activity responses collected during the main research cycles as shown in Table 3.9. Hull et al (2013) concluded that "problem-solving rubrics should be revised or repurposed to more accurately assess problem-solving expertise." Therefore, a rubric to analyze the written responses was adapted (Mason et al., 2010; Tall, 2013) and revised by going through several iterations. The initial rubric is shown in Figure 3.11. The final and refined rubric and the sample-activity response analysis are presented in Chapter 6 (Section 6.2.6). The description of codes used in the rubric is given in Appendix E.

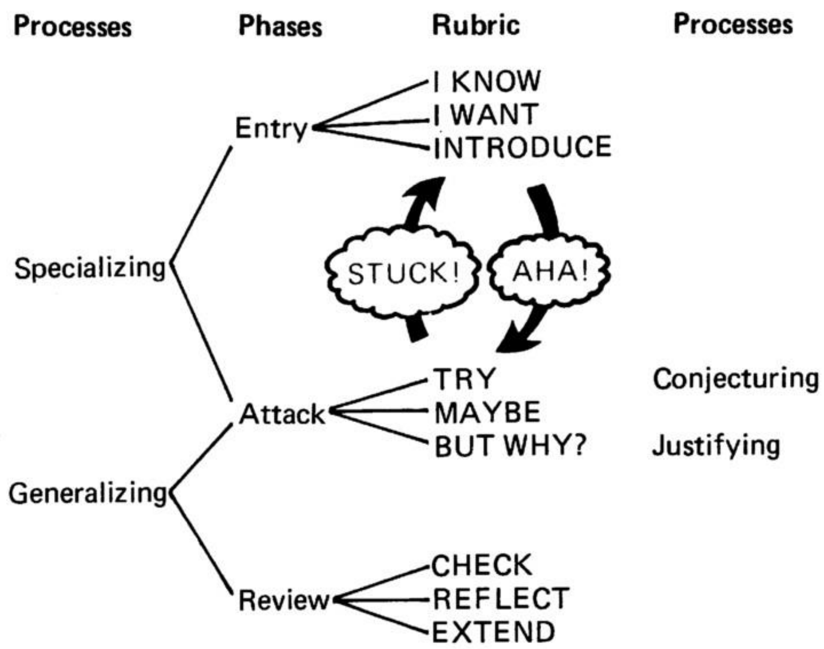


Figure 3.11: Initial Rubric to assess the mathematical thinking (Mason et al., 2010)

3.9.1 Pre-identified Deductive Coding Scheme

The pre-identified deductive coding scheme is used to develop the final rubric for the assessment of mathematical thinking activity responses as given in Table 3.10. The associated codebook is given in Appendix E. Three phases of evidence-based problem solving strategy (Mason et al., 2010) have been employed in this research as described in the following paragraphs:

During the "Entry" phase, students were instructed to:

- summarize "everything that is KNOWN and WANTED";
- represent "the question in some form that is concrete and confidence inspiring"
- take "advantage of the specializing already carried out";
- re-reading or re-digesting "the question while looking for alternative interpretations."

During the "Attack" phase, students were advised to:

- "do enough special cases systematically to reach a conjecture"
- "suggest altering a plan of Attack by trying to generalize in a different direction"
- "seek alternative underlying patterns."

During the "Review" phase, students were required to:

- critically check of the solutions
- reflect on the ideas used and the important moments during the process of mathematical thinking
- try to create new examples, questions and cases having similar concepts.

Table 3.9: Pre-identified deductive coding scheme adapted from (Mason et al., 2010; Tall, 2013)

Pre- identified Deductive coding Scheme for Problem Solving Phases	
Entry (E)	• I Know (know) (E1) • I Want (want) (E2) • I Introduce (introduce) (E3)
Attack (A)	• Pre-Procedure (A0) o No solution (A0a) or o Partial solution or initial action(s) prior to building a procedure (A0b) • Procedure (Ai) o Step by Step procedure to carry out an operation or step by step solution of a routine Problem • Multi-Procedure (Aii) o Several different procedures (or representations) to carry out the same operation, with a choice of the most efficient or choice of solutions for increased efficiency • Process (Aiii) o Equivalent solutions with various alternatives or Equivalent procedures as a single process • Procept (Aiv) o A single thinkable concept represented by equivalent symbols operating dually as process or concept
Review (R)	• Check the Resolution (Check) (R1) • Reflect on Resolution (Reflect) (R2) • Extend the Concept (Extend) (R3)

Table 3.9: Continued

Pre- identified Deductive coding Scheme for Worlds of Mathematical Thinking	
Worlds of MT	• Conceptual Embodiment (CE) • Blended Embodied and Symbolic (BES) • Operational Symbolism (OS)
Pre- identified Deductive coding Scheme for Mathematical Thinking Processes	
MT Processes	• Specializing (S) • Generalizing (G) • Conjecturing (Cj) • Justifying (Seeking Why) (J) • Convincing (Explaining Why) (Cv) • Imagining (Im) • Expressing (Ex) • Stressing (St) • Ignoring (Ig) • Classifying (Cl) Characterizing (Ch)

Questions, prompts, and suggestions are used during the preliminary, pilot and main action research cycles and compiled in Table 3.12 to illustrate their usage during different phases of problem solving activities. In order to think mathematically, multiple processes should be activated concurrently or in series. “Where do these processes lead to” are further described in Table 3.11.

Table 3.10: Process and “where does it lead to” adapted from (Mason et al., 2010)

Process	Where does it lead to?
Specializing (S)	It leads • randomly, to get the feel of the question systematically • to prepare the ground for generalizing • artfully, to test the generalization
Generalizing (G)	GENERALIZING means detecting a pattern leading to • WHAT seems likely to be true (a conjecture) • WHY it is likely to be true (a justification) • WHERE it is likely to be true, that is, a more general setting of the question (another question!) • Generalizing can sometimes bring about a change of perspective on the original problem
Conjecturing _Cj	Conjecturing is the recognizing of a burgeoning generalization. Once conjectures begin to flow, they tend to come like a cloud of butterflies, elusively flying off when approached. It is wise at this time to try to capture some of them and to recall the cyclic process Conjecturing is less of an activity and more of an attitude to the ideas that I have or to the statements that others make, so, too, questioning is an attitude, an approach to life.
Justifying (Seeking Why)_J	Once developed, the internal enemy can be extremely useful during other phases of thinking apart from justifying, because hidden assumptions can block progress at Entry as well as Attack.
Convincing (Explaining Why)_Cv	Checking the justification to see if it is convincing can be extremely difficult. Cultivating a healthy, positive skepticism of your own conjectures, actively searching for examples, which refute the conjecture, and learning to be critical of both your own, and other people’s arguments are essential. The three levels of convincing: convince yourself, convince a friend, convince an enemy (Skeptic)
Imagining_Im	Imagery alone is solipsistic. It is possible to use material objects, diagrams and pictures, voice tones and gestures, words and symbols to express discerned objects, recognized relationships and perceived properties.
Expressing_Ex	The act of expressing one’s feelings helps to distance him from the state of being stuck. It frees him from incapacitating emotions and reminds him of actions that he can take.
Stressing_St & Ignoring_Ig	Stressing leads to focus on most relevant information whereas ignoring helps to filter out the unnecessary information. In that sense, it’s a manifestation of amplifying and diminishing
Classifying _Cl & Characterizing_ Ch	It leads to the classification of an object based on its characteristics.

Table 3.11: Questions, prompts and suggestions if stuck for Entry, Attack, and Review Phases (Mason et al., 2010)

Entry Phase	
Questions	**Entry 1:** What do I KNOW? What do you KNOW from the question? From the past experience (Prior Knowledge) **Entry 2:** What do I WANT? What do you WANT from the problem? **Entry 3:** What can I INTRODUCE? What could you INTRODUCE to be able to express what you WANT succinctly? **Entry 4:** Ask Yourself, Is it the best way to do it?
Prompts	• Careful reading (to avoid overlooking information and to notice ambiguities) • Identify type of information that the question contains • List down how it might be used. • Formulate a more precise WANT • Trying to reconstruct the question (not necessarily in detail) • Writing down the essentials in your own words • INTRODUCING diagrams, symbols and charts can substantially help to get into the question • INTRODUCING a notation, a means of recording or a representation, puts you in a good position to begin the Attack phase.
Suggestions if Stuck	• The best advice is to specialize • Organize the results of specializing • Try simple cases • Do not give up just because you cannot picture it. Find a way! • Specialize. Heavily! Look for physical objects that might help. • Introduce a means to record your simple examples.
Attack Phase	
Questions	1) Have you tried generalization? 2) Are you able to make any conjecture? 3) Have you seen anything similar (in essence) before? 4) How would you justify your conjecture? 5) Have you convinced yourself, your friend and enemy? 6) What is the essence of the strategy? Does it always preserve what it claims to preserve? 7) Have you got all the possibilities? 8) Have you got repetitions? 9) What can you deduce?
Prompts	Do enough special cases systematically to reach a conjecture. Suggest altering a plan of Attack by trying to generalize in a different direction, to seek alternative underlying patterns.
Suggestions if Stuck	• Start somewhere! Make an Assumption! • Keep going! • Write down what you are doing to generate new ideas • Try the subsidiary question • Try to find a simpler way to represent the given information • Look for a pattern in the numbers, and in the diagram

	• Do enough special cases systematically to reach a conjecture • Try to find a systematic way to proceed • Try to simplify methods. • Introduce some pictures, diagrams or symbols. • Make a conjecture, however wild. • Now check your conjecture, looking for why it is right/wrong. • You may find yourself making and modifying several conjectures before you find one succinct statement that covers all cases. • Work backwards. Look at how you finish rather than how you start. • Record your various deductions so that you can CHECK them later • Try to combine special conjectures for special cases into one statement. • TRY supposing that you had done it.
Review Phase	
Questions	1) What you have done and why? 2) Did you come up with some ideas for improving your resolution? 3) Can you extend it to solve other problems? 4) Which concept did you learn new today? 5) Did you enjoy the way you learn new concepts in this class? 6) What is the strength of your group? 7) How to improve the effectiveness and experience? 8) What kind of skill you have learned today?
Prompts	**Review 1:** CHECK the resolution **Review 2:** REFLECT on the key ideas and key moments **Review 3**: EXTEND to a wider context
Suggestions if Stuck	• write up your resolution for someone else to read • Try to reflect on the way you had used specializing process of mathematical thinking and extend the question • Even if you get totally stuck, go over what you've done before consulting your facilitator!

3.10 Persona development Process

Persona development process in this research is adapted from Lene Nielsen (2013) used in design domain and contained four different main parts: data collection and analysis of data (Steps 1 and 2), persona descriptions (Steps 4 and 5), scenarios for problem analysis and idea development (Steps 6), and involvement of the participants and peer researchers (Steps 3, 7, and 8).

Step 1: Collecting the data

In the first step, the researcher collected as much knowledge about the student as possible. Data were collected from many different sources like demographics surveys, online open-ended google surveys, and current knowledge state using ALEKS, students' met-befores, formative assessment during the class and MTL, and classroom observations during the entire semester.

Step 2: Formulating Assumptions (Formulating Selection Criteria)

Based on Stage I data analysis, the researcher formulated a general idea of the various students within the focus area of study (Mathematical thinking during problem solving through blended learning among first year engineering students); including in what way the students differ from one another that is based on their attitudes, behaviors, mind-sets, and performances in engineering mathematics and their participation in MTL. The Stage II data analysis helped the researcher to filter some emerging personas.

Step 3: Discussing with Peer Researchers

In this step, the goal was to justify the assumptions about the differences among the users. This happened by confronting peer researchers with the assumptions and backed it with evidences.

Step 4: Deciding number of personas (Applying Selection Criteria)

In this step, the researcher decides the final number of personas. Eight archetypal student characters were emerged based on affective (emotions, behaviors, attitudes), cognitive (thinking processes) and psychomotor (what do they do in Class and in the MTL) skills of the students and researcher's prolonged engagement in the field.

Step 5: Describing the personas

The purpose of describing personas was to highlight the overlooked aspects of teaching and learning practices. It is also meant to prepare persona descriptions that express enough understanding and empathy with students. These personas have potential to help the practitioners and researchers to understand the challenges faced by students and help them by changing the way they teach.

Step 6: Preparing situations

The method was directed towards creating scenarios that described the solutions. The situations were taken as precursors to scenarios. In this, a number of specific situations were described that could trigger the use of research findings. Every situation or a number of situations was the basis of a scenario. The performance comparisons of all personas were also highlighted in their respective situations. The performance during the formative assessment in the class, the participation-based assessment in the MTL, persona's current knowledge state using ALEKS and their relative positioning in the classroom performance are shown in Figure 5.1 to 5.4.

Step 7: Peer Checking

To involve the research participants in the development of the personas, the researcher shared these personas through emails and asked the participants to check if they want to edit their profiles. With minor edits, the personas were then described (Section 6.2.4).

Step 8: Disseminating knowledge

In order for the method to be used in the research, the knowledge of the persona descriptions should be disseminated to the engineering education research community. The researcher decided to include persona descriptions in her thesis, planned to conduct a workshop for peer researchers, and intended to publish the research articles based on the findings.

The first two stages of the persona development process are reported in Section 6.2.4, the third stage is discussed in Section 6.2.5 by providing different situations for creating scenarios. The situation-based scenarios are created followed by the identification of the challenges and the idea development for their solutions in the same section. Stage 4 is also partially employed during this research by involving participants for member checking through emails. The researcher intends to conduct a series of workshops to introduce personas to peer researchers as her future research activity.

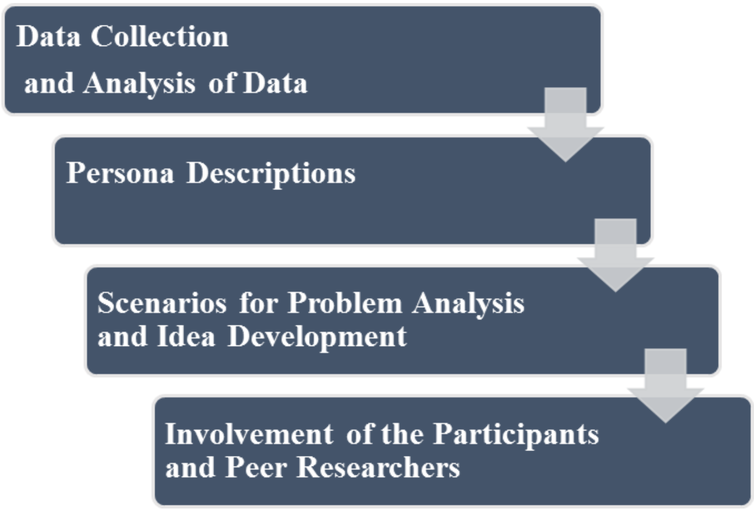


Figure 3.12: Persona development Process adapted from Lene Nielsen (2013)

3.11 Quality of Research

This section is inspired, adapted and modified from the latest research work employing interpretivist paradigm (Siddiqui, 2014). Various worldviews of qualitative research demand multiple ways of assessing qualitative research (Guba and Lincoln, 2005; Moss et al., 2009) but the best practices and criteria are still common to any qualitative study (Tracy, 2010). Significance of research topic, researcher's reflexivity, and prior teaching experience are some of the considerations in designing this research. The researcher started reviewing the literature over a wide scope and later engaged in discussions within the engineering education community that helped her to identify significant opportunities for action research in the area of mathematical thinking. She contextualized the study from the perspective of engineering education where mathematical thinking is a theme in discussions. The researcher adopted continuous reflectivity through all the stages of the study for rigor and meaningful coherence. Keeping journals of notes and memos constituted an important element for the quality of the research. Reflectivity was a dominant act in critically interpreting data and literature and in assessing the researchers' interpretations through the study's iterative process. The guiding questions during data analysis were "what do the data tell us and what do the data not tell us?" How strongly and how well does the data support our claims? The "interpretive validity" (Cohen et al., 2007) requires the reflectivity of this nature with the goal of having a "fidelity" to the participants' accounts. The researcher discussed her interpretations with her supervisors and presented those, on a few occasions, to peer researcher requesting them to challenge and question the analysis from the perspective of validity. This process of peer reviewing or debriefing is one of the techniques used to achieve validity in qualitative inquiry (Creswell and Miller, 2000). Research logs are also documented for the rigor of the research process (Creswell and Miller, 2000). The configuration aspects of the study design also included quality considerations. The study is situated in well-defined theoretical and methodological frameworks, both of which the researcher presented comprehensively in this chapter to inform readers on the perspective which underlined the study. The participants in the study consist of first year engineering students with similarities and diversity over a variety of dimensions, and in sufficient numbers to capture the richness and complexity of the activation of mathematical thinking processes during problem solving through blended learning. The researcher selected the participants purposively for their relevant and diverse educational background. The study also included measures for ensuring the accuracy of interview transcripts, as the researcher checked the accuracy of the transcripts herself and shared the transcripts with the participants for member checking. Triangulation process of corroborating evidence from different data collection methods is used to enhance the accuracy of the study (Creswell and Miller, 2000; Creswell, 2012). During the "process of conducting an external audit," the researcher obtained the services of a senior researcher who was not involved in this research in order to review different aspects of the research. The auditor reviewed the different elements of the study and provided a written evaluation (Appendix K) of the study (Creswell and Miller, 2000; Creswell, 2012) . Throughout the research process, the researcher took into consideration and was conscious of the ethical aspects. Reflectivity during the study included being critical in taking into account ethical considerations in relation to her actions and decisions. The researcher was mindful of being respectful and understanding to the participants and being attentive to the participants' voice when they were relating their experiences. The study included measures for protecting the anonymity of the human subjects of the study. The concept of accuracy of the post-positivist paradigm is not applicable to the interpretive qualitative research, which looks beyond the deterministic patterns and takes into account the complexity of a phenomenon with a holistic perspective. Instead, this interpretive qualitative action research involved the plausibility in terms of how much the researcher's claim correspond to reality (Cho and Trent, 2006). In relation to the complexity theory as a paradigm of inquiry, Davis and Sumara (2006) noted that "complexity thinking is more oriented towards truths that are viable, reasonable, relevant, and contingent" (p. 26). Plausibility is subjectively determined within the triangular relationship of the researcher, the study's participants, and the audience of the study (Patton, 2002). Researchers carried out their roles with an awareness of inter-objectivity in interpreting their observations. The researcher's goal in presenting the results was to be transparent to the readers, to let them listen to the voice of the participants. She provided thick (detailed) description of the participants' narratives and written responses along with her interpretations to allow the readers assess the plausibility of her claims and their relation with the reality examined in the study. As the assessment of plausibility is subjective, depending on the perspectives of the readers, the researcher expects different readers to find different levels of plausibility in this work. A study like this, with multiple goals and situated in an interpretive paradigm of inquiry, adapting action research involves a transgressive dimension for quality that emphasizes a higher degree of self-reflexivity (Cho and Trent, 2006). This research also covered the social validity drawn on Habermas (1976) using his ideas on the four validity claims in reaching mutual understanding i.e. comprehensible communication, providing evidence for assertions, revealing the normative background of communication and revealing authenticity in interaction through time.

The quality of research is also related to the constructivist perspective on mathematical thinking of future engineers, playing a role in facilitating makeover in the engineering mathematics context focusing on the development of mathematical thinking. An indicator of the quality of this work is in answering how much impact the study will make by bridging the gap between research and practice and how to facilitate the replicability in other research settings.

3.12 Summary

This chapter was concerned with methodology and its implementation process for the current research. The replicability of the research design and implementation process (Heinze, 2008) helped the researcher to align the research objectives, research questions, the context of engineering education research, data collection methods, methodology, data analysis and the researcher's philosophical assumptions. After discussing the two stages of data analysis, the process to integrate Masons' problem solving strategy with BLOSSOMS modules is described following by written activity response analysis and persona development process. In the end, the quality of this research is addressed.

The next chapter will target to meet the research objective meant for the development and implementation of blended learning environment.

CHAPTER 4

DEVELOPING AND IMPLEMENTING BLENDED LEARING ENVIRONMENT

4.1 Introduction

The previous chapter focused on the research methodology justifying the researcher's philosophical assumptions, employing the research methods, and describing the types and sources of data collection. The previous chapter also outlined the implementation of the research method by explaining the research setting and describing the participants of the study. The data collection methods, techniques, and two-staged data analysis were also explained in the last chapter. This chapter will describe the preliminary, pilot and main phases of action research, which were conducted during the second academic year of this research.

During the preliminary cycle, the researcher developed problem-solving activities by extracting the context and content from the BLOSSOMS modules and integrating them with Mason's problem solving strategy in the form of worksheets. There were three activities per module and every module was covered in a single worksheet. The worksheets were reviewed for pilot study based on the results of students' written responses and observations during the preliminary study. Two complete cycles were conducted during the pilot study to check the related issues of the intervention and to finalize the research methods and techniques for the main cycles of the research. Pilot study focused primarily on the integration process of Mason's problem solving strategy with BLOSSOMS modules to develop the blended learning environment conducive to mathematical thinking as shown in Figure 4.1. The main study will deal with the implementation process of blended learning, as highlighted in Figure 4.1.

The modified stages of action research: initial idea, reconnaissance, initial planning, initial therapeutic intervention (implementing & acting and monitoring & observing) and diagnosis (evaluating & reflecting, and reviewed planning) are described in detail for preliminary action research phase whereas therapeutic intervention (implementing & acting and monitoring & observing) and diagnosis (evaluating & reflecting, and reviewed planning) are discussed for the pilot and main action research phases. Guided by the preliminary and pilot studies, the researcher explored all the available resources to make the main study more effective and informative for future action research. The modified stages of action research: therapeutic intervention (implementing & acting and monitoring & observing) and diagnosis (evaluating & reflecting, and reviewed planning) are discussed in detail for the main study cycles. Before describing the preliminary and pilot cycles, the initial idea, reconnaissance, and initial planning stages of this action research will be discussed.

4.2 Initial idea

The initial idea is extracted from literature comprising journal and conference papers, books, academic and research reports, PhD dissertations, web pages, and personal communications with local and foreign researchers. The literature is reviewed in Chapter 2 whereas the research problem based on research gaps is formulated in Chapter 1. The next section will describe the initial investigations to draw guidance for this research.

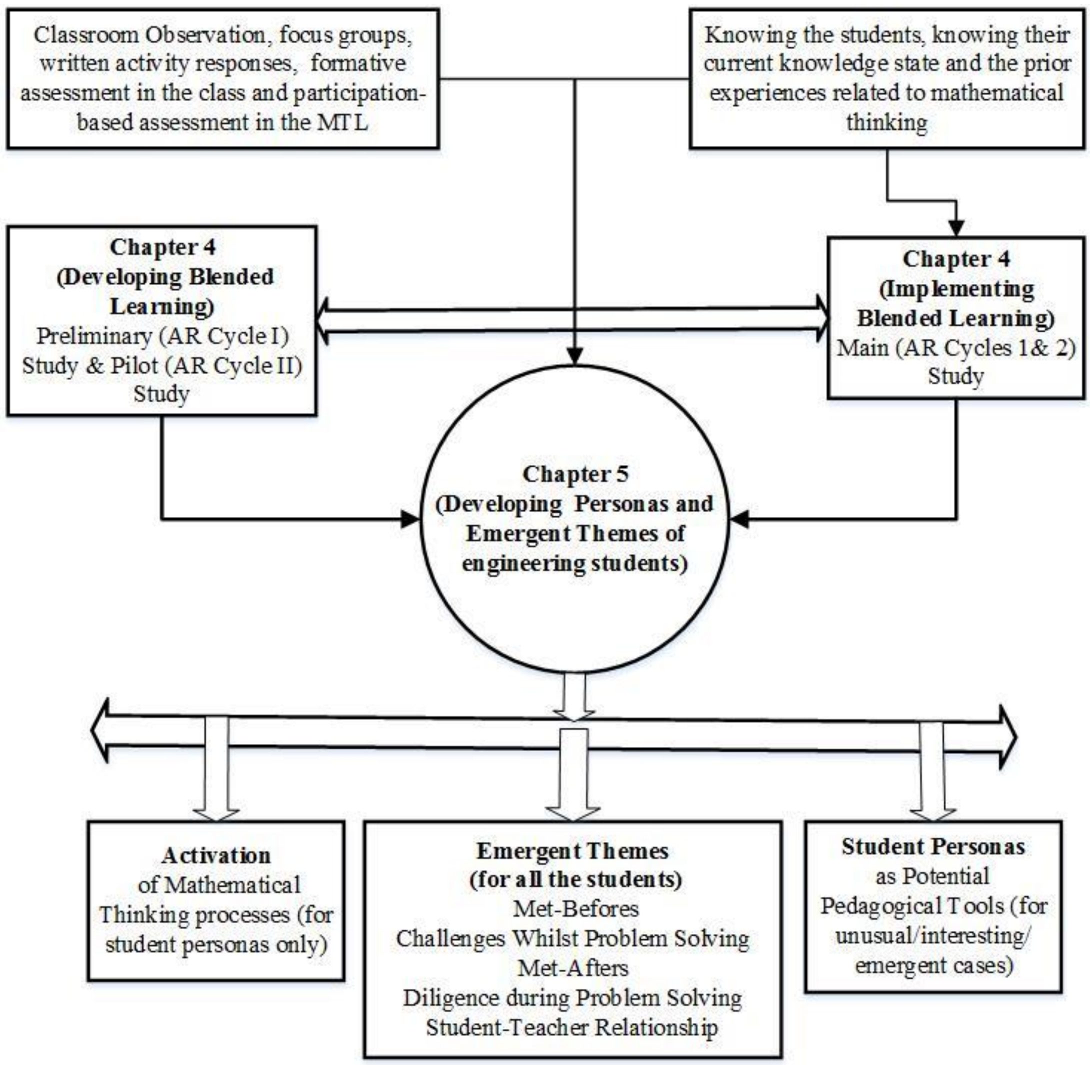


Figure 4.1: Chapters 4 and 5 and their relationships

4.3 Reconnaissance

An initial exploratory investigation is carried out to comprehend the main concepts, answer the initial queries, and extract the practical guidelines and to support the rationales of using BLOSSOMS modules and Mason's problem solving strategy in this study by analyzing:

- the interview transcript of the dean of mathematics department of a local university related to the importance of mathematics for engineers conducted on the 26th of April 2013.

- the transcript of an online interview with Alan Schoenfeld sharing his vision for a healthy & productive mathematics classroom retrieved on the 15th of March 2013 (Schoenfeld, 2013).
- take away messages from a MOOC "EDUC115N How to learn maths" taught by Jo Boaler from Stanford University and completed by the researcher on the 28th of September 2013 (Boaler, 2013)
- take away messages from a MOOC "Introduction to Mathematical Thinking" taught by Keith Devlin from Stanford University and completed by the researcher on 3rd of November 2012 (Devlin, 2012).

The researcher conducted and transcribed the first interview, whereas the transcription of the second interview and video lectures of the MOOCs were already available online. The researcher reviewed all the transcripts for errors and imported them to NVivo 10 for data coding and analysis. Table 4.1 describes the responses of important queries related to this study during the reconnaissance stage. The researcher reordered them to help readers make sense of reconnaissance stage. In the second column, the researcher tried to interlink all the answers in an interpretive manner by quoting selected excerpts from the data.

After grasping all the basic concepts: mathematics, mathematical thinking, and problem solving, the researcher focused on the importance of mathematical thinking for engineers. The Dean of the Mathematical Sciences Department showed his concern about the lack of importance given to mathematical thinking in engineering education and gave a few examples (Table 4.1) where mathematical thinking was essential to solve practical problems in this technological world. He further emphasized that mathematical thinking skills should be developed during P12 (secondary) education. He shared that current exam-oriented system is one of the hindrance in teaching mathematical thinking skills at P12. To avoid the low grades, many of the teachers take the safest way by complying with what the ministry wants and that is another hindrance. Later, Boaler (2013) explained the relationship of design thinking and mathematical thinking that added up to the importance of mathematical thinking for prospective engineers.

Table 4.1: Important queries and related answers during Reconnaissance stage

Important Queries	**Related Answers using Excerpts from Interviews and MOOCs**
What is Mathematics?	"Mathematics is the ***science of patterns***. According to that description, the mathematician identifies and analyzes abstract patterns. They can be numerical patterns, patterns of shape, patterns of motion, patterns of behavior, voting patterns in the population, patterns of repeating chance events, and so on. They can be either real, or imagined patterns, visual or mental, static or dynamic, qualitative or quantitative, utilitarian or recreational, they can arise from the world around us, from the pursuit of science, all from the inner workings of the human mind. Different kinds of pattern give rise to different branches of mathematics. For example, arithmetic, or number theory, studies the patterns of counting a number. Geometry, studies the patterns of shape. Calculus, allows us to handle patterns of motion. Logic, studies patterns of reasoning. Probability theory deals with patterns of chance. Topology studies patterns of closeness and position. Fractal geometry studies the self-similarity found in the natural world, and so on, and so on, and so on." (Devlin, 2012)
What is mathematical thinking and Problem Solving?	"In particular, a key feature of mathematical thinking is thinking outside the box and it helps you develop a valuable mental ability, a powerful way of thinking that people have developed over 3000 years. For problem solving the first key step is learn to stop looking for a formula to apply or a procedure to follow. If a student cannot solve a problem by looking at a template to follow or a formula to crunch some numbers into or a procedure to apply, what do you do? The answer is, you think about the problem a certain way. Not the form of the problem that is probably what you were taught to do at school and it served you well there. Rather, you have to look at what the problem actually says. Sounds as though it ought to be easy, but most of us initially find it extremely hard and very frustrating. It does not come quickly or easily, you have to work at it. You are going to have to accept going a lot slower than you do are used to. Most of the time, you won't feel as though you're making any progress. Your goal has to be understanding, not doing." (Devlin, 2012)

Table 4.1: (continued)

Important Queries	**Related Answers using Excerpts from Interviews and MOOCs**
What is the vision (characteristics and dimensions) for a healthy & productive Mathematics classroom?	**Characteristics of a Productive Classroom** "I would like to see classroom in which the students are engaged with the help of the teacher in really doing mathematical sense making and doing things appropriate for their grade level that involve them either taking real-world situations and mathematizing them using the mathematics to understand more deeply or doing the same with abstract mathematics." (Schoenfeld, 2013). **Dimensions of a Productive Classroom** ***First-focus and coherence*** of the mathematics: That's not surprising a class that is typically incoherent where the students and teacher, all over the place is obviously not going to be mathematically powerful but it also speaks to mathematical connections because a classroom in which students are doing procedures without understanding. Where the procedures come from without the sense making is also incoherent. For us, a coherent classroom is one in which there is the sense making so the procedures kids are using, make sense to them. ***Second-cognitive demand:*** you were talking about struggle and there's a strong body of research that says the typical teacher reaction in the United States when a student is in difficulty, is for the teacher to say here do it this way and what that does is, it romps the students the opportunity to engage in the mathematics and do the thinking themselves on and from our perspective the really productive zone is not where the teacher the students are given things where they just flounder and don't know what to do so the teacher does need to provide enough support to get the student to appoint where the student can engage with the problem productive way but then we have this phrase productive struggle that you don't want to do all the work for the student, you want to bring the student to the point where the student understands what the issue is and then have some honest sense-making to do. ***Third-equity:*** How many students were involved in the discussion, the teacher always call on the top five students who can provide the explanation that will move the lesson forward or the teacher had various techniques for making sure that all the kids are engaged and none of them is off the hook. ***Forth- agency, authority, and accountability:*** Do the students feel they can do mathematics? Can they move forward with mathematics? If I had one measure to capture what I thought was a productive classroom for students it would be the number of times that a student had the opportunity to say a second sentence in a row in explanation and you can contrast the Japanese and American tapes on that dimension. So do students have an opportunity to speak mathematics, to develop expertise, to be known for that proficiency in the accountability part, is do they do so in accordance with the rules of mathematics, the authority isn't the one who just stands up and talks the loudest, it's the one who speaks the most sense mathematically like that kid who produced the proof that the sum of two odd numbers is even. **Fifth-the uses of assessment:** we talked a lot earlier today about formative assessment on. Assessment serves the students when the teacher asks questions that reveal what the students understand and then the teacher can build on the understandings the students reveal and ask questions that help on deal with things that the students do not understand. (Schoenfeld, 2013)

Table 4.1: (continued)

Important Queries	Related Answers using Excerpts from Interviews and MOOCs
Why Mathematical Thinking is important for engineers?	"Dubai has several iconic buildings and one of them is Burj Dubai, the Sultan requested the engineers and architectures to address an issue. It defies gravity and it defies much more than the desired. The building was declining like the Pisa Tower and it was against the rules of mathematics so some new rules have to be devised and it is another type of innovation and without mathematics it is impossible to build such type of buildings. In another example, during the construction of a building when it was about to complete, Sultan requested engineers that building should have to bear the weight of a helipad too, Here comes the "wind tunnel analysis" where wind currents of certain level can cause the building to collapse, here engineers again have to stretch more than ordinary level to prevent collapse by the use of mathematics and it is another different kind of innovation and that is why it is important for engineers." (Interview excerpt of the Dean) "There's a process whereby you might try something, realize it doesn't work, refine it or revise it. That process is an essential component of design thinking as taught by Stanford's D School. But it's also an essential part of mathematical thinking. And then super-importantly, does the answer make sense? You may need to stop and ask that at various stages of a problem." (Boaler, 2013)
Do we really need to change the way we teach mathematics to engineers?	"There is one chapter in the book "Engineers for 2020" dedicating or requesting to change the way mathematics should be taught in the engineering course and you can get the idea from there. A new kind of mathematics is required now. You know how people many years ago build air craft, space shuttle and now they build it parts by parts and different parts are designed in different countries and it requires high order thinking like modelling, conjecturing, forecasting and these require high level thinking when you are not dealing with physical things but you are dealing with something else." (Interview excerpt of the Dean)
What kind of tasks, problems, and activities should be used?	Take away messages from the interviews and online courses related to mathematical thinking tasks are summarized below: a) Tasks should activate thinking processes. b) Complex problems should replace straightforward problems. c) Tasks should provoke critical and creative thinking. d) Diagnostic Feedback should be used instead of grading on problem solving responses. e) Tasks should have multiple solutions. f) Ill-structured problems should replace structured problems. g) Activities should have low entry but high ceiling point during problem solving. h) Tasks should be relevant to real-life problems.

Table 4.1: (continued)

Important Queries	**Related Answers using Excerpts from Interviews and MOOCs**
What are the current issues and efforts in using new teaching and learning methods?	Efforts for using Student Centered Learning (SCL) and Problem Based Learning (PBL): a. Student Centered Learning (SCL) and Problem Based Learning (PBL) are promoted b. Individual efforts of using new methods are not discouraged (few exemplars in PBL are available c. Success of one teacher motivates others d. Teacher's Competency in SCL is required While exploring the current issues related to teaching engineering students to think mathematically, the researcher short-listed the followings issues from the above discussion: • Countable few examples of bridging the gap between research and practice • Gap between written and implemented outcomes • Not enough practice and training to teach how to think mathematically • Drastic change encountered while moving from pure content based to the process driven learning for students • Low mathematical thinking skills of students at the time of joining the university • Many of the teachers still think that they cannot completely run away from content based approach
What are the current assessment and grading methods?	"Students should be assessed formatively to inform learning and not summatively to give a rank with their peers. Students should regularly receive diagnostic feedback on their work, instead of grades or scores. Summative assessments are best used at the end of courses." (Boaler, 2013)
What is the role of communication in mathematics classroom?	The teachers should communicate with students in the mathematics classroom to promote growth mind-set: "Students should be given ***growth mind-set messages*** at all times, through the ways they are ***grouped together***, the ***tasks they work on***, and the ***ways they are assessed and graded***." (Boaler, 2013)
How the relationship of inquiry with mathematics is developed?	"To develop an inquiry relationship with mathematics, approaching math with curiosity, courage, confidence, & intuition" (Boaler, 2013) is essential.

The reconnaissance stage informed the researcher that lacking perception about the importance of mathematical thinking is one issue but the next challenge is more serious in nature i.e. we do not have enough local experts to teach how to think mathematically. The dean has reported some of the individual efforts in implementing problem-based and student-centered learning in mathematics. Although the Research University (RU) did not discourage the individual efforts but he admitted that a planned change is required in teaching engineering mathematics. He also highlighted some content-related, student-related and teacher-related issues along with the need to bridge the gap between research and practice in teaching engineering mathematics.

The rationales of using Mason's problem solving strategy (Section 2.6.4) and BLOSSOMS modules (Section 2.4.4) are supported by the discussion during the reconnaissance stage of this study (Table 4.1).

4.4 Initial Planning: Integration of Mason's Problem Solving Strategy with BLOSSOMS Modules

The previous section dealt with exploratory investigation that enabled the researcher to look forward and plan the integration of Mason's problem solving strategy with BLOSSOMS Modules prior to the implementation of blended learning. This section will elicit the prospective process of selecting BLOSSOMS modules, extracting the content and context from the videos, integrating them with Mason's problem solving strategy, and developing the first version of activity based worksheets.

As a ***first step,*** the structure of BLOSSOMS video lessons and the employed pedagogy (MIT-LINC, 2013) are reviewed as described below:

- BLOSSOMS are Browse-able, download-able, carefully crafted, and ready to use lessons in the classroom.
- Every 50-minute lesson is a complete resource built on math fundamentals by relating abstract concepts to the real world that includes 3 to 4-minute

video segments, a teacher's guide, download-able hand-outs, and a list of additional online resources relevant to the topic.

- BLOSSOMS modules are built on shared teaching named as a "Teaching Duet. Over half of the lesson time involves in-class problem-focused learning with the students, with the video turned off.
- BLOSSOMS video modules are not intended to replace an existing curriculum but rather to enhance the teaching of certain lessons by the lively video presence of a gifted "guest teacher".
- Each video is designed for viewing in brief segments, allowing the in-class teacher between segments to engage the class in an active, goal-oriented hands-on exercise built from the video segment and provided by the guest lecturer. The lessons intersperse video instruction with planned exercises that engage students in problem solving and critical thinking, helping students build the kind of gut knowledge that comes from hands-on experience.
- By guiding students through activities from beginning to end, BLOSSOMS lessons give students a sense of accomplishment and excitement.
- After the learning objective is accomplished, the video is turned on again for another short segment.
- This iterative process continues until the exercise is over, usually lasting a full class session.

The **Second Step** is to maintain a database for 36 video lessons along with their summaries, grade levels and subject area as shown as a sample in Appendix F. The final selection of the video lessons is based on:

1 the relevance of the topic to the engineering mathematics course (Table 4.2a and 4.2b)
2 level of complexity suitable for first year engineering students
3 interesting real life contextual mathematical problems

The **Third step** is to select the two most appropriate modules and record the information in tabular form related to their introductions, objectives, outcomes, prerequisites and related activities. The selected modules “The Power of Exponentials, Big and Small” and “The Flaws of Averages” along with all the extracted information are shown in Tables 4.3a and 4.3b.

The **Forth step** was to examine video transcripts carefully and extract the content and context from different video segments, integrate Mason’s problem solving strategy with activities and develop the worksheets. Descriptions of all the scenes in the video segments 1 and 2 of the module “The Power of Exponentials, Big and Small” are provided in the Table 4.4a and 4.4b. Three phases of Mason’s problem solving strategy along with their descriptions and related questions are extracted (Mason et al., 2010) in the Table 4.5.

The **Fifth step** was to develop the first version of worksheets (Table 4.6) by integrating the context and content from BLOSSOMS module and three phases of Mason’s problem solving strategy.

Table 4.2a: Relevancy of BLOSSOMS module "The Power of Exponentials, Big and Small" to the current academic situation extracted and modified from BLOSSOMS online resources (MIT-LINC, 2013)

<table>
<tr><th>BLOSSOMS Module</th><th>The Power of Exponentials, Big and Small</th></tr>
<tr><td colspan="2">
➢ Motivation in choosing module on exponentials:
The "topic of exponential growth, and perhaps especially exponential growth as compared to polynomial growth, is a topic that really interested everyone" on the research team, "which is what motivated" them to use this module for this study. Exponential function is one of the key functions used in engineering mathematics as well.
➢ Applications of Exponentials
1. "Exponential growth is a topic that's keenly applicable to a number of different real world problems. For instance, population growth is often well modeled as an exponential function. Population growth is one of the huge long-term issues facing policymakers around the world."
2. "Another example, and one that really interested, deals with algorithm run times. Algorithms are used to process data all over the world for many important tasks. If you have an algorithm that runs in an exponential runtime, your hands are kind of tied, you can't give it too much data, or you're simply never going to get an answer back, because exponential functions grow so rapidly."
➢ Comparing exponential functions with polynomial functions
"Providing insight into just how fast exponential functions grow as compared, for instance, to polynomial functions, it is a very useful thing that's often not covered in a typical high school or perhaps even college curriculum. Of course, the in-class teacher has a key role in motivating the activities and kind of tailoring it to her classroom and making it the most accessible to her students."
</td></tr>
</table>

Table 4.2b: Relevancy of "The Power of Exponentials, Big and Small" to the current academic situation adapted from BLOSSOMS online resources (MIT-LINC, 2013)

BLOSSOMS Module	**The Power of Exponentials, Big and Small**
➢ Motivation in choosing the module "This module is not trying to send the message that averages are inherently bad. Averages are often worthwhile representations of a set of data by a single descriptive number. Our objective with this module is simply to point out a few pitfalls that could arise if one is not attentive to details when calculating and interpreting averages. " ➢ "The three flaws of Averages: #1: The average is not always a good description of the actual situation #2: The function of the average is not always the same as the average of the function #3: The average depends on your perspective" ➢ Other flaws of averages: "These are not necessarily the only three flaws of averages, and" researcher encourages students to "include other flaws of averages that they have encountered and find relevant when discussing this material with the class." ➢ "Designing the Module" "When designing the Flaws of Averages module, the goal was to make the video segments as clear as possible conceptually so that most students at any level in high school could understand the concept of these three flaws of averages. The essential prerequisite knowledge for the Flaws of Averages module is the ability to calculate an average from a set of numbers. More optional prerequisite knowledge that would be helpful for going into more depth in between the video segments is knowing how to calculate the area of a circle and also familiarity in working with the functions in general." ➢ Flexibility "In between the video segments, the choice of what material to cover and at what depth is entirely up to the instructor. This will clearly depend on students' background and knowledge of mathematics. The total length of the four in-class video segments is only 12 minutes, leaving lots of time in a typical class session for instructor to work with the students on learning examples to firm up their understanding of the ideas presented on the flaws of averages."	

Table 4.3a: Introduction, Objectives, Outcomes, Prerequisites and Activities of the BLOSSOMS module "The Power of Exponentials, Big and Small" extracted from BLOSSOMS online resources (MIT-LINC, 2013)

Introduction	Module Objective	Module Outcome	Prerequisite Knowledge	Activities
Exponential growth is keenly applicable to a variety of different fields ranging from cell growth in biology, nuclear chain reactions in physics to computational complexity in computer science.	To compare exponential growth to polynomial growth and to develop an insight about how quickly the number can grow or decay in exponentials through various examples and activities. To reflect on the magnitude of growth and decay of exponential functions.	1) students will develop an insight about how quickly the number can grow or decay in exponentials 2) students will be able to compare exponential growth to polynomial growth 3) students will be able to reflect on the magnitude of growth and decay of exponential functions	A basic knowledge of scientific notation, plotting graphs and finding intersection of two functions is assumed. It would be better if the students have done pre-calculus, though this is not a requirement.	Activity 1: Reward: "1 grain of rice for the 1st square of the chessboard, two for the second, four for the third, and so on. Every square must have double the number of grains as the previous one." Questions: "Do you think the award was very small? Was the treasurer able to pay off the mathematician with a single bag of rice? Was the queen able to pay off the mathematician at all?"
				Activity 2: Process: "Tear paper into half, and stack." Questions: "What do you think? How many tears would we need to reach the height of a person? How many tears would John need to reach the moon? Do you think we should call NASA with our new discovery?"
				Activity 3: Scheme: John gets paid 10,000 rolls a day. Swati gets paid twice the number from the previous day, starting with 1 on the first day. Questions: "On what day of the month, would we have given each other the same number tootsie rolls? And how many more tootsie rolls should John have asked us for in order to have the same or more number of tootsie rolls by the end of the month?"
				Activity 4:Scheme: John gets paid 10,000n2 rolls by the nth day. Swati gets paid twice the number from the previous day, starting with a 1 on the 1st day. Questions: "What do you think? On what day of the month would they have exchanged the same number of tootsie rolls? What would have happened if we had paid John 10,000n50 by the nth day, which means by the second day, we would have paid him 11 quintillion, that's 11 billion billion. And by the third day, we would have paid him seven octillion. That's 7 billion billion billion by day three. Do you think we would have a point in future when swati has been paid more Tootsie Rolls than John?"

Table 4.3b: Introduction, Objectives, Outcomes, Prerequisites and Activities of the BOLOSSOMS module “The Flaws of Averages” extracted from BLOSSOMS online resources (MIT-LINC, 2013)

Introduction	Module Objective	Module Outcome	Prerequisite Knowledge	Activities
This learning video presents an introduction to the Flaws of Averages using three exciting examples: the "crossing of the river" example, the "cookie" example, and the "dance class" example..	To simply “point out a few pitfalls that could arise if one is not attentive to details when calculating and interpreting averages.”	Students will learn about three flaws of averages: “(1) The average is not always a good description of the actual situation, (2) The function of the average is not always the same as the average of the function, and (3) The average depends on your perspective.”	“The ability to calculate an average from a set of numbers, how to calculate the area of a circle, also familiarity in working with functions.”	Activity 1: Flaw of Average #1 : “the average is not always a good description of the actual situation”
				Activity 2: Flaw of Average #2: “the function of the average is not necessarily the same as the average of the function”
				Activity 3: Flaw of Average #3: “the average depends on your perspective”

Table 4.4a: Content and context extraction from video segment one of “The Power of Exponentials, Big and Small” module

Video Segment # 1
A situation is modeled to raise the curiosity about the topic and to introduce the problem related to exponentials in the form of a story.
Story Problem embedded in Segment # 1:
Scene 1*: [Two students Nataly and Swati talking to each other while struggling with homework related to the concept of exponentials]*
Swati: Nataly, I just hate doing this homework.
Nataly: I know. Exponentials are a huge drag.
Swati: Yeah, well, now that you mentioned it, let me tell you a story my grandmother once told me about exponentials.
Scene 2*: [A queen and a mathematician is shown playing the game of chess]*
Background narration: There used to be a queen in India, and she got really bored playing the routine games. And so she asked all the mathematicians in the country to come up with a new game to amuse her.
And there was a poor mathematician who after years of carefully working through different ideas came up with the game of chess. And the queen, she was so pleased, and asked the mathematician to name her price.
Queen: Tell me, my math wizard, any reward you feel worthy of. I love your game of chess.
Mathematician: My lady, I'm a poor woman. All that I need is to have enough grains of rice to feed my family. I would like to have one grain of rice for the first square of the chessboard, two for the second, four for the third, and so on. Every square must have double the number of grains as the previous one.
Queen: Ha. A chessboard has only 64 squares. That's a triviality for a mighty queen like me.
Scene3: *[Queen ordering his treasurer where as treasurer is nodding the head]*
Queen: Treasurer, please give this young mathematician all the grains she's asked for.
Background narration: The queen seemed to think that the award was very small, a triviality, and that she would be able to pay the mathematician with a single bag of rice. Do you think the award was very small?
Scene4*: [Showing 2 students again and asking audience (who are in the actual classroom) some questions related to problem posed in the above scenario]*
Nataly: Hm. I think I need some help here. Do you think the award was very small? Was the treasurer able to pay off the mathematician with a single bag of rice? Was the queen able to pay off the mathematician at all? Discuss with your neighbours and with your teacher, and we'll be back in a few minutes.
After the first video segment, the face 2 face instruction will start and control will be transferred to the class teacher.

Table 4.4b: Content and Context extraction from video segment two of "The Power of Exponentials, Big and Small" module

Video Segment#2
Scene1: *[Treasurer explaining the queen mathematically that they are unable to pay the reward]*
Treasurer: My lady, I think there is a problem.
Queen: What is it?
Treasurer: Well, we've used hundreds of bags of rice, and we have not even covered half of the squares of the chessboard.
Queen: How can that be? We just started with one grain of rice.
Treasurer: Well, as you can see, at the beginning, we just needed one grain of rice. And then three and seven. Each time we added the rice for a new square, we added one more grain than the amount on all of the previous squares combined.
On some of the early squares, I noticed that the total we had paid out up to that square was 2 to the n minus 1 for the square n. For instance, we paid 2 to the third minus 1 or seven grains up through square three. Since we added eight more on square four, we had paid a total of 2 to the fourth minus 1, or 15 grains through that square.
Through a technique called mathematical induction, we can show that we paid 2 to the n minus 1 grains through the nth square no matter the value of n.
Queen: But what caused us to pay so much rice. Even though we doubled from the previous square, we have only 15 grains of rice till now.
Treasurer: Well, the problem, Your Majesty, is that after the first few squares, our payouts grow very rapidly. While at the end of the first row, we have only placed down a few hundred grains, and by the second, we have still placed down less than a full bag. We reached a million grains paid by the third row, and this is when the payments started getting big quickly. We had already placed down a billion grains of rice, more than 1,000 sacks by the fourth row when we still had more than half of the board to go. Had we been able to keep bringing the rice, we would have paid a trillion grains by the fifth row, a quadrillion by the seventh row, and according to my calculations, more than 18 quintillion grains of rice in total. That heap of rice would have been larger than the largest mountain in the world.

Table 4.4b: (Continued)

Queen: Curse you, wise mathematician, you tricked me. I have gone from the richest queen in the world to the poorest woman.
Scene 2: *[Showing Swati and Nataly again making sense of the mathematical concept of exponentials and student 3 named John jumps in to introduce the next activity]*
Swati: See, ignore exponentials, and you could lose a fortune.
Nataly: Well, that make sense. But I am not a queen, and I have no mathematicians in my employ, so I am still stuck with my homework.
John: Guys, guys, oh my, let me tell you. I just made a fabulous discovery.
Swati: Hey, John, what's going on?
John: Nataly, let me tell you. So I found when you take a stack of paper and you tear it in half and put one half on top of the other, it becomes twice as tall with each tear, which means in no time flat this pile is going to be enormous, very tall. So let me tell you my friends, today, I am going to the moon. See you.
Nataly: OH geez, John must have had too much coffee today.
Swati: It seems strangely similar to the one our queen faced in the story. There with every new square, the amount of grain doubled, and here with every new tear, the height of the stack doubles. So since we know exponentials grow so fast, you never know, starting with a 0.1 millimeter thick sheet, you might actually reach the moon.
Nataly:: I don't buy that. The moon is too far away. Let's think of a height of a person first.
What do you think? How many tears would we need to reach the height of a person? How many tears would John need to reach the moon? Do you think we should call NASA with our new discovery? Think about it a little bit, and we will come back shortly.
[Control will again be passed to the class teacher and she will engage the students in problem solving activity]

Table 4.5: Three phases of Mason's problem solving strategy along with their descriptions and related questions

Phases	Description	Related Questions
Entry_E	This first phase of tackling a question begins when someone first encounter the question, and ends when someone has become involved in attempting to resolve it. Work in the Entry phase often begins with specializing in order to get to grips with the question. The Entry phase work is largely in formulating the question precisely and in deciding exactly what I want to do. The other activity, which often takes place during Entry, is to make some technical preparations for the main attack, such as deciding on a notation or a means of recording the results of specializing.	Entry 1: What do I KNOW? What do you KNOW from the question? From the past experience (Prior Knowledge). Entry 2: What do I WANT? What do you WANT from the problem? Entry 3: What can I INTRODUCE? What could you INTRODUCE to be able to express what you WANT succinctly?
Attack_A	The major effort to resolve a question occurs in the Attack phase. This may lead ultimately to a complete resolution, or it may terminate in an incomplete resolution consisting of conjectures and unresolved questions. In either case, activity should not cease until after a final phase of Review. The states, which are particularly associated with Attack, are STUCK! and AHA! And the fundamental mathematical processes called upon are conjecturing and justifying convincingly. These in turn depend on specializing and generalizing. Attempts to resolve difficulties may stay within Attack or may lead back to Entry. Before leaving a question it is essential to carry out a third phase, Review. Discovery of an error or inadequacy may lead back to Entry or to Attack, and if an interesting new question is uncovered, perhaps through generalizing the resolution, the whole process begins again. During Attack, several different approaches may be taken and several plans may be formulated and tried out. When a new plan is being implemented, work may progress at a great rate. On the other hand, when all ideas have been tried, long periods of waiting and mulling for new insight or for a new approach may characterize the phase.	1) Have you tried generalization? 2) Are you able to make any conjecture? 3) Have you seen anything similar (in essence) before? 4) How would you justify your conjecture? 5) Have you convinced yourself, your friend and enemy? 6) What is the essence of the strategy? Does it always preserve what it claims to preserve? 7) Have you got all the possibilities? 8) Have you got repetitions? 9) What is the essence of the strategy? Does it always preserve what it claims to preserve? 10) What can you deduce?
Review _R	(Check, Reflect and Extend) It involves both looking back, to CHECK what you have done and to REFLECT on key events, processes and difficulties, and looking forward to EXTEND the processes and the results to a wider context. Discovery of an error or inadequacy may lead back to Entry or to Attack, and if an interesting new question is uncovered, perhaps through generalizing the resolution, the whole process begins again.	1) What you have done and why? 2) Did you come up with some ideas for improving your resolution? 3) Can you extend it to solve other problems

Table 4.6a: First page of worksheet "The Power of Exponentials, Big and Small"

BLOSSOMS MODULE_The Power of Exponentials, Big and Small

(Worksheet)

Greetings and Introduction to the Module

Assalmualikum and welcome to our module on "The power of exponentials, big and small.

Module Entry (required to raise the curiosity about the topic)_watch video segment#1

Activity 1:

Reward:

1 grain of rice for the 1st square of the chessboard, two for the second, four for the third, and so on. Every square must have double the number of grains as the previous one.

Questions:

Do you think the award was very small? Was the treasurer able to pay off the mathematician with a single bag of rice? Was the queen able to pay off the mathematician at all?

Assumption:

- Number of grains of the rice in a bag = 800,000

Instructions

- Please record your responses in the space provided under each question.
- Prompts will help you to come out of any STUCK state.

Entry Phase:

Entry 1: What do you KNOW from the question and also from your past experience?

- 64 squares on a chess board (example)

Entry 2: What do I WANT?

Entry 3: What can I INTRODUCE? (e.g. images, diagrams, symbols)

Table 4.6b: First page of worksheet "Flaws of Averages" used in Preliminary Cycle

BLOSSOMS MODULE_FLAW OF AVERAGES_Worksheet

Greetings and Introduction to the Module

Welcome to our Blossoms module on the flaws of averages. We begin our module with two illustrations of some limitations of averages.

Module Entry (required to raise the curiosity about the topic)

Example 1: On Thursday, I attended a party, on campus, in white dress. Then, Friday night, I went to UTM's elegant graduation dinner in my favorite black dress. For Saturday night, I was going to visit some friends and I decided to wear something that was the average color of the two dresses. So what did I end up wearing, which has neither the elegance of the black dress, nor the simplicity of the white dress?

Example 2: I'd like to tell you a quick story about a day that I had recently. On that day, I spent four hours in the morning here at UTM. Then, in the afternoon, I spent four hours across the river in Danga Bay. So, if I were to stand in my average location over those eight hours, where would I be? Yep, I'd be standing on the river. At least there was a sailboat for me to stand on! As I said earlier, today I am here to talk with you about the flaws of averages.

Clarification

Now before we start we do want to be clear that we're not trying to say that averages are bad. In many situations, averages provide a very good, single descriptive number of a situation. For example, what's the average height of all the students in your class? I'm sure you can come up with a number of other situations where the average number is a good description. The main point of this BLOSSOMS module is simply to point out a few pitfalls that could arise if you're not attentive to detail when you're using and interpreting averages.

Activity 1: Flaw of Average #1

"Our first flaw of averages is that the average is not always a good description of the actual situation."

Model: let's imagine that you're at the edge of a river that you want to cross. But, there's a sign. The sign says, "**Average river depth one meter**." Now, given this sign, would you cross the river?

Entry Phase:

Entry 1: What do I KNOW from the question and also from my past experience?

Entry 2: What do I WANT?

Entry 3: What can I INTRODUCE?

4.5 Preliminary Action Research Cycle

This cycle is used to validate the construct of the developed worksheet (Section 4.4) and assess the potential issues in the development of blended learning environment in the next cycles.

4.5.1 Therapeutic Intervention

After producing the first version of worksheets as described in the previous section, the researcher requested the volunteers from various faculties to participate in the preliminary cycle of this action research. Nine students from mixed faculties volunteered and cooperated during this intervention session. They were all in the first year of their academic programs. The researcher reported as:

> "I started the session in the hostel's activity room in an informal environment. The students are served with drinks and short cakes. They are briefed about the research and the consent forms are distributed. The students read the consent forms and handed me over after signing them." (Excerpt from field notes taken on June 13, 2013)

We decided to conduct the session without playing the BLOSSOMS video with the purpose of validating the worksheets. Therefore, the contextual text is extracted from the module "Flaws of Averages" and embedded in the worksheets. The first page of the sample worksheet is given in Table 4.6b. The date, venue, time, respondents, and other related information is recorded in the log file as shown in Table 4.7.

Table 4.7: Log file entry for preliminary action research cycle

Date	Venue	Time	Respondents	Facilitator	Module	Data Collected	Video	Photos
13th June, 2013	Activity Room Kolej 9	10:30 PM-12:00 AM	9 (7 females, 2 males) students from Mixed Faculty	Researcher	Flaws of averages without Video	Consent Form, Activity responses on worksheets	Yes	Yes

Classroom setting are listed below:

- Activity room Kolej 09 with a big meeting table and chairs reserved for the session
- Flexible seating arrangement helped students to change their positions during the session
- One big white board is available
- Room is spacious with ample open space
- Temperature is low
- Atmosphere is pleasant
- Friendly and informal environment

The beginning of the session is described through the video observation as follows:

> "The facilitator asks the volunteer to read the introductory part of the BLOSSOMS module from the worksheet and asks the other students to discuss, "Which color do you want to wear?" She asks again, "do you know how to find out the average?" A student replies, "The average of two numbers can be found by adding both and dividing the sum by two." The facilitator asks, "Now one color is black and the other is white, how would you get the average of both?" After discussing with each other, they responded with the answer "Grey". The facilitator asks, "Are you sure," now they are rethinking. The facilitator prompts, "which formula did you use to get the average of both the colors?" try to write it down on the worksheets and see how does it work? [They all write the formula as (Black+White)/two on their worksheets]. Giving

> them a chance to struggle further with the concept, she asked the volunteer to read the next example. One student volunteers and reads aloud the example given on the worksheet. The facilitator asks, “Did you get any idea what we are talking about?” Flaws of average! What are they? The facilitator is reading aloud the description given on the worksheet. While she is reading, most of the students are following the text on the worksheets and only one student is looking at the facilitator. After introducing the module, the facilitaor presents the activity by verbally repeating whatever is written on the worksheets. Students try to make sense of the problem but they are stuck. The facilitator prompts with a question, “think of a swimming pool, what about its ground surface? Try to visualize different depths in a swimming pool and then map that visualization to the riverbed. She repeats the prompts given on the worksheet and asks them to think about the prompts carefully. She adds, “You have to draw different shapes of the riverbed.” Then she explains that two different responses are required, “response 1: when you are able to keep your head above the water” and “response 2: when you are unable to keep your head above the water.” Students start to record their responses on the worksheets.”

The rest of the intervention session was kept deliberate and controlled. The researcher (who was the facilitator of the above session) tried to play the role of a practitioner with careful and thoughtful variation in her own teaching practice. Although the actions are not fully controlled by prior practice and plan, yet the teacher’s guide facilitated by BLOSSOMS and prompts and questions suggested by Mason (2010) guided the rest of the session. The session is made interesting and dynamic by making some instant decisions and practical judgements using just-in-time teaching technique.

We documented the intended and unintended effects in the form of field notes and self-reflections are recorded right after the session. The session was also videotaped to observe the students during problem solving activities and to capture the action process. Written activity responses were collected in the form of worksheets for activity response analysis.

Some of the Excerpts from the MTL observations on June 13, 2013 are listed below:

> "Students 1, 4, 5 and 7 are busy in discussion whereas students 3, 6 , 8 and 9 are not communicating with their partners" , "Students are reading the text written on the worksheets", "Students are relaxed in the informal setting".

4.5.2 Diagnosis

The researcher's reflections were retrospective, recalling actions as captured in video observations (recorded during the session) whereas activity response analysis was carried out during the stage-II data analysis. Some of the researcher's reflections related to behaviours and attitudes of the students are recorded below:

> "Small student group showed high engagement level but would it be true for large group of students as well? To check that I must choose a bigger group to compare the two responses. Students showed their willingness to learn new problem solving strategy but would it be the same for new group of students so I will have to investigate for the similar or different behaviors in the new setting and with a new group of students. Students showed full cooperation with me and with other students as well. I was expecting that due to the Malaysian culture of showing respect to the instructors. Students were engaged, relaxed and motivated. They communicated frequently with their neighbors and with me but they looked tired after two activities [Mathematical thinking activities are time consuming and require more time than procedural mathematics] so I am anticipating the same response from the other groups as well."
>
> (Excerpt from reflections taken on June 13, 2013)

Some of the findings from reflections related to the cognitive aspect of the problem solving activities are:

> "The students have the tendency to attack the problems without even thinking deeply about what is being asked, what is required to know and what they can contribute to solve the problem. The activity responses of all the students in response to activity 1 for "Flaws of averages" are shown in Table 4.8. Instead of deeply thinking about the problems, they all tried to solve the problem without using mathematical approach, for example; one student suggested using raft and the other suggested using life jacket to cross the river. All the responses showed their struggles in solving the problem while their thinking initiated in conceptual embodiment. The written activity responses showed lack of mathematical thinking skills. However, the students were not hesitant to record even the wrong answers on the worksheets." (Excerpt from reflections taken on June 13, 2013)

After the evaluation of the preliminary cycle, the practitioner decided that for the next cycles:

- Researcher should make sense of processes, problems, issues, and constraints made evident in strategic action.
- Researcher should take account of the variety of perspectives during the pilot and main cycles.
- Inquiry should be aided by discussion among participants as frequently as possible.
- The interpretations should have descriptive aspect covering the new understanding, a richer picture of what actually happened during the MTL session, what worked and what did not, what is the same and what is different in terms of cognition and attitudes among different students.

Guidelines for the review to further refine of activity-based worksheets and blended learning environment are based on the overall preliminary cycle's experience as recorded in Table 4.9.

Table 4.8: Preliminary cycle-Activity Responses for Activity 1 - “Flaws of Averages”

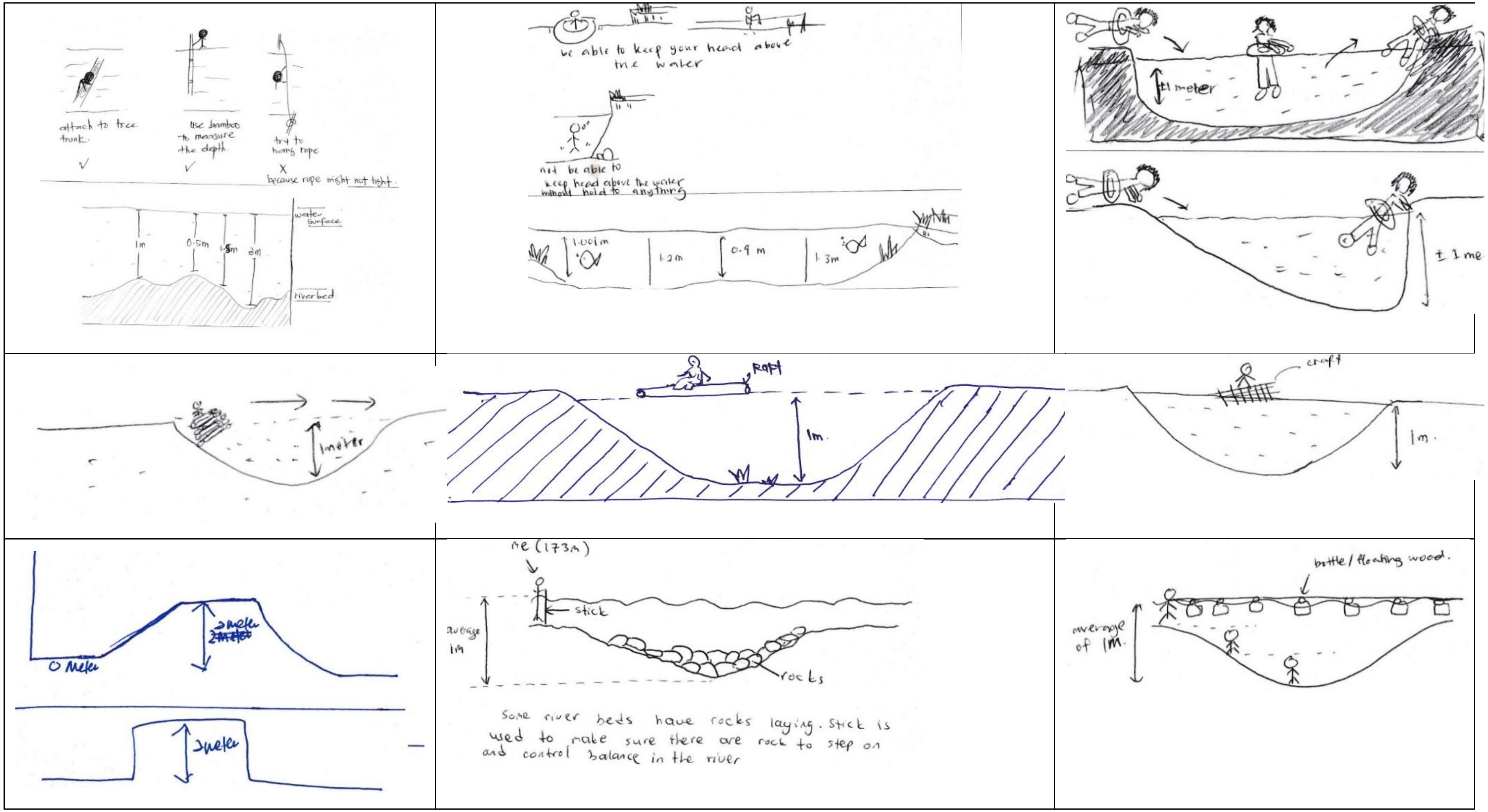


Table 4.9: Preliminary action research- cycle summary

Implementation & Monitoring
▪ At least two hours are required to complete one BLOSSOMS module in the class. ▪ High Engagement, cooperation, and positive attitudes are evidenced during context-rich problem solving activities.
Evaluation & Planning
▪ The worksheets are validated and revised based on the students' responses and researcher's reflections. ▪ BLOSSOMS videos should be used in pilot cycles along with worksheets. ▪ New group of students from first year engineering for pilot study should be considered as potential informants. ▪ At least two research assistants are required to videotape the lessons and observe the classroom activities. ▪ At least two modules are required to be conducted during pilot in the actual classroom setting ▪ The working response space for different phases of Mason's problem solving strategy (Entry, Attack, and Review) should be allocated on the worksheets. It would help the researcher to classify the specific responses under different phases. ▪ Excessive textual information should be eliminated when using video modules in the classroom. ▪ The prompts, questions and themes should be repeated verbally to reinforce even if they are written on the worksheets ▪ Time and space constraints should further be taken into consideration. ▪ The way I conduct blended learning activities in the actual classroom should be kept flexible. ▪ The way I group students should be kept flexible. ▪ Video recording and classroom observations are subject to the availability of the supporting staff but are highly recommended. ▪ The pilot study should be strategic to facilitate the researcher acting as practitioner as well, to act more effectively over a greater range of circumstances, more wisely and more prudently.

4.6 Pilot Action Research Cycle 1

We revised the action plan for the pilot cycles based on the reflections made during the preliminary cycle. The researcher decided to use BLOSSOMS as an online component of blended learning and Mason's problem-solving approach as a face-to-face component of blended learning to harmonise the use of the worksheets, to optimize classroom communication, and to highlight emerging queries. The summary of the Preliminary cycle is given in Table 4.9.

4.6.1 Therapeutic Intervention

Worksheets containing problem-solving activities were revised based on the results of the preliminary cycle (Table 4.9). The researcher requested a few faculty members who were teaching first year engineering mathematics to allow her to work with their students. One senior lecturer at the local research university (RU) agreed to allow intervention in her class of "Foundation of Engineering Mathematics." Then the researcher started communicating with her about the pilot cycle of this action research. She provided the researcher with a list of all the students pursuing Diploma in Mechanical Engineering during their first year.

The meeting with the class instructor and delivery of the first segment of the module is described in the next paragraphs. The researcher scheduled a meeting with the classroom instructor to know about the students, course, ethics, and norms of the classroom. During that meeting, the researcher introduced her research to the senior lecturer and requested her to sign the consent form.

The class instructor informed that chapter one of the textbook covering Indices, Surds, and Logarithms has already been taught. She allowed the researcher to use the tutorial session to conduct the research. The researcher then introduced the BLOSSOMS modules "The Power of Exponentials, Big, and Small" and both agreed on the content. The researcher asked the instructor to further support the research by motivating the students to cooperate and to convey them that it is in their own interest to learn how to think mathematically. The researcher also shared the concern that students may not be willing to learn something that is extra than their syllabus. The lecturer showed her agreement and added:

> "This is because all the students are not at the same level. The students are here from SPM level. Since they are coming from the school so they are having problem of changing the way they use to learn at school. Sometimes they cannot accept the change but hopefully this group of students is better because their basics in mathematics are good. My other section is not very good, they are slow. They cannot even do a plus b whole square. I have to conduct more tutorial classes for them. In this university the students in mechanical are usually good." (Interview Excerpt, June 27, 2013)

The class instructor accompanied the researcher to the classroom. Plain sheet of papers, the consent forms, and worksheets were distributed among the students. Students were requested to read and sign the consent form. One research assistant videotaped the session, one was assisting the researcher to setup the video projector, and distribute the worksheets whereas another research assistant recorded the classroom observations. Students read the consent and signed it. The class teacher was also inspecting the session. Researcher started playing the first segment of the video module; students were engaged in story based (context-rich) problem. After the first segment, control is transferred to the researcher, and she inquired if they understood the problem and they all nodded. The researcher asked them to start discussing with their group members and record their responses on the worksheets. The class instructor who was constantly inspecting what the students were doing in the class, requested the researcher to explain how to record the number of rice grains in different boxes. The researcher showed them how 2 by 2, 3 by 3 and 4 by 4 matrix is filled with grains. Students were then asked to extend it to 8 by 8 to replicate the chessboard and try to fill in the grains or number of grains in each box of the chess. She asked whether the reward is small or not. Students answered, no. She then asked, "Can you explain why?" What pattern can you see? Students were advised to record their responses on the worksheets and the rest of the session went on smoothly.

Classroom setting and environment related information is listed below:

- Venue (see log file in Appendix L).
- Fixed seating arrangement
- White board available
- Big auditorium like room
- Temperature high
- Timings (see log file in Appendix L)
- Formal environment
- Students sitting in group of fours
- Video projector and video camera were facilitated

The Research assistant who was recording the classroom observations reported:

> *Time Management*
> Senior Lecturer-Researcher: The discussion started half an hour late at 9:30 AM due to the late arrival of the classroom teacher.
> Student-Researcher: The session should have started at 10:00 AM, but the students entered the lecture hall at 10:30 AM, which caused 30 minutes of delay. They reasoned that they had just finished their previous class.
> *Classroom Environment*
> For these kind of activities, the lecture hall is not an appropriate place. The ascending students' locations made it difficult for the practitioner-researcher to approach each of them. It was also difficult to have frequent communication in this situation.
> *Students' engagement and interest*
> Students were not prepared for the activities and in fact were puzzled. However, after 15-20 minutes, they started showing their interest to learn more about the technique.

The research assistant also sent some of her suggestions through email on June 28, 2013 to improve the overall practice:

> "Overall, that was such a great technique and approach for the students to apply their knowledge into the application and real life activity. However, here are some ideas for improvement.

1. Give stress on the quality of time and punctuality.
2. Change the location conducive to classroom discussion environment.
3. Increase the intervention time. The allocated time is not enough for them to show their interest and knowledge."

4.6.2 Diagnosis

The researcher's self-reflections related to behaviours and attitudes of the students after the session are described as:

> "It was difficult to manage a big group of students in mathematical thinking sessions like today's. Students were fully engaged during the video segments. They were hesitant to respond verbally in the class. They discussed with neighbouring friends but less frequently. Sitting arrangement was not flexible and ideal for group discussion and collaborative work. Students were willing to learn new problem solving strategy. Students showed cooperation with me and with other students. Students were engaged, but not very relaxed [the presence of their class teacher can be one of the reason]. Time was not enough to cover the whole module." (Excerpt from researcher's Journal on June 27, 2013)

Some of the findings related to cognitive aspect of the problem solving activities are:

> "Students did not record sufficient written evidence to show their mathematical thinking."

> "Only five students showed partial Blending Conceptual Embodiment and Operational Symbolism (BES) as the response to first activity."

A sample worksheet is shown in Appendix M.

After the evaluation of Pilot Cycle 1, the researcher reported the following ways of proceeding:

> "I need to request the classroom instructor to allow longer intervention time per session. I need to explore different ways to make the classroom environment friendly and open. I need written evidences to gauge the cognitive aspect of the problem solving activity responses. Videos should be used in the next cycle. Revise the worksheets for next cycle on the same guidelines as for cycle 1. Sitting arrangement should be flexible. I can allow the groups of three or four students but I will ask them to record their individual responses on the worksheets."

Hence, the plan is reviewed based on the overall cycle's experience.

4.7 Pilot Action Research Cycle 2

The first pilot cycle did not provide sufficient evidence to support the activation of mathematical thinking processes. However, it guided the researcher to make some changes as suggested in the previous section. This cycle is conducted after about a month's time from cycle 1 as listed in the log file (Appendix L).

4.7.1 Therapeutic Intervention

The phase of implementation started as:

“Students entered the classroom at the designated time. They left the first two rows and started sitting from the third one. The teacher asked, "Why did you leave the first two rows?" They started smiling and mumbling. “Are you afraid of me?” The teacher asked to break the ice. The facilitator distributed the worksheets among the students and encouraged them to be open in writing the responses on the worksheets and requested them not to leave the worksheets blank. She asked, "Did I tell you earlier that mistakes are good? I will not deduct any marks for wrong answers but I will not give you any credit for blank responses.” The facilitator distributed the worksheets and students started checking those worksheets. They showed interest in class. The researcher asked the students to work in a group of three because of the sitting arrangement in the class. Some students entered late. The attendance sheets were circulated. The students filled in their names, date, and class. They were in good mood and seemed comfortable [The class teacher is not here today]. Then the instructor asked again, “who will answer why the mistakes are good?” One student answered, “We learn something new when we do mistakes.” Then the researcher shared a research finding that because of a mistake, new connection of neurons build up and the brain grows in size. [Researcher used the information from MOOC “How to learn Maths” shared by Jo Boaler (2013)]. The instructor then asked, " Do you know the concept of Average?. One student replied, "We add two numbers and divide by two to get their average. The teacher asked, " Have you ever used the function of average in your real life?" Another student replied, " I calculate average marks in exams." One student added, " Temperature". The facilitator said ," Can you explain how?" He was unable to explain, when the students laughed, the teacher encouraged the student to speak up. Then he tried again, "finding average temperature of the week". What about

> average run rate of a player in cricket?, the facilitator asked. They all smiled. Later, the first segment of the video module "Flaws of averages" was displayed using a video projector. All the students were fully engaged in watching and listening the video. The first activity was presented and the facilitator asked, “If the sign board besides the river says that the average depth of the river equals to 1 meter, would you cross the river?". First, they all said, "NO" then after a moment, they changed their answers to "YES.” The facilitator directs them to justify their answers and try to go through phases of entry, attack, and review during problem solving [She explained all the three phases of problem solving at this point]. Then the students started working with their group members and recorded their responses on the worksheets. The rest of the session continued in the same manner.”

The above description of the beginning of cycle 2 showed the researcher’s careful and thoughtful variation of practice from the previous sessions.

The research assistant who recorded the classroom observations reported later through email as:

> “The session started on time. The instructor tried to reach every student and kept walking back and forth [to improve their attention]. Individual worksheets were distributed instead of sharing one with group members [gave them the opportunity to answer individually and discuss with their friends at the same time]. The students were losing attention at the end of the class. “The BLOSSOMS video is easy to understand and interesting” as per most of the students. One group managed to sit in a circle at the back of the class and they were discussing among their group members more frequently in comparison with the students who were sitting in rows. Students were relaxed and constructing knowledge at the same time. Two hours are too long for a session like this. Most of the students were active and responded to the facilitator’s queries.” (Excerpt from researcher’s Journal on July 29, 2013)

Written activity responses were collected in the form of worksheets at the end of the session and will be analyzed during the stage-II data analysis.

The classroom related observations are listed below:

- Lecture rooms (see log file Appendix L).
- Fixed seating arrangement (due to the space constraint)
- White board available
- Spacious room
- Temperature Normal
- Atmosphere pleasant
- Timings (see log file Appendix L)
- Semi-Formal environment
- Multimedia available

4.7.2 Diagnosis

The researcher's self-reflections related to behaviors and attitudes of the students after the session are copied below:

> "It still is a challenge for me to manage a large group during inductive way of teaching. The students were fully engaged during the video segments. They were comfortable in responding to my queries in today's class. They discussed with neighbouring friends more frequently today. Sitting arrangement was not flexible due to the unavailability of the free space in the room. Students were willing to learn new problem solving strategy. Students showed cooperation with me and with other students. Students were engaged, and enjoying the session. Time was not enough to cover the whole module. Students looked tired at the end of the session."
>
> (Excerpt from researcher's Journal on July 29, 2013)

Some of the findings related to the cognitive aspect of the problem solving activities are described as:

> "Students record written evidences to show their mathematical thinking processes using worksheets. The evidences of Conceptual Embodiment (CE), Blending Conceptual Embodiment and Operational Symbolism (BES), Operational Symbolism (OS) are classifiable but I need a rubric to maintain the uniformity in assessing the responses."
>
> (Excerpt from researcher's Journal on July 29, 2013)

A sample worksheet is shown in Appendix M.

The following general questions emerged after pilot study and helped researcher to review the plan for the main study.

- Whom to teach? (Knowing the students, their profiles, current knowledge state and met-befores)
- Who is teaching? (Researcher's profile and expertise)
- Where teaching and learning happen? (Space)
- What to teach and why? (Content)
- How to teach? (Teaching method and approach)
- How to group the respondents in the class? (Class management)
- How to collect and manage the data? (Data management)
- How to group and record findings? (Data display)
- What is the researcher's lens? (Biases and view point)
- How to assess the embodied and symbolic activation of mathematical thinking processes? (Drawing conclusions)

The plan to conduct the main cycles is reviewed based on a summary extracted after pilot Cycles 1 and 2 and given in Table 4.10.

Table 4.10: Pilot action research- Cycle (1 and 2) summary

Implementation & Monitoring
▪ Two-hour session became hectic for students. ▪ The video modules are tested and results related to engagement were positive. ▪ High Engagement , cooperation and positive attitudes are evidenced during the problem solving activities
Evaluation & Planning
▪ Revisions are based on students' responses and researcher's reflections ▪ Videos and worksheets should be used in the main cycles. ▪ First year engineering students should be preferred for the main study. ▪ At least two research assistants are required to videotape the lessons and observe the classroom activities ▪ At least two modules are required to be conducted during the main study in the blended learning environment ▪ Implementation of blended learning should be adaptable towards unexpected and emerging effects and constraints, and thus need to be very flexible. ▪ Research design should be strategic means the real constraints should be recognizable and researcher should be well-prepared for emerging risks. ▪ Different components should be selected in a way to allow the practitioner to act more effectively over a greater range of circumstances, more wisely and more prudently.

The researcher critically reflected upon the emergent questions as highlighted in the previous sub-section and explored all the available resources to take action based on her initial reflections in response to the emergent queries:

Whom to teach? (Knowing the students, their current knowledge state and current mathematical thinking and problem solving skills)

Initial Reflections: "I should investigate the online methods to conduct the demographic surveys collecting as much information as possible related to the attitudes, expectations, and perceptions of the students. Some diagnostic assignment should be used to gauge their current mathematical and thinking skills. Online assessment methods should be used to know the current knowledge state of the students. Informal, unplanned, and short conversations between student and teacher, and classroom observations can help to know their met-befores related to mathematical thinking and problem solving."

Actions Taken: Google Forms are developed and used to collect the demographic data, attitudes, expectations, academic background, and reflective feedback of all the students. An initial diagnostic study is planned to gauge the conceptual understanding related to functions as described in the next section. The researcher used **A**ssessment and **LE**arning in **K**nowledge **S**paces (ALEKS) to know the current knowledge state of the students related to engineering mathematics. ALEKS is a web-based, artificially intelligent assessment and learning system that uses adaptive questioning to quickly and accurately determine exactly what a student knows and does not know related to a concept. Informal, unplanned, and short conversations between student and teacher are encouraged to understand their prior experiences related to mathematical thinking. The researcher also decided to observe the students in their traditional classroom during the engineering mathematics class to inform her own practice in the mathematical thinking lab (MTL) and during the overall data analysis process. The prolonged engagement in the research setting and spending long periods with the students will also contribute towards the validity of this research (Creswell and Miller, 2000).

Who is teaching? (Researcher's profile and expertise)

Initial Reflections: "I should critically review my past teaching practices and plan a strategy to work on my professional development by attending workshops on new trends in engineering education."

Actions Taken: Researcher reflected on her previous practice of teaching. She intentionally attended some workshops during her research to train herself to the new trends in engineering education. These workshops contributed towards her professional development and helped her to grow as an engineering educator.

Where teaching and learning happen? (Space)

Initial Reflections: "Classroom should be spacious to facilitate students to change their sitting positions more frequently."

Actions Taken: A spacious computer lab is allocated to conduct the weekly mathematical thinking lab (MTL) sessions.

How much time per session? (Time)

Initial Reflections: *"*Decision related to time per session is subjected to permission from the related authority."

Actions Taken: The class teacher agreed to give one hour per week throughout the semester to conduct the MTL sessions.

What to teach and why? (Content)

Initial Reflections: "I should elaborate on the relevancy of the content used in the current academic situation for the respondents."

Actions Taken: The researcher extracted the relevancy of the content to the current academic situation of the respondents from the teacher's guide available with the BLOSSOMS modules as shown in Table 4.2a and 4.2b.

How to teach? (Teaching method and approach)

Initial Reflections: Constructivist teaching is recommended based on literature review

Actions Taken: Constructivist teaching and collaborative learning is employed in this action research

How to group the respondents in the class? (Class management)

Initial Reflections: *"*Students will work collaboratively in groups of 4's with a designated role in that group."

Actions Taken: Students are grouped based on their academic backgrounds. They will collaboratively work in groups of 4's with an assigned role in a group.

How to collect and manage the data? (Data management)

Initial Reflections: *"*I should investigate the ways of collecting and managing data through multiple ways in order to address the validity of the research. At least two research assistants are needed to videotape and to observe the classroom activities."

Actions Taken: Two research assistants are appointed to help in data collection. Systematic Qualitative Data Management (SQDM) process is used to manage the bulk of data. The summary of the SQDM process is given below:

1. Selecting the appropriate software programs
2. Pre-processing data outside NVivo
3. Recording the steps for conceptualization of the research
4. Recording researcher's reflexivity using framing interviews
5. Maintaining the repository of literature using Mendeley
6. Selecting the appropriate data collection techniques

7. Transcribing the interviews in word processing software before import to NVivo
    - apply uniform transcription format
    - do not use headers or footers
8. Data anonymizing in word processing software before importing to NVivo
9. Selecting uniformly structured file names
10. Planning how best to organize data files (sources) in folders within Internals
11. Importing data in appropriate import formats
    - .doc or .rtf for text
    - .xml for spreadsheets
    - .mp4 for video
    - .jpg for images
12. Deciding whether to include images, or video as internals or externals
13. Organizing all External files in a single folder on PC or server
14. Organizing essential documentation files of the research in 'documentation' folder in Memos
15. Using memos to document research
16. Using classifications to document and annotate sources, persons interviewed, study cases, etc.
17. Enabling time-stamped project event log
18. Recording consent for data sharing amongst person attributes
19. Storing NVivo project file and external files securely
20. Arranging regular back-up of NVivo file and externals
21. Arranging read/write permissions for the project file
22. Considering that all users have access to all the content of an NVivo project file

How to record, group and report the findings? (Data display)

Initial Reflections: *"*Look closely for the emergent themes during the intervention and try to ideate the multiple ways to communicate the findings."

Actions Taken: Emergent profiles and emergent activation of mathematical thinking processes are guiding the discussion and conclusions presented in the following chapters.

What is the researcher's lens? (Biases and view point)

Initial Reflections: "I am well aware of my logical shifting in epistemological viewpoint from positivist to interpretivist."

Actions Taken: The researcher was well aware of her own shift in epistemological viewpoint from positivist to interpretivist. However, there were some instances where the researcher has realized that her positivist attitude dominated the situation.

How to assess the embodied and symbolic activation of mathematical thinking processes? (Drawing conclusions)

Initial Reflections: "A rubric should be adapted or modified to assess the written activity responses."

Actions Taken: A rubric is adapted and modified from the book "Thinking Mathematically" to assess the written activity responses (Mason et al., 2010).

4.8 Main Action Research Cycle I

This cycle is used to run the full implementation of blended learning guided by the previous action research cycles and initial diagnostic analysis. Before illustrating the therapeutic and diagnostic components of the main Cycle 1, The researcher described the respondent-related themes in the quest of knowing the students, their current knowledge state through ALEKS and their prior mathematical thinking and problem solving skills through a diagnostic process.

4.8.1 Knowing my Students

We selected a batch (52 students; 47 males and 5 females) from first year Mechanical Engineering at the beginning of their Engineering Mathematics I course. These 52 students were grouped in fours (13 groups) based on their academic backgrounds. The initial demographic data were collected during the first two sessions of the course through Google Forms. All the participants joined the engineering program from three mainstreams, Type I students came in with high school certificate (15 out of 52), Type II students completed matriculation studies (35 out of 52) and

Type III (only 2 out of 52) students joined after achieving their diploma in Mechanical Engineering along with some work experience. Group identifications replace actual names for anonymity concerns with students' consent. The survey response analysis will help the researcher to know the respondents as well as to further categorize sub-themes into student's met-befores related to their mathematical thinking and learning presented in the next chapter.

Every parent node shows number of occurrences in the format x(y) where x is the total occurrence included the occurrences at child nodes and y means the occurrences of the parent node. If the value of y is zero, it means the parent node is created to group some of the child nodes. Hence, it is not directly emerged from the actual data rather created to group the nodes in a meaningful way.

The next sub-section will describe the factors affecting students' mathematical thinking, leaning and problem solving: perceptions, strengths, and weaknesses, Bad and good experiences, study and exam preferences, expectations, fears, needs, and difficulties.

4.8.1.1 Perception about using maths in future

Categorization of responses for this query in main and sub-themes along with their occurrences are listed in Table 4.11.

Table 4.11: Main themes related to perception about using maths in future

Main theme	Sub-themes	Occurrences
For Problem Solving	Creative Problem Solving (1), Engineering Problem Solving (2), For daily (real) life Problem Solving (5)	17(9)
For Future	For my Job (7), In Engineering Field (6), To become a great engineer (3)	18(2)
For Calculations	Calculations for invention (1), For Engineering Calculations (2)	9(6)
For designing purposes	To build a model of Invention(1)	3(2)
For Teaching Mathematics	-	2
No Response	-	5
Total Occurrences		**54**

Table 4.11 shows that there are four main nodes. Seventeen students responded with problem solving, 15 for future use, nine for calculations and three students think that they will use mathematics for designing purposes. There are two students who wanted to use mathematics for teaching in future. Five students did not respond to the question.

4.8.1.2 My strengths in math

Categorization of responses in main themes and sub-themes along with their occurrences are listed in Table 4.12. It clearly shows that the main theme is related to mathematics concepts because 36 students reported different concepts as their strength in mathematics. Five students think that computation is their strength, five students think that their strengths are related to their memory skills, five of them think their strength is problem solving whereas four students reported attitudinal strength. One response is irrelevant, one reports no strength, one says he is not sure whereas two students did not record their responses.

Table 4.12: Students' Strengths

Main themes	**Sub-themes**	**Occurrences**
Conceptual (Topic specific)	Algebra (4), Averages (2), Calculus (2), Differential Equation (1), Differentiation (9), Function (1), Graphs (1), Integration (6), Matrices & Vectors (1), Permutation & Combination (2), Pythagoras Theorem (1), Simultaneous Equation (1), Statistics (3), Trigonometry (2)	36(0)
Computational	Fast at calculations (3), Good at Calculations (2)	5(0)
Memory skills	Memorizing Equations (2), Memorizing Formulas (2), Memorizing Steps of Calculations (1)	5(0)
Solving Problems	Good in Problem Solving(1), Inter-relating Topics to Solve Problems (1), Solving Easy Questions (1), To solve any Problem (1), To Solve Mathematics Question (1)	5(0)
Attitudinal	Confident (1), Loving to do Maths (3), Fast Learning (2)	6(0)
Others	-	6
Total Occurrences		**63**

4.8.1.3 My weaknesses in math

Categorization of responses in themes and sub-themes along with their occurrences are listed in Table 4.13.

In the table below, conceptual weakness emerged as a main theme with 44 occurrences whereas 2 occurrences are found for computational weaknesses, three are related to memory, 4 are related to problem solving, 3 are related to lack of management, 5 responses are related to attitudes, and the rest of the nodes are kept open and grouped under "others" due to low occurrences.

Table 4.13: Students' Weaknesses

Main themes	Sub-themes	Occurrences
Conceptual (Topic specific)	Algebra(1), Calculus (2), Circle (3), Differentiation (4), Functions (1), Graph (2), Integration (6), Limit (1), Normal Distribution (1), Permutation & Combination (2), Probability (9), Statistics (1), Trigonometry (9), Vectors (1), Poor concepts (1)	44(0)
Computational	Calculations with decimal numbers (1), In long calculations (1), Proving Equations (2)	4(0)
Memory Related	In memorizing the equations (1), In memorizing the formula (2)	3(0)
Solving Problems	Difficulty in problem solving situations (1), Difficulty in understanding (1), In applying the formula (1), Using correct method to solve problems (1)	4(0)
Lack of Resource Management	Lack of time management in exam (1), Wasting time while solving a question (2)	3(0)
Attitudinal	Careless (4), Lazy to think (1)	3(0)
Others	-	5
Total Occurrences		**66**

4.8.1.4 One Example of Bad Experience

Categorization of responses in themes and sub-themes along with their occurrences are listed in Table 4.14.

The emergent theme as a response to one example of bad experience, is "failing in test" with 13 occurrences. Seven reported as "fail to meet the expectation." "Unable to solve math question" is reported by four students whereas three students had a bad experience of "getting bad results." The rest of the nodes cannot be grouped together under any theme so kept as open nodes and three students did not record their responses.

Table 4.14: Students' Bad Experiences

Main themes	Sub-themes	Occurrences
Failing in Test	-	13(13)
Fail to meet self-expectation	Did not get A grade (3), Got A- grade (3)	7(1)
Unable to solve math question	Could not answer the question (1), Could not answer trigonometry question (1), Could not solve probability question in exam (1)	4(1)
Got bad result	Got C grade (1), Got very low marks (1)	3(1)
Got Stuck in a question for long hours	-	3
Made mistake due to carelessness	-	2
Used wrong method	-	2
Others	-	15
Not available	-	3
Total Occurrences		**52**

4.8.1.5 One Example of Great Experience

Categorization of responses in themes and sub-themes along with their occurrences are listed in Table 4.15.

Table 4.15: Students' Great Experience

Main themes	Sub-themes	Occurrences
Got A unexpectedly	Got A (12), Good marks in final exam (2), Got full marks (2), Got good result (1), Got highest marks (1), Highest marks in final exam (1)	20(1)
Solving Problem	Solved a mathematical problem fast (5), Solving Hard questions (5), Solving maths problems (3)	13(0)
Worked as a facilitator	Became mentor during matriculation (1), Selected as a math tutor (2), Teaching friends (2), Made my friend understand a topic (2)	9(2)
Answered the question unexpectedly	Answered the exam question (1)	2(1)
Others	-	7
Total Occurrences		**51**

Table 4.15 shows the emergent themes of students' great experiences. The main emergent theme is related to "getting A unexpectedly" in the exam with 20 occurrences. The next theme is solving problem with 13 occurrences and "worked as facilitator" with 9. Two students reported, "Answered the question unexpectedly" whereas the rest of the nodes are kept open due to low occurrences of the said responses.

4.8.1.6 How do I study Mathematics?

Categorization of responses in themes and sub-themes along with their occurrences are listed in Table 4.16.

Table 4.16: How Students study for Maths

How I study Mathematics	Sub-themes	Occurrences
Practice	Extensive Practice (17), Practice last year paper (2), Practice with Music (1), Regular Homework (2), Regular Practice (4)	39(13)
Conceptual Understanding	-	8
Follow lecture	Concentrate in Class (5), Follow lecture notes (1)	7(1)
Group Study	Comparing with friends (1), Group discussion (1), Study with friends (2),	4(0)
Need reading	-	3
Teaching others	-	2
Others	-	7
Total Occurrences		**70**

It is evident from Table 4.16 that 39 students are habitual of "practice" based study, eight students prefer "conceptual understanding," 7 "follow lectures", 4 study in groups and the rest of the responses are kept as open nodes due to their low occurrences.

4.8.1.7 How I prepare for exams in maths

Categorization of responses in themes and sub-themes along with their occurrences are listed below:

Table 4.17: How Students prepare for exam

Main themes	Sub-themes	Occurrences
Practice	Extensive Practice (27), Practice Past Exams (8), Doing hard questions (1), Regular Practice (1)	37(0)
Do Revision	Last minute Revision (1)	20(19)
Teaching others	-	3
Memorizing important formulas	-	2
Consulting teacher	-	2
Study hard	-	2
Others	-	5
Total Occurrences		**71**

"Practice" again emerged with high occurrence and preferred by 37 students for the preparation of exam. Twenty preferred to do revision, three preferred to teach others and the rest of the students reported different items, and their responses were kept under open nodes due to their low occurrences.

4.8.1.8 My expectations from this course

Categorization of responses in themes and sub-themes along with their occurrences are listed in Table 4.18:

Table 4.18: Students' expectations

Main themes	Sub-themes	Occurrences
Getting good score	Getting A (4), Getting A+(3), Getting Full marks (1), Scoring (2)	11(1)
Hard, Complicated, Difficult	Adapting with maths as it becomes harder (1), Terrible and tough (1)	8(6)
Excelling in Mathematics	Be good at Maths (1), Doing well (1), Improvement in Mathematics (1)	5(2)
Enjoying and loving it	Enjoy (1), Fun (3)	5(1)
Interesting	-	3
Learning and Understanding all the topics	Understanding to solve (2), Understanding the lecture (1)	4(1)
Challenging but possible	Ability to solve challenging problems (1)	2(1)
Fast learning	Quick understanding (1)	2(1)
Easy to understand	-	2
Learning new things	-	2
Becoming an expert	-	2
Others	-	14
Total Occurrences		**60**

The emergent themes of the students' expectations are "getting good score", "Hard, Complicated, Difficult", "Excelling in Mathematics", "Enjoying and loving it", "Interesting", "Learning and Understanding all the topics", "Challenging but possible", "Fast learning", "Learning new things", "Becoming an expert" with occurrences of 11, 8, 5, 5, 3, 4, 2, 2, 2, 2 respectively. The rest of the 14 nodes are kept open.

4.8.1.9 My fears about this course

Categorization of responses in themes and sub-themes along with their occurrences are listed in Table 4.19. Students' fear emerged as mainly "exam related with 20 occurrences, "workload" with 5 occurrences, "Negative self-concepts" with 4 occurrences, "Unable to understand" with 6 and "Unable to catch-up all the topics" with 4 occurrences. The rest of the responses are kept under open nodes because of their low occurrences.

Table 4.19: Students' fears

Main themes	Sub-themes	Occurrences
Exam related fears	Failure (7), Getting bad grades (4), Cannot answer the question (1), Cannot remember formula in exam (1), Confused in exam (1), Feeling scared (1), Forgetting basic concepts (1), forgetting learned concepts in exam (1), Forgetting the formula (1), Not getting A (1), Unable to use the correct method (1)	20(0)
Workload	Many Assignments (3), lots of derivations (1), Many Projects (1)	5(0)
Negative Self Concepts	Carelessness (1), Laziness (1), Lazy to do Practice (1), Slower than others (1)	4(0)
Unable to understand	-	6
Unable to catchup all the topic	-	4
Others	-	18
Total Occurrences		**57**

4.8.1.10 My need about this course

Categorization of responses in themes and sub-themes along with their occurrences are listed in Table 4.20. Fifteen students needed to do "extensive practice", 12 of them needed "Support from others", 11 of them needed "Resources" whereas three students just needed "Good results" and two needed to "cover all topics". The rest of the nodes were kept open with low occurrences.

Table 4.20: Students' needs

Main themes	Sub-themes	Occurrences
Extensive Practice	Exercises done during the class hours (1), Exercises with answers (1), Lots of examples (1), Lots of exercise (1), More exercises to help (1)	15(10)
Support from others	Support from everyone (1), Friends & Classmates Support (4), Discussions with classmates (1), Friend to teach me (1), Getting help from classmates (1), Group Study (1), Help (1), Helpful and Supportive teacher (2)	12(0)

Table 4.20: Continued

Main themes	Sub-themes	Occurrences
Resources	Books and note (1), Detailed notes (1), Free Textbooks (2), Good reference book (1), Internet (1), Fast Internet Connection (1), Notes and exercises (2), Smart notes (1), Study Notes (1)	11(0)
Good results	Getting A (1), Getting better result (1)	3(1)
Covering all topics	Covering all in short time (1)	2(1)
Focus	-	2
Past year papers	-	2
Other		16
Total Occurrences		**63**

4.8.1.11 My difficulties about this course

Categorization of responses in themes and sub-themes along with their occurrences are listed in Table 4.21. Based on the response analysis, 12 students have negative self-concept like being careless and lazy. Six students face difficulty in memorization of formulas, 4 students face difficulty in resource management like wasting time. “Forgetting what I have learned,” “lack of focus in class”, “Language Barrier” and “Complex Questions” are some other themes with 3, 3, 3, 2 occurrences. Rest of the themes are kept under open nodes.

Table 4.21: Students’ Difficulties

My Difficulties	Sub-themes	Occurrences
Negative Self-Concept	Being Careless (2), Being Lazy (4), Lazy to attend math tutorials (1), Lazy to do exercise (2), Lazy to Practice (1), Lazy to Study (1), Lazy to try out (1)	12(0)
Memorizing	Memorizing formulas (5)	6(1)
Resource Management	Not having a computer to access eLearning (1), Spending More time to solve problems (1), Time Management (1), Wasting time (1)	4(0)
Forgetting what i have learned	Forgetting if no revision (1)	3(2)
Lack of focus in class	Lack of focus in afternoon class (1), Loosing focus in the class (1)	3(1)
Language Barrier	Learning in English (1), Understanding English (1)	3(1)
Complex Questions	Answering hard questions (1)	2(1)
Sleepy	-	3
Others	-	23
Total Occurrences		**59**

4.8.1.12 What is your current GPA?

This question's response shows that 19 students join engineering mathematics with 4 GPA, 15 students have GPA between 3.90 to 3.98, 11 students' GPA lie in between 3.84 and 3.89. Only two students with diploma background join with 3.42 and 3.46 GPA. Six students did not share information related to their GPAs.

Table 4.22: Current GPA of Students

What is your current GPA?	Frequencies	What is your current GPA?	Frequencies
4	19	3.88	1
3.98	1	3.87	1
3.96	7	3.86	1
3.94	1	3.84	1
3.92	4	3.46	1
3.91	2	3.42	1
3.89	6	N/A	6

4.8.1.13 What is your previous experience with mathematical thinking?

Given four options, 51 out of 52 students responded as:

Table 4.23: Student's previous experience with MT

Previous experience with mathematical thinking	Frequencies
I have no experience with mathematical thinking	32
I have only heard or read about mathematical thinking	10
I have participated in many mathematical thinking activities	8
Never heard about mathematical thinking and never use it	1

4.8.1.14 Did the mathematics course(s) that you have taken, require activities related to applying course material in the real world?

Thirty-six students responded with “YES”, 15 students with “NO” and one student did not record his response.

4.8.1.15 I prefer to work in teams

The agreement to work in teams in higher than the its disagreement.

Table 4.24: Students’ preference to work in teams

Strongly Agree	8
Agree	28
Somewhat Agree	12
Somewhat Disagree	2
Disagree	2
Strongly Disagree	0

4.8.1.16 I prefer to work individually

The responses show the more agreement to work individually rather than its disagreement.

Table 4.25: Students’ preference to work individually

Strongly Agree	2
Agree	14
Somewhat Agree	22
Somewhat Disagree	5
Disagree	6
Strongly Disagree	3

4.8.1.17 Aspect of mathematical thinking course looking forward to or excited about

A majority of the students (24) did not respond to the query where as eight students looked forward to or excited about “problem solving”, seven about “thinking”, four about “real life mathematics” and two of them were excited about gaining “new knowledge”.

Table 4.26: Aspects of mathematical thinking course looking forward to or excited about

Main Themes	Sub-themes	Occurrences
Not available	-	24
Problem Solving	Correct MT skills to Solve Problems (1); Finding new ways to solve problems (1); Giving best solutions (1); How to solve Complex Problems (1); Interpreting Problems (1); Learning Method to Solve Problems (1); To solve Everyday Problem (1);	8(0)
Thinking	Creative Thinking (3); Meta-cognitive Thinking (1); Thinking Critically (1); Thinking Out of the Box (2);	7(0)
Real Life Mathematics	-	4
New Knowledge	-	2
Others	-	13
Total Occurrences		**58**

4.8.1.18 Reservations or concerns about attending mathematical thinking lab

Eleven students were no sure about their reservations or concerns whereas four showed their reservations and concerns in “learning something new throughout the course” and the same number reported “thinking”, “Problem Solving” and “having fun” with three occurrences each were some of the reservations and concerns. The remaining responses were kept under open nodes due to their low occurrences.

Table 4.27: Student reservations or concerns about attending Mathematical Thinking Lab

Main Themes	Sub-themes	Occurrences
Not sure	-	11
Learning something new throughout the course	-	4
Thinking	Critical Thinking (2), Thinking Skills (1), Thinking smart (1)	4(0)
Problem Solving	Being able to solve problems (2), Increasing my ability to solve problems (1)	3(0)
Having Fun	-	3
Improve socialization	-	2
Improved performance in future	-	2
Others	-	32
Total Occurrences		**61**

All the above open-ended questions and their respective open and axial coded responses will help the researcher to further group them into emergent themes based on the overall findings during the stage II analysis after action cycles of the main study.

4.8.2 Knowing the Current Knowledge State Through ALEKS

Only 29 students used ALEKS whereas 23 did not even try to utilize the resource provided. The initial assessment in ALEKS clearly showed that the knowledge states related to the concept of function of a majority of students are not sufficient to prepare them in the learning of transcendental functions that were already planned to be taught as the first topic in Engineering Mathematics I course. Only 2 Out of 29 students managed to score more than 50% during their first assessment whereas the rest of the students showed low performance. A snapshot of ALEKS report is displayed in Figure 4.2.

All	Student Id (Login|Name)	Performance Course Mastery Show: Percent / Topics	Knowledge per Slice							
			Algebra and Geometry Review (156 topics)	Functions and Graphs (72 topics)	Polynomial and Rational Functions (80 topics)	Exponential and Logarithmic Functions (33 topics)	Trigonometry (120 topics)	Systems of Linear Equations and Matrices (35 topics)	Sequences, Series, and Probability (23 topics)
1	Alpha1	35 %	67 %	35 %	28 %	27 %	13 %	43 %	17 %
2	Alpha2	*							
3	Alpha3	14 %	35 %	14 %	4 %	0 %	3 %	11 %	0 %
4	Alpha4	11 %	28 %	15 %	3 %	0 %	3 %	6 %	4 %
5	Beta1	*							
6	Beta2	32 %	62 %	46 %	18 %	27 %	11 %	23 %	13 %
7	Beta3	*							
8	Beta4	46 %	75 %	54 %	33 %	45 %	26 %	51 %	9 %

Figure 4.2: Snapshot of Students' performance shown through ALEKS

4.8.3 Knowing the Prior Mathematical Thinking and Problem Solving Skills

A task "Function in real life" is designed to test the students' embodied and symbolic world views of mathematics related to the concept of function. The description of the task is given below:

> Find an example of a function (in table, graph, or equation form) from a newspaper, magazine, or the internet. Cut it out or print it and answer the following questions on a separate piece of paper. Be sure to include your source or reference.
>
> A. How do you know that this table (or graph, or equation) represent a function?
>
> B. This table (or graph, or equation) represents ________as a function of ___________?
>
> C. What is the domain of the function? What is the range?

The class instructor marked the submitted work and handed over to the researcher to analyze the responses. After scanning the written responses, the researcher transferred the digital images to NVivo 10 to further analyze them. At first, the responses were analyzed one by one and then all the preconceptions, misconceptions, and idiosyncronises for all the 13 groups (52 students, 4 students per group) were compiled.

Data analysis started with deductive coding for each of the data collected during the research, meaning thematic coding was pre-identified (Fereday and Muir-cochrane, 2006). A description of codes is given in Table 5.19. Axiomatic formalism was not emphasized in the previous curriculum as *"it builds formal knowledge in axiomatic systems specified by set-theoretic definition, whose properties are deduced by mathematical proof"* mentioned by Tall so that is not included in our coding scheme. For each of the group work submitted, two coders analyzed students' written responses for the instances in accordance with Tall's three worlds of mathematics. They also worked together to code the rest of the instances in order to develop the inductive themes showing the supportive and conflicting met-befores. The findings further guided them to retrace student's historical mathematical thinking experiences. The coders used a consensus coding approach in which they used to read and code separately, first a small portion of data and then all sets of data, afterwards they worked and discussed in collaboration to reach the mutual consensus by going through several iterations (Patton, 2002). The researcher highlighted the preconceptions, misconceptions, idiosyncrasies of the students based on the autobiographies, electronic profiles, classroom observations, video-recorded lectures, informal interviews and current knowledge state reported by ALEKS. The interpretive analysis also helped the researcher get the insights of the problems related to their mathematical thinking skills during the time of transition to engineering mathematics.

Table 4.28: Pre-identified Deductive codes used in data analysis

Code	Description
Conceptual embodiment	It builds on human perceptions and actions developing mental images that are verbalized in increasingly sophisticated ways and become perfect mental entities in our imagination.
Operational symbolism	It grows out of physical actions into mathematical procedures. While some learners may remain at a procedural level, others may conceive the symbols flexibly as operations to perform and also to be operated on through calculation and manipulation

Some samples of the initial diagnostic analysis are illustrated in the next sub-sections.

4.8.3.1 A typical submitted group work and initial diagnostic analysis

Three students of this group joined university after matriculation and one after high school certificate. Their GPAs are between 3.84 and 3.39, and all are males. Their submitted work shown in Figure 4.3, clearly displays their misconceptions related to the concept of functions.

The selected image from a magazine by the group clearly showed a histogram of telecom laser diode revenues from year 1999 to 2004 and a high but declining growth rate equalling 22% per year over the next five years. The students are unable to identify the variables as well as the relationships between the variables as a function. The teacher gave a comment "refer to definition of function" on their submitted work. The researcher discovered that this was the first time they have been asked to choose a function from the real life so they got confused in the selection of the appropriate function and thus ended up giving unclear responses for the proceeding questions as well. In the response to the last part of the question, they were unable to write the intervals of domain and range. It is evident that the linkage between conceptual embodiment and operational symbolism is not well established and thus ends up with an unclear interpretation of the graph. Their met-befores related to function did not help them to select an appropriate function and hindered their ability to identify the relationship between the variables for the function being selected.

According to the division made by Tall further for practical, theoretical and formal mathematics as shown in Figure 4.4, helped to understand the students' lack of skills to transfer knowledge from practical to theoretical mathematics. The formal mathematics was out of scope in their previous mathematics curriculum so that high level was not explored. Their GPAs can partially reflect their theoretical knowledge that seems enough for getting good grades in school mathematics but during their encounter with engineering mathematics, they need to develop skills to make their transition smooth from embodied to symbolic world on a rebuilt foundation of practical and theoretical mathematics as shown in Figure 4.4.

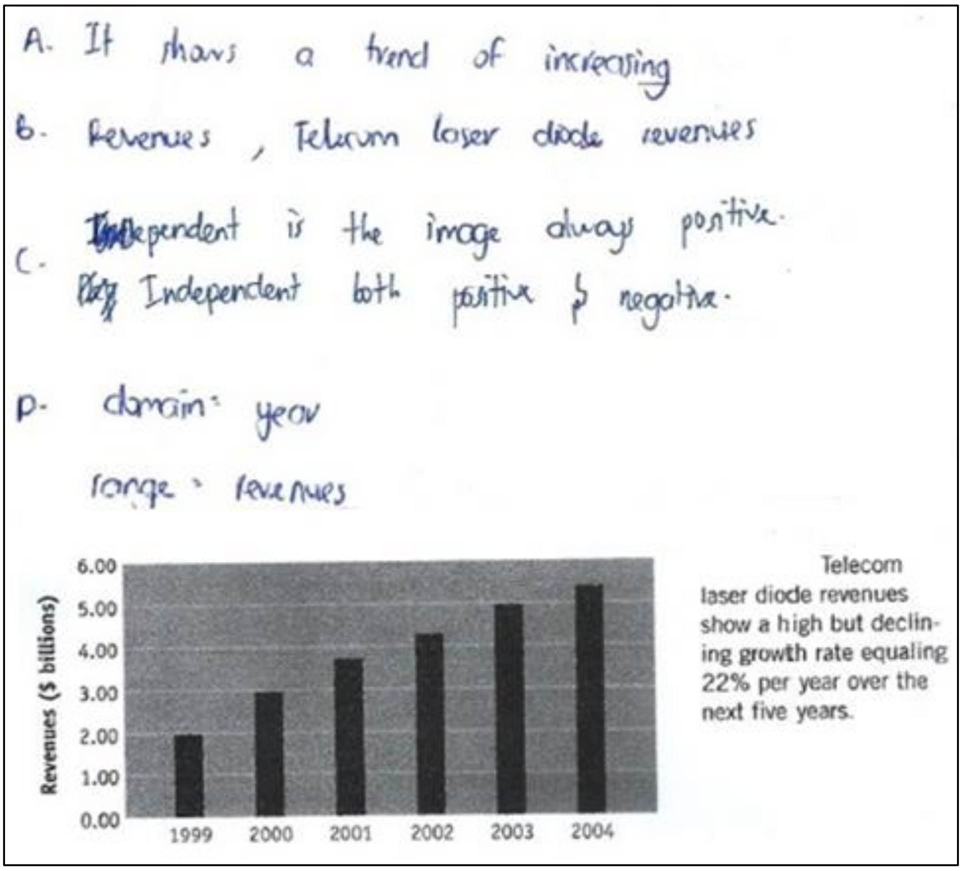


Figure 4.3: A typical submitted group work

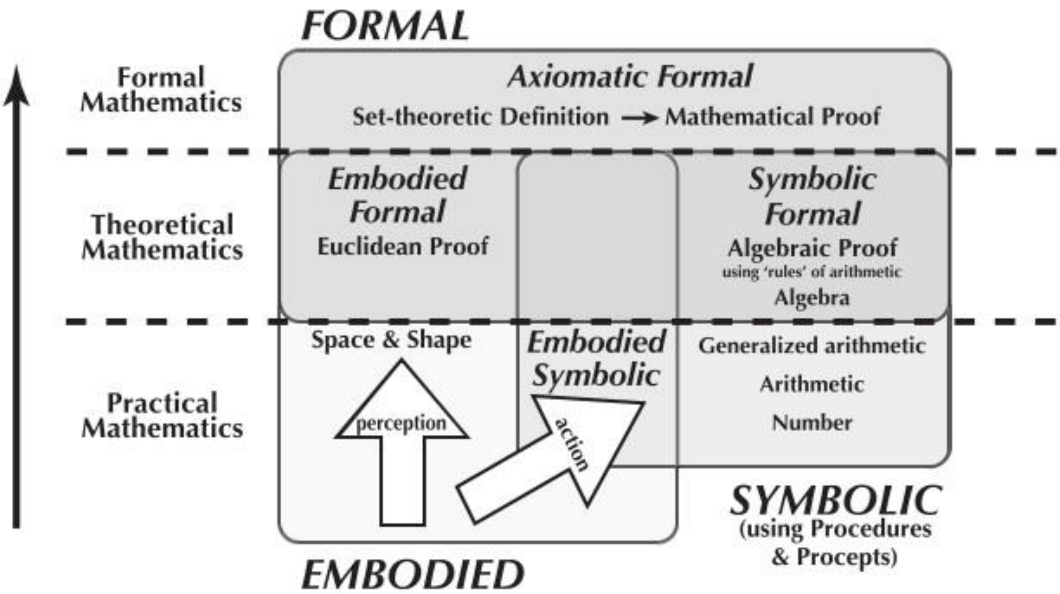


Figure 4.4: Practical, Theoretical and Formal Mathematics (Tall, 2013)

4.8.3.2 Another typical submitted group work and its initial diagnostic analysis

Another group consisted of three students who had already done Engineering Mathematics during their matriculation studies and one of them studied mathematics during high school foundation engineering. One got GPA of 3.98 and the rest did not report their GPAs.

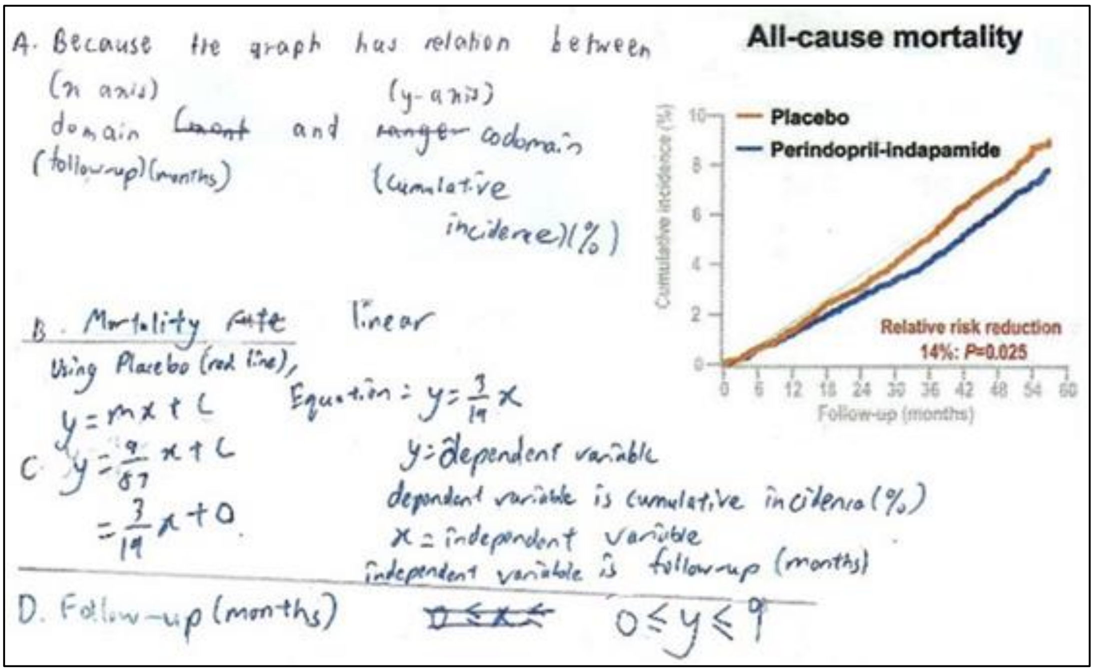


Figure 4.5: A typical submitted group work

They selected a figure labelled "Mortality" showing the comparison between two functions as displayed in Figure 4.5. Both the curves showed functions having an independent variable called follow up (months) and a dependent variable called cumulative incidence (percentage). They were able to identify the relationship between the two variables but could not identify that there were two functions, one for "placebo" and the other was for a particular medicine. They mentioned that the curve shown in the image was linear which was wrong and based on the misconception related to the concept of the slope of a curve. They did not answer part d of the task asking for the domain and range of the under-investigated function. The descriptive nature of the submitted work helped the researcher to understand the students' mathematical thinking skills.

The researcher analyzed the rest of the group and the analysis of the written work showed that the conceptual understanding of student groups can be categorized into different levels as shown in Table 4.29.

Table 4.29: Levels of Mathematical Thinking related to the concept "Function"

Levels of Conceptual Understanding	Description of the levels	Number of groups attaining levels of Conceptual Understanding
Level 0	Group is unable to select an appropriate table (or graph, or equation) from the embodied world to represent a function	6
Level 1	Group can select an appropriate table (or graph, or equation) from the embodied world to represent a function	-
Level 2	and can read and interpret variables from table (or graph, or equation)	-
Level 3	and can describe function as a relationship between independent and dependent variables	-
Level 4	and can represent the function symbolically	5
Level 5	and can identify the domain and range of the function	-
Level 6	and can write the axiomatic formal form of the function	6

The key findings of this diagnostic analysis are provided in next paragraphs.

Preconceptions, misconceptions, and idiosyncrasies

Most of the students have difficulty in reading the graph of a function and they usually interpreted data incorrectly in graphs showing functions (Tall, 2002). Their preconceptions are not consistent and most of them were unable to translate practical mathematics to theoretical mathematics, thus were not ready to learn advanced mathematics. The classroom observations showed that the students were not asking questions and that is also validated through informal interviews with two students:

Student A: "I don't like to ask questions in front of the whole class, but I can see the teacher later in her office to get my concepts clear."

Student B: "We were not encouraged to ask so many questions in the class so we usually keep quiet even if we do not understand the concept."

Their misconceptions related to the concept of function clearly showed that they have never been asked to do that kind of mathematical tasks before. It also reflected that grades alone cannot be the criteria for evaluating the students' conceptual building and upon further data exploration; the researcher observed that grades could partially reflect the assessment for routine problems and exercises. The schemas for doing mathematical computation without involving sense-making are well-practiced and well-developed; thus hindering the ability of the students to think mathematically in a new situation.

Met-befores for mathematical thinking related to functions

From the electronic profiles submitted by 51 out of 52 students, the researcher came to know that 32 students are not aware of mathematical thinking, 10 students have only heard or read about mathematical thinking whereas only 8 students have participated in mathematical thinking based activities (Section 5.3.13). A snapshot of their responses collected through google forms is shown in Figure 4.6.

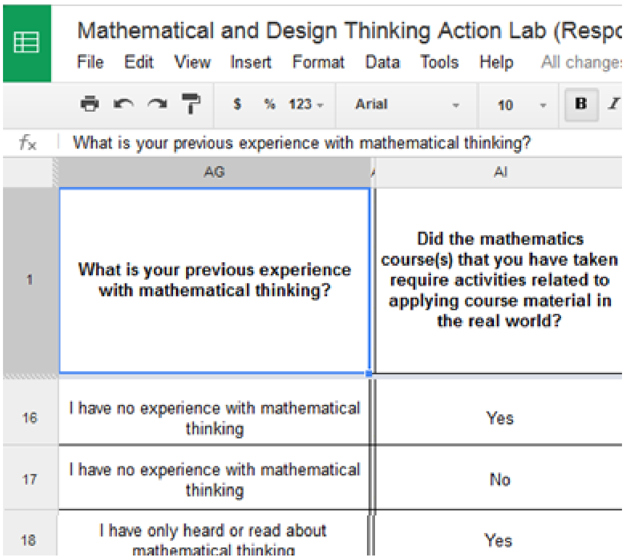


Figure 4.6: Snapshot of Students Responses

Building upon the data collected relating to the students' prior experience with mathematics, it was inferred that very few students were able to think mathematically, even doing a simple task related to function. Only two groups out of 13 managed to attain level 6 according to Table 5.20. They were able to select the appropriate functions as well as describe the relationship between the independent and dependent variables. They were also able to identify the domain and range of the function under investigation through symbolic manipulation. Five groups managed to select the appropriate functions but were unable to identify the domain and range of the functions. So they were ranked at Level 4. The rest of the six groups were at level zero and have shown very low performance demanding to intervene with their conceptual accommodation.

Performance, Awareness, Mathematical Sense and Attitudes

The performance was represented by the marks and comments given by the class teacher after assessing their group works showed the high probability of experiencing conflicting met-befores in other concepts too where as supportive met-befores are expected to be fewer than past occurrences. Students' mathematical sense-making and attitudes towards problem solving appeared to be inactive.

4.8.4 Therapeutic Intervention

After reviewing the action plan and worksheets as guided by the previous cycles, the researcher conducted this cycle with a batch of 52 students of first year Mechanical Engineering as mentioned in the previous section.

Below is the tentative outline of the ***course stru***cture:

Online and Outside of Class Activities:

- Practice on ALEKS (at least 4 hours/week)

In Class Activities (60 minutes, once a week)

- Mental ramp-up period (10 minutes): Facilitator asks questions about students' activity to gauge understanding.
- BLOSSOMS Modules (30-50 minutes) with 5-7 minutes segments along with Mathematical thinking oriented problem-solving activities.
- Group quiz (10-20 minutes, 4 quizzes per semester): Teams of four students work on group quizzes (10% of grade).
- Solution of group quiz (5 minutes): Instructor reveals strategies to solve the problem and the solution is distributed.
- Individual quiz (10-20 minutes, 2 per semester): Individual quiz based on ALEKS is given to each student to gauge their understanding of the subject material (10% of grade).
- Solution to the individual quiz (5 minutes): best strategy to solve the problem is presented and the solution is distributed.
- Preview of the next class session (5 minutes)

In the first MTL session, the students were briefed about the research and consent forms were distributed. Students read and signed the consent forms. The researcher directed them to fill in demographic information through online Google forms. After self-introduction, the researcher conducted an introductory session about mathematical thinking and Mason's problem solving strategy. A guidebook describing the mathematical processes, phases of problem solving strategy and summary of mathematical thinking was developed and distributed among students.

We allocated a spacious computer lab for the MTL sessions in the Department of Mechanical Engineering. The MTL environment was kept friendly with a mild sense of formal outlook. The students were permitted to change their sitting positions and to make small clusters during the problem solving activities.

Classroom setting during the MTL sessions is described as follows:

- Computer Lab with about 50 computers
- Flexible seating arrangement

- White board available
- Multimedia Available
- Spacious room
- Moderately low Temperature
- Pleasant Atmosphere
- Days and Timings: (Tuesdays 2:00 PM to 3:00 PM)
- Semi-Formal environment

Date, venue, time, respondents, and other related information for main cycle 1 and 2 were recorded in the log file as shown in Appendix J. The first page of the revised worksheet “The Power of Exponentials, Big and Small” is given in Appendix A.

The Second MTL session started by providing the students with mathematical thinking guide (Appendix R) along with the rubric (Figure 6.5) to guide them on how to record their written responses on the worksheets. Then the researcher played the video module “The Power of Exponentials, Big and Small.” The following paragraphs described the different activity segments used in MTL session 2 and 3 to cover the respective module. The description of all the activities is extracted and modified from the teacher’s guide provided with the respective BLOSSOMS Module (MIT-LINC, 2013) and presented by the researcher here:

> “In the ***first activity*** segment, the lesson deals with an Indian queen, who is paying out rice, one for the first square, double for each additional square? The key understanding to being able to manipulate this problem is to know the number of grains of rice that have been played down by the nth square on the board. Students are having a tough time understanding exactly how the problem is set up, Researcher starts explaining with a two by two, a three by three matrix instead of an eight by eight, which is a bit large, and let them actually work out for each square the number of pieces of rice that have been laid out on the board. Specializing is facilitated by letting them list those pay-outs for n equals 1, 2, 3, 4, 5 next to the numbers 2 to the 1, 2 to the 3, 2 to the 4, 2 to the 5, help the students understand that the general formula

by the nth square is 2 to the n minus 1, which is again the key comprehension to be able to proceed and answer a question like could the queen actually pay out 64 squares of the chessboard. The ***second activity*** segment deals with the problem of tearing and stacking paper on top of each other. The researcher specifically ask a number of questions related to just how many times you need to do this operation of tearing paper in half and stacking it to get a stack of paper to a certain height. Therefore, the key activation of mathematical thinking process from specializing to generalizing enables students to answer any question of this sort and facilitates them to generalize how many papers we have in our stack after the nth tear. Some of the students quickly get this depending on their level of mathematical sophistication, but some of them do not, the researcher folds a sheet of paper, keeps on folding in halves, and scaffolds students to get the trend quickly. The fact that after n tears, there are two to the n pieces of paper in the stack or two to the n no of folds in the stack. Once they understand that, the questions require a bit of insight for the kind of key base question, which is just how many pieces of paper do you need in a stack to get to a certain height. The researcher also gives them a piece of data, which is that a piece of paper is 0.1 millimeters thick, which is 10 to the negative fourth meters. However, the researcher scaffolds them to figure out themselves that they need to know the typical height of a human being and the distance to the moon. Some students have a way to easily guess; they figure out roughly how tall human beings are, about 1.5 to 1.75 meters tall, but some of them have trouble figuring out how far it is to the moon, the researcher gives them that piece of information, it's right around 384,000 Kilometers away. So once students have figured out exactly how many sheets of paper they need to reach a certain height, it is expected from them to record their written responses on the worksheets. However, they still need to actually convert that to a number of tears or folds to answer the problem. So of course, this is really an logarithm, the log base 2 of the number of sheets of paper that they need, and that would give them the answer which is the number of tears. The researcher also provides them with powers of 2 table to make it easier for them to just look up the table and figure out the number of tears/folds. In this case, they look for the number “n”, at which finally they exceed the number of sheets of paper that they need to reach a certain height. Then, the researcher presents the material at the beginning of the next section.

In the ***third and fourth activities***, the module introduces the concept of which function grows faster. Is it an exponential or is it a polynomial function such as a linear or a quadratic function. In addition, when asked on a range, like on the time range of a 30-day month, the module addresses it by plotting the functions. The module particularly uses functions that are somewhat easy for students to plot and the researcher assume that students should be able to plot the graph of two to the n versus 10,000n without too much trouble. Of course, if they have graphing calculators available [not available during the session], another option would be to plot on the domain of these two functions and to use the graphing calculator's ability to intersect the functions to find an exact point of intersection, which of course would correspond to the exact day at which there's a pay-out kind of breakeven point and when they start making more tootsie rolls, Swati starts making more tootsie rolls than John. So the ***fifth activity*** segment is related to the concept of an arbitrary polynomial function and an arbitrary exponential function with base greater than one, which one ends up growing faster as “n” approaches infinity. So there are a couple of concepts here that are very intimately related to calculus. And some of the students listening to the lecture have not already taken a calculus course. There is a kind of natural link to limits, for instance, and to the ratio test. Researcher thinks that if teachers of calculus classes, who want to look at alternate derivations of this fact, for instance, using repeated application of L'Hopital's rule that could be a really interesting and useful exercise for students.”

Some of the sample responses related to the above activities are presented in Appendix G. The MTL sessions 2 and 3 are kept deliberate and guided by prompts and questions (Table 3.11) suggested by Mason’s strategy. The researcher tried to play the role of a practitioner with her carefully reviewed teaching practice. Prompts and questions guided the mathematical thinking struggles of the students. The session is made interesting and dynamic by making some instant decisions and practical judgements during the session.

> "The fifth activity is the one that's most mathematically involved. Due to the time constraint, and sensing the low engagement of students, I will just introduce it without asking them to do the related activity."
>
> (Excerpt of Researcher's Field Notes taken on September 24, 2013)

The researcher documented the intended and unintended effects during the previous stage in the form of field notes during the session and self-reflections that are recorded right after the session. A research assistant carried out classroom observations and the session is videotaped to observe the students during problem solving activities and to capture the action processes. Written activity responses are collected in the form of worksheets for further activity response analysis.

The observations are documented (MTL session 2-main cycle 1) and a sample description is given in Table 4.30.

Some of the key classroom video observations are listed below:

> "Students are fully engaged during the video displays."
> "Students are engaged in problem solving activities."
> "Some students are not paying attention to the facilitator and are busy with their computer terminals."
> "Students are enjoying the new way of learning."
> "Most students show full cooperation during the whole session."
>
> (Video Transcript, September 24,2013)

Students in group Alpha were collaborating in the form of exchanging ideas and discussing the possible solutions with their respective group members. Their conversation during Activity 1 is described as:

> Alpha 1: What do we want from the question? I think we want the amount of rice awarded.

Alpha 2: I think we have to calculate the amount of grains that the queen needs to pay the mathematician.

Alpha 3: and we also have to tell that do the grains in the chess board exceed the bag of rice.

Alpha 4: We need summation of all the grains in each square and check whether that exceeds the grains in the bag of rice

The engagement is sensed more at the start of the session compared to the end of the session as described below:

"Students look tired and dis-engaged during Activity 4"

(Video Transcript, September 24, 2013)

Table 4.30: Documented Observations (MTL session 2-main cycle 1)

Date: 24 September 2013 (Tuesday) Time: 2.00 – 3.00 pm Venue: C24 408 (Computer Lab)	
Time (PM)	Activity description
2:00	Students occupy their computers and sit in their respective groups. Instructor informs them to open the Facebook group (MTL) to crosscheck the material uploaded by her.
2:05	Each student is given a worksheet.
2:10	Before showing the BLOSSOMS video, the MTL instructor explains what is there in the worksheets. She explains different phases of mathematical problem solving by making the students reflect what they have been doing in their past practices. She is walking forward and backward in the middle (comparatively the widest space in the middle of the lab). She asks the students, "Who has the habit of checking the solution after solving a problem?" Some students (about ten) raise their hands. She explains why checking the solution is important and they also need to develop a habit to reflecting back on their resolution and as the last and most important step , they need to create new examples replicating the concept learn after every problem solving activity. She encourages the students to ask questions whenever they feel stuck. The students start communicating with each other

	in their assigned groups. The instructor allocates 5 minutes to review the content on the MTL guide. The students respond back with a positive attitude. Two-way communication is in progress (Instructor-students). Students showing positive attitude [as they understood what the instructor said]
2:15	The instructor walks back and forth in the middle while explaining the problem solving strategy to the students. She asked, "Who has the habit to check the solution in exam", some of the students raise their hands. She explains why it is important to check your resolution, to reflect on key ideas and to extend the topic.
2:20	The instructor plays a video module titled "The Power of Exponentials, Big & Small". The students need to do the first problem solving activity (Group discussion is allowed, allotted time: 3 minutes)
2:25	The 1st Activity is introduced through video as "1 grain of rice for the 1st square of the chessboard, two for the second, four for the third, and so on. Every square must have double the number of grains as the previous one. Do you think the award was very small? Was the treasurer able to pay off the mathematician with a single bag of rice? Was the queen able to pay off the mathematician at all?" Some students did not understand the problems. Therefore, the instructor explains again, what the problems is and gives some prompts (by drawing a chessboard) on the white board.
2:30	The instructor moves from one group to another to see their progress.
2:40	The instructor plays the next video segment to show the resolution of the first activity.
2:45	The 2nd activity is introduced through video as "Given a paper. Fold it. What do you think? How many folds would we need to reach the height of a person? How many folds would John need to reach the moon?" Assume the thickness of paper as 0.01 mm. Students start folding the paper and recording their resolutions on the worksheets
2:50	The student are attentive while video segment for the solution of 2nd activity is shown. Students look excited while doing the activities.
2:55	The instructor ends the session and informs the students that the remaining activities from the same module will be resumed next week.

4.8.5 Diagnosis

The researcher's reflections were retrospective (looking back what she did), backed by the classroom observations and field-notes recorded during the session and analyzed during the Stage I analysis. Whereas activity response analysis are intended to carry out afterwards during the Stage II analysis. Some of excerpts from the researcher's reflective journal related to the attitudes of the students and her own practice to help her improve it are recorded below:

> "The students entered the lab with a smile on their faces. Some of them joined late. I previously thought that managing a big group of 52 students would be hard but the open spaces in the middle of the computer lab and at the left and right partitions made it easy for them to move around whenever required. The students seemed interested in learning new ways of problem solving after the introduction of what to do in MTL. They talked to each other and moved their chairs to be in their respective groups. The purpose for this informal setting was to encourage them to communicate with their neighboring friends and with me on a frequent basis. The first two activities of today's MTL went fine with lots of discussion around but students were looking tired after that. I was also expecting the same because these activities were time consuming and required more time than procedural mathematics. Today I felt more confident than the preliminary and pilot cycles. I also felt gravitated towards learning about my students' behaviour. One thing, which I really admire in the Malaysian culture, is that the students respect their teachers and that helped me to work with them on a partnership basis. One pattern that I observed is that they enjoy more if I give them manipulative (as today I facilitated them with a paper and asked them to fold it several times and then unfold it to see the pattern. I noticed that some students, who were lost in "what to do," started sharing ideas after carefully provoked with questions and prompted. One thing, which I observed in my own practice, is that I used to do lots of "Just-In-Time" teaching and getting more comfortable with that in this third iteration of MTL. The teams having diploma students had

many ideas to share within their group in comparison with the other groups. Prompts and questions really worked fine whereas one hour was not enough for them to think consistently of the new activities."

(Excerpt from researcher's reflective journal on September 24, 2013)

Some of the important points are extracted from field notes and observations related to the cognitive aspect of the problem solving activities as listed next:

- The students have the tendency to attack the problems without even thinking deeply about what is being asked, what is required to know and what they can contribute to solve the problem.
- Most of the activity responses are providing evidence that their entry phases of problem solving started in conceptual embodiment (Some of the student responses are presented in Appendix G).
- Students are not hesitant to record even the wrong answers on the worksheets (Appendix G)
- The written responses on individual worksheets are quite similar within one group (Appendix G) [mutual discussions were allowed].

After the evaluation of main Cycle 1, the researcher decided the following ways of proceeding:

- One worksheet per group is enough to address the issue of replicability in the written responses.
- The prominent students' profiles should be taken into account during the next cycle too.
- Inquiry should be aided by discussion among participants as frequently as possible.
- The interpretations should have descriptive aspect covering the situational insights and making sense of emergent challenges while introducing a new context during the next sessions.
- The prompts, questions, and themes should be used whenever required.
- Manipulatives like paper helped students to make sense of patterns by folding and unfolding them (this was more engaging than cutting the paper and stacking)

4.9 Main Action Research Cycle 2

Action plan for the main Cycle 2 was revised based on the reflections recorded during the main Cycle 1. The online and face-2-face components are kept the same as used during the Cycle 1 The summary of Cycle 1 is outlined in Table 4.31.

Table 4.31: Main action research- cycle I summary

Implementation & Monitoring	▪ One session is not sufficient to complete one BLOSSOMS module. Thus, multiple sessions are used. ▪ High Engagement , cooperation and positive attitudes are evidenced during problem solving activities
Evaluation & Planning	▪ Action plan is reviewed based on the students' verbal and written responses and researcher's self-reflections ▪ The instructor should motivate them to be expressive in verbal and written responses. ▪ Video recorder and observer are advised to capture group discussions and teacher-student interactions

4.9.1 Therapeutic Intervention

After reviewing the action plan and worksheets (as guided by the previous cycle and conducted before the mid-semester break), the researcher conducted this cycle with the same respondents after the mid-semester break (starting from week 10 of the current semester).

In the seventh MTL session, students are welcomed back after the semester break with a quiz (Appendix C). The questions are adapted from the book "Thinking Mathematically." The written responses are analyzed.

The eighth MTL session is designed to improve the perception about mathematical thinking and its relationship with engineering. Students are requested to fill in some open-ended questions showing their perception. Then three videos are shown in the class. The first was an animated movie showing the role of mathematics in the way of scientific inventions, the second an animated video explaining what engineering is and in the third video the current project to build up an elevator from earth to moon was documented. The data during this session is collected through Google Forms and the results will be drawn during stage II analysis. .

The classroom setting during the MTL sessions in main Cycle 2 is the same as in main Cycle 1. The date, venue, time, respondents, and other related information for main Cycles 1 and 2 are recorded in the log file as shown in Appendix J. The first page of the worksheet is shown in Appendix B.

The ninth MTL session started by reminding the students to use mathematical thinking guide along with the rubric to guide them how to record their written responses on the worksheets. Then the researcher played the video module "Flaws of Average." The following paragraphs describe the different activity segments used in MTL sessions 9 and 10 to cover the respective module. The description of all the activities is extracted and modified from the teacher's guide provided with the respective BLOSSOMS module (MIT-LINC, 2013) and presented by the researcher as:

> "The two of the opening segments, Dan on the river, Rhonda in her dresses, both are designed to help illustrate the flaws of averages. To begin with, Rhonda mentions that they are not trying to say that averages are bad! Averages do have their place. She gives example, "the average height of the students in your classroom." That is just one example of how the averages could be very helpful. In this module, a few of the pitfalls that a student could run into if he is not careful with using averages.
>
> The researcher starts off with ***the first flaw of averages***, that the average value may not always be a good description of the actual situation. One of the concepts that the instructor uses is that the average value may not be an actual outcome of the situation. So what does it mean? "Let's take a coin. If I flip a head, I am going to give you a dollar. If I flip a tail, I am going to give you zero dollars. Therefore, if I were to flip this coin many times, all I would be giving you is 50 cents on average. But wait a minute, 50 cents is not one of the outcomes. The outcomes are either

a dollar or zero dollars." So that's conceptually what this module is trying to convey. Another concept the researcher discusses with her students is that depending on the situation, the average may be exactly the same, but the distribution may be different. She discusses the normal distribution, exponential distribution, whatever kinds of distributions the students want to look at. For example, think of the crossing of the river. We had the flat line, we had the sloped line, we had the sort of double table top line. They are essentially three different distributions that all have the same average value.

The second flaw of averages is that the function of the average is not always the same as the average of the function. In the next video segment of this module, an example of cookies is introduced. Plate A with two cookies and plate B with two cookies, where the average diameter for plate A of the two cookies was 7 cm and the average diameter of the two cookies on plate B was 8 cm. And as Rhonda helped the class beautifully illustrate, intuitively they would think that the plate B set of cookies would be bigger in terms of area since the diameter is bigger. But the students saw, that the cookies on plate A were actually bigger when we uncovered them. So that was just a nice way to illustrate this point that the function of the average is not always the same as the average of the function.

The last flaw of averages is that the average depends on your perspective. So from the main video the example was Rhonda's dance classes. One of the examples, though, about Lake Woebegone. Now Lake Woebegone is a fictional town from a radio program in the US, and one of the key phrases about Lake Woebegone is that "all of the children are above average." Now this may sound like an impossible statement, but again it all depends on your point of view. If the average that you're discussing is a national average, it could be possible that Lake Woebegone's children could all be above average."

MTL sessions 9 and 10 are kept deliberate and guided by prompts and questions suggested by the Mason's approach. The researcher tried to play the role of a practitioner with her carefully reviewed teaching practice. Prompts and questions guided the mathematical thinking struggles of the students. The session is made interesting and dynamic by making some instant decisions and practical judgements during the session.

> "I changed Rhonda's dance class example with my teaching class example to make it more relevant to the Malay values."
>
> (Excerpt of Researcher's Field Notes taken on December 3, 2013)

The researcher documented the session as she did in the previous cycle. A research assistant carried out classroom observations and the session is videotaped to observe the students during problem solving activities and to capture the action process. Written activity responses are collected in the form of worksheets for in-depth activity response analysis. The sample observations are documented (MTL session 2-main cycle 2) and described in Table 4.32.

Table 4.32: Documented Observations (MTL session 9-main cycle 2)

Date: 3 December 2013 (Tuesday) **Time**: 2.00 – 3.00 pm **Venue**: C24 408 (Computer Lab)	
Time (PM)	**Activity Description**
2.05	Only a few students are present in the lab today and they are using computers. The students are still coming into the lab. The instructor said, "Please, group coordinators! Come and take your worksheets for this session." Then she added "Please write down the name of your groups and the name of all the group members in front of their respective roles"
2.10	Some of the students are facing problem while logging in the computers. Three students just arrived. The students are filling the required information in the worksheets. She asked the students to be attentive. She then asked, "If we want to find the average of two numbers, how will we do that?" The students started responding simultaneously so the instructor asked them to raise their hands to answer the query. Then, one student tries to explain the way to find the averages of the numbers. He said, "add those numbers and divide the answer by two." "Ok, if you have more than two numbers, then?" she asked. "Why do we need to calculate the average of numbers or values? Can you give

	some practical examples from your daily life? The student started responding with multiple answers like, "average height of students, average temperature in a week, and average shoe size of adults in a family. Instructor asked for any other example. She added, "What about the GPA?" All the students responded with "Yes".
2.15	The instructor played the first video segment. Some of the students are not paying attention to the video. They are engaged with the computers in front of them. A late student just entered the Lab.
2.20	The instructor asked the students "Would Dan cross the river having an average depth equals to 1 meter." Some students said no and some said yes. Instructor added, "Whatever is your answer, try to justify with examples. Can you record your answer on your worksheets?" Two minutes are allocated for the discussion and response. Students start discussing the problem and started writing on the worksheets. One student is working alone. The instructor walked around to see the progress. The instructor added, "Discuss with your friends, try to convince them, try to conjecture, conjecturing should be implicit, understandable, justifiable." The students started solving the problem. After the allocated time the instructor, resumed the video for the resolution of the problem. This time all the students are fully attentive and engaged.
2.25	The instructor said, "Can you think of some other examples when Dan is able to keep his head above the water, Also think of some examples "when Dan is not able to keep his head above the water" and I will give you 2 minute to record two examples." The students are engaged in discussion with group members. The instructor asked, "Can somebody read for me the extended concept 1?" One student volunteers and reads it.
2.30	The instructor is walking all around the lab to see the students' work.
2.35	The next video segment is played. All students are fully attentive and engaged. After watching the segment, they started discussing and solving the next task.
2.40	The instructor asked a volunteer to solve the problem solving activity on the board. One student comes forward to write the solution on the white board.
2.50	The student explains the solution to the class. The instructor also discusses the solution with the class. The next video segment is played. One student is

	talking to his friend. All the students try to figure out the solution for the task given.
2.55	Some of students are talking to each other while the instructor explains the answer. The next activity is in progress. The students are recording their responses on the worksheets.

Some of the excerpts from video transcripts (December 3, 2013) are listed below:

> "Some students are not paying attention to the video segment 1."
> "Students are fully engaged during the rest of the video segments."
> "Group Gamma is engaged in problem solving activities."
> "The engagement in the classroom activities is decreased during the second half of the class and most of the students are not paying attention"
> "Except for four students, the rest are fully cooperating during the whole session."
> "Students in Alpha have enjoyed the module and Alpha 4 added, "This module made me think about averages in a different way."

4.9.2 Diagnosis

The researcher's reflections are retrospective, recalling action as it is captured in video observations recorded during the session whereas activity response analysis is carried out in the next chapter. Some of the researcher's reflections are related to the attitudes of the students and her own practice are recorded below:

> "The students were curious to know the solutions of the problems and showed their cooperation with me and other students. Most of the students were engaged, motivated and collaborated with group members during the video segments and problem solving activities. They looked tired after two activities. MTL activities are time consuming and most of them are new to this way to learning. I tried to use the prompts and questions during the activity sessions. I also took

> some immediate decisions to handle the emerging issues like one student was not contributing in the discussion so I announced in the lab that if someone is found not contributing in discussion then I will give no grade to that student."
>
> (Excerpt from Researcher Reflective Journal, December 3, 2013)

Some of the findings related to the cognitive aspect of the problem solving activities are:

- The tendency of attacking the problems without spending time on the Entry phase is still prevalent.
- Most of the responses are showing evidences of the students working in conceptual embodiment.
- The written activity responses show better mathematical thinking skills than the previous cycle in terms of their written expression and logical sequencing.
- Students are not hesitant to record even the wrong answers on the worksheets.
- Collecting one worksheet per group limits the researcher in gauging individual contribution to the solution of the problem.

After the evaluation of the main cycle 2, the researcher decided the following ways of proceeding:

- A rubric should be used to assess the written activity responses of the students during all the cycles.
- The researcher should highlight the emerging profiles as a result of observing the students throughout the semester in engineering mathematics class and MTL.
- Highlight what works and what does not in the main cycles.
- Highlight what is the same and what is different in terms of cognition and affect.

4.10 Guidelines for Stage II analysis

During the main action research cycles, eight student profiles have emerged. These profiles will be documented in the next chapter based on observations during the engineering mathematics class and MTL throughout the whole semester. The students' current knowledge state of all the prerequisite concepts should also be extracted from ALEKS and displayed in the next chapter. Formative assessment during the semester could help the researcher to position the emerging profiles. Participation of students during the MTL should also be graded and displayed. The disparities of learners' knowledge should be highlighted to make sense of their performances during the semester. Written activity responses should be assessed using a rubric for mathematical thinking. Contextual impact of blended learning should also be discussed along with social, cognitive, and attitudinal impact.

4.11 Summary

This chapter has outlined the preliminary and pilot cycles to address the question, "How Masons' problem solving strategy and BLOSSOMS are integrated for Blended Learning?" It is described that the context-rich problem solving activities are created by extracting context and content from BLOSSOMS modules and integrating it with Mason's problem solving Strategy. Then the blended learning environment is implemented by using BLOSSOMS as a pedagogical tool, Mason's problem solving strategy along with themes, prompts, and questions and context rich problem solving activities in the form of worksheets. Some of the queries emerged from the preliminary and pilot phases of the study and highlighted the practical issues for the implementation of blended learning. The emerging issues and queries of the preliminary and pilot cycles are summarized in Table 4.33. The first cycle validated the worksheets containing the problem solving activities by integrating Mason's problem solving strategy with BLOSSOMS modules. During the preliminary and pilot cycles, some tactics were not effective like running the session for two hours.

The main action research cycles will predominantly focus on the process of implementation of blended learning to foster mathematical thinking among first year engineering students.

Table 4.33: Emerging issues from the first three action research cycles

	Preliminary Cycle	Pilot Cycle 1 & Pilot Cycle 2
Emerging Issues	• Time and Space • Unknown current knowledge state of the students • Shifting from content-based to process-based learning	• Time and Space • unknown current knowledge state of the students • Disparities of learners knowledge • Shifting from content-based to process-based learning

This chapter has outlined the main cycles to address the question, "How blended learning environment is implemented for mathematical thinking?" Some of the themes emerged from the main phase of the study e.g. classroom engagement, positive attitudes, time and space, student's profiles etc., which will help the researcher to describe the pragmatic implications of blended learning. Table 4.34 shows the summary of the main cycles. Stage I analysis of the implementation process highlighted "what works and what does not" and "what is the same and what is different in terms of cognition and affect."

In depth, analysis (Stage II) is carried out after the completion of the main study. The next chapter will document the emerging themes and student personas as a result of Stage II analysis.

Table 4.34: Summary of the main action research cycles

Main Cycle 1 & Main Cycle 2	
Emerging Issues (Evidence provided in the discussion above)	Time and Space, Disparities of students' knowledge , Moving from content-based to process-based learning, Conflicting Met-befores, Lack of support system for at risk students
Anticipated Emerging Themes	Students' Met-befores, Student's Met-afters Implications of BL

CHAPTER 5

EMEGENT THEMES AND STUDENT PERSONAS

5.1 Introduction

The preceding chapter described the five cycles of action research, which commenced during the entire study. The preliminary and pilot cycles were concerned with the process of integrating Mason's problem solving strategy with MIT-BLOSSOMS to develop a blended learning environment conducive for mathematical thinking. Whilst the main two cycles were related to the practical process of implementing blended learning, focusing on factors related to the contextual understanding of the practice, the respondents, and their written activity responses showing mathematical thinking processes. This chapter will focus on the second stage of data analysis, which is drawn on all the data collected during the main study. In relation to the implications of blended learning environment for mathematical thinking, the findings are described in the form of emerging ***students' met-befores*** in mathematical thinking and learning, the ***Challenges whilst problem solving***, the impacts of blended learning in the form of ***student's met-afters in mathematical thinking, and the diligence during mathematical problem solving, student-teacher relationship,*** the emergent ***students' personas and the activation analysis of their mathematical thinking processes*** during problem solving activities. In this chapter, the emergent themes are described as the result of interpretive analysis using inductive as well as pre-identified deductive coding scheme An adapted rubric from the books "Thinking Mathematically and " How Humans Learn to Think Mathematically: Exploring the three worlds of Mathematics" (Mason et al., 2010; Tall, 2013) is used to analyze the activation patterns of mathematical thinking processes of students' personas during problem solving through blended learning.

5.2 Emergent Themes

Emergent themes from inductive coding and interpretive analysis are highlighted after examining 52 hours of transcribed classroom observations, 52 hours of taped classroom videos (for corroboration), 52 students' survey responses using Google Forms, 13 focus group transcripts, 16 informal conversations (participants-researcher), 70 pages of field notes, self-reflections (researcher) and sense-making conversations (researcher-expert). All the data were collected at the local campus of a research university (RU), with 52 first year Mechanical Engineering students during their first semester. The data were stored and analyzed in NVivo 10. Each of the 52 hours of taped classroom videos during the Engineering Mathematics I and Mathematical Thinking Lab (MTL) were recorded using a video camcorder, transcribed and then imported into NVivo as .mp4 and .docx files. All the transcripts and written notes, field notes, and online surveys were also imported into NVivo project. Throughout the analysis process, data were imported from Excel and Word to NVivo, to transfer spreadsheets and transcripts, and exported from NVivo to Excel, to produce charts and tables. The initial open and axial coding is carried out during Stage I analysis of the main study as reported in Section 4.8.1. Those main themes are further grouped in higher order themes during Stage II analysis in the form of emergent themes presented in this chapter. Guided by Stage I analysis (Sections 4.8.1, 4.8.2, 4.8.3), the researcher will discuss the higher order themes: students' met-befores and met-afters through blended learning, implications of blended learning and challenges whilst problem solving for all the 52 students. To further investigate, the challenges related to mathematical thinking and problem solving at individual level, the researcher developed student personas. The narrative style in describing student personas would help the researcher to communicate the research findings to the community of practice. The researcher also observed these students in different situations: in the classroom, in the MTL and will investigate their current knowledge state to address their specific challenges.

5.2.1 Students' Met-befores

The students' personal and social met-befores are concluded after analyzing informal student-researcher conversations, classroom observations, and students' online demographic surveys and the results of Section 5.1 as tabulated next:

Table 5.1: Student's Met-befores

Student's Met-Befores	Individual experiences of students in life that lead to the personal development of mathematical thinking	Occur rences	Example
Passive in Asking Questions	Students were not encouraged to ask questions in the class	17	"we have not been encouraged to ask lots of questions in the class"
Individual Task Assignments	Students used to work individually	12	"I worked on individual assignments"
Drill and Practice	There was a great emphasis on drill, practice, rote learning and memorization	28	"Practice makes Perfect"
Teacher-Centered Learning	There was a predominance of passive rather than active learning, with teaching as the delivery of facts rather than the promotion of learning and understanding	11	"I used to focus on the lectures and we only have lectures, sometimes I feel sleepy"
Didactic Teaching	It is a process whereby knowledge is considered to be 'imposed' on the learner. This places a great emphasis on the teacher and how he/she constructs the teaching process. The role of the teacher is that of the 'sage on the stage'.	16	"No, I never had a chance to do interactive activities during the math class, there were lectures based on drill and practice"
Traditional Learning Styles	There was a reliance on a very limited range of learning styles	6	"I used to practice a lot and this is what we all do"
Minimal Exposure to Mathematical Thinking	There was a limited exposure to mathematical thinking processes	42	"I have just heard of mathematical thinking but never used it"
Grade-Based System	Students joined university from grade-based and exam-focused educational systems	40	"I always got A and expects to get A in future as well"
Rare Feedback on Work	Students rarely received constructive feedback on their work	6	"Teacher never told me about what I did wrong on my solution"
Problem Solving Strategies	Limited exposure to various problem solving approaches	6	"I used to read the problem and start solving it"
Pressure of examination failure	The pressure of examination failure is dominant and severe	48	"I am afraid of failing in this course"
Focusing Short Term Goals	Short-term goals e.g. getting good grades are more in focus than long-term goals e.g. to be an effective thinker at workplace	43	"I want to get good grades and I think good grades are enough"
Language Barrier	Limited practice to communicate in English in schools. [The local culture is not English; it is Malay so students are not comfortable in expressing their feelings and ideas in English]	9	" I am afraid to talk to you in English"

The above table shows the students' met-befores in terms of their "habits of mind" based on their prior experiences like their passive attitudes towards asking questions in the class even if they do not understand the concepts, habit of doing individual assignment, and the habit of drill and practice associated with mathematics. The emergent themes also highlighted the didactic teaching and teacher-centered learning without focusing on different learning style with a few instances of exposing students to mathematical thinking and problem solving strategies. Grade-based system is prevalent reinforcing the fear of failures in exam and focusing on short-term goals like getting good grades in exam. Students rarely get constructive feedback on their wrong or incomplete solutions. The classroom conversations were mostly carried out in their national language (Malay) and they did not feel comfortable to communicate in English.

5.2.2 Implications of Blended Learning

The followings are the pragmatic implications of the blended learning that emerged during Stage II data analysis.

5.2.2.1 Students' Met-afters

Table 5.2 shows the students' met-after experiences of mathematical thinking during context-rich problem solving through blended learning.

The themes categorized under students' met-afters show that students started asking questions actively during the MTL. Since the activities were designed for group work, the students had to collaborate with their team members for problem solving. The researcher has found some evidence where students' sense making of what was happening during problem solving was obvious. The constructivist teaching and student-centered learning is also evidenced. Duet Teaching is supported through online BLOSSOMS modules. The students are given a chance to record their responses of mathematical thinking in written as well as in verbal form, thus covering multiple learning styles. They have been exposed to explicit mathematical thinking process by introducing Mason's problem solving strategy along with prompts, questions and themes wherever appropriate. They were assessed based on their participation in the MTL instead of just assessing them on their computational skills. Most of the students enjoy the activities. However, there are some students who still like the traditional way of teaching. During the focus groups, the students have been given the freedom to record their responses verbally in their own native language and high engagement is observed during those sessions. The overall goal of this intervention turn out to be long term in the sense that the researcher tried to make them perceive the importance of mathematical thinking for their future career.

Table 5.2: Students' Met-afters

Student's Met-afters	**Individual experiences students have during the MTL that lead to the personal development of mathematical thinking**	**Occurrences/ Evidences**	**Examples**
Active in asking Questions	The students started asking questions in the class	12	"I couldn't understand this; can you tell me how to extend the concept?"
Group Activities	The students work in groups of four to do problem solving activities	34	Video Evidences
Sense-Making	Students are thinking mathematically to make sense of the problem posed	6	Video Evidences
Student-Centered Learning	There was a predominance of active rather than passive learning, with teaching as promotion of learning and understanding rather than the delivery of facts	12	Video Evidences

Table 5.2: Continued

Student's Met-afters	**Individual experiences students have during the MTL that lead to the personal development of mathematical thinking**	**Occurrences/ Evidences**	**Examples**
Constructivist Teaching	It is based on the assumption that learning is a process whereby a student is actively engaged with constructing their knowledge. The role of the teacher is as a facilitator of that learning. This is contrasted with the didactic teaching process, where the emphasis is on the teacher's activity and not the learner's.	10	Video Evidences
Duet Teaching and multiple Learning Styles	One teacher was teaching online and the other was taking control of the class to scaffold multiple learning styles	13	Video Evidences
Enhanced Exposure to Mathematical Thinking	The students are exposed to multiple mathematical thinking processes	10	Video Evidences
Participation-Based assessment System	The students are assessed based on their participation in the MTL during activities	12	Assessments
Feedback on Work	The students receive constructive feedback on their work	4	Video Evidences
Problem Solving Strategies	The students are taught how to adopt a problem solving strategy explicitly	8	Classroom Observations
Enjoying the activities	The students are enjoying the activities	20	"I enjoy a lot today during the MTL"
Focusing long Term Goals	Long-term goals e.g. to be an effective thinker at workplace is focused in the class as well as during focus groups	13	Focus Groups
Language Freedom	The students are communicating in Malay	13	Focus Groups

5.2.3.2 Diligence during mathematical problem solving

The students' diligence during problem solving activities through blended learning was enhanced as reported in Table 5.3. Fifteen evidences were found where the students showed higher levels of attentiveness by listening to the videos, the facilitator, and their team members. The students were highly engaged in watching video segments and working within teams during the pilot and the main study. The students were encouraged to ask question so they started asking questions during the MTL, compared to the actual classroom response, they were still hesitating in asking questions in front of the class. Our initial assumption that all students can think mathematically is backed by the evidences during students' problem solving activities in the MTL.

Table 5.3: Diligence during mathematical problem solving

Diligence during mathematical problem solving	**Evidences of Persevering applications (assiduity)**	**Occurrences/ Evidences**
Student Attentiveness	Students showed increased attentiveness in MTL	15
Classroom Engagement	Students are fully engaged during the online (video displays) component of blended learning	14
Team work	Students showed a greater capacity to work in teams;	12
Asking questions	Students asked questions and worked together more cooperatively after a few MTL sessions;	10
Persistence	Students showed persistence in solving the problems after a few sessions of MTL	4
Ability to think Mathematically	Students showed ability to think mathematically	16

5.2.3.3 Student-Teacher Relationships

After Stage II data analysis, some of the themes emerged, that could be grouped under a higher order theme labelled as "student-teacher relationship." Constructivist teaching in a blended learning environment also affects the student-teacher relationship. In a semi-formal environment, students can friendly communicate with the class teacher. They are encouraged to debate and discuss during the MTL sessions and are given freedom to collaborate with their peers. The researcher is scaffolding the students' learning and thinking processes, empathizing with at-risk students, listening their concerns and opinions acknowledging their positive behaviors and supporting low achievers to improve.

Table 5.4: Student-Teacher Relationships

Student Teacher Relation ship	**Teacher as mentor, facilitator and motivator**	**Occurrences/ Evidences**
Friendly Relationships	friendly relationships between teacher and students;	14
Democratic Classrooms	the establishment of more democratic classrooms;	5
Empowering Students	teacher giving students more power by allowing them to collaborate with peers;	4
Scaffolding	teacher scaffolding students during learning to think mathematically;	8
Acknowledgements	teacher seeking opportunities to acknowledge and reinforce appropriate behaviors and contributions during the blended learning sessions;	6
Empathizing	teacher seeking ways to show empathy to "at risk students"	4
Support	teacher seeking ways to support students especially low achievers in traditional class	7
Listening	teachers listening to students and responding to their concerns and opinions;	8

5.2.3 Challenges Whilst Problem Solving

Challenges whilst problem solving are concluded after analyzing informal student-researcher conversations, classroom observations, students' profiles, ALEKS performance, the results of Section 4.8.1, 4.8.2 and students' written activity responses (Section 4.8.3) are listed in Table 5.5. The emerging challenges whilst problem solving are “lack of confidence”, “lack of intrinsic motivation”, “lack of perseverance”, “lack of retention”, “different levels of participation”, “aversion to word problems”, “eagerness for formula”, “different speeds in computations and thinking”, “disparity in prior knowledge”, “lack of resource management”.

Table 5.5: Challenges Whilst Problem Solving

Challenges Whilst Problem Solving	Examples/Observations/Filed Notes/Reflections	Occurrences
Lack of confidence	Students do not want to lose face in class	2
	Students are shy and afraid of making mistakes	6
	They do not ask questions in class.	16
	They are afraid of writing incomplete and wrong answers	4
	They are afraid of being lazy	25
	They are afraid of failure in exam	21
Lack of intrinsic motivation	Students are less motivated to learn something that does not reflect on their grades	23
Lack of perseverance	Tendency of answering a problem without thinking	52
	Fluctuating interest in learning	5
Lack of retention	Students cannot stay focused during long procedural/conceptual learning sessions	16
	Students cannot concentrate during long problem solving sessions	13
Different levels of participation	Some students are willing to learn new ways of learning whereas some want to learn the old and traditional way.	6
	Some students enjoy team work whereas others prefer to work independently	8
Aversion to Word Problems	Students do not like long descriptive problems	2
Eagerness for formula	Students want to know the formula to solve problems	8
	They want a quick recipe to solve a problems	2
Different Speeds in computations and thinking	Some students can compute fast where as some can think fast for eample, Ismail is a fast thinker whereas Zain is fast in computations	8
Disparity in prior knowledge	Students coming from 3 different academic main streams showing disparity in their prior knowledge through ALEKS	52
Lack of Resource Management	More than 50% of the students did not practice on ALEKS to improve their prior knowledge, thus did not properly utilize a given resource.	25

5.3 Student Personas

In this section, the personas are described followed by specific situations (Section 6.2.5) in the form of bar charts shown in Figures 5.1 to 5.4. The interpretive analysis of two extreme-case personas' written activity responses will be described in Sections 6.3.3 and 6.3.4. The criteria to select the personas is based on their affective (emotions, behaviors, attitudes), cognitive (thinking processes) and psychomotor skills (what do they do in Class and MTL).

The following personas emerged during Stage II analysis along with their subsequent profiling in the next sub-sections:

Table 5.6: Student Personas and their Pertinent Characteristics

Student Persona ID	Student Persona Pseudonym	Persona's Pertinent Characteristics
Persona 1	Zain	Zain is the non-persistent, high achiever, careless but confident
Persona 2	Chen	Chen believes in practice and readymade solutions for problems.
Persona 3	Abdullah	Abdullah is a persistent performer throughout the semester but didn't enjoy the non-traditional way of teaching during MTL
Persona 4	Ismail	Ismail is gravitated towards sense making in the world of conceptual embodiment during MTL but he failed in Engineering Mathematics with alarmingly low grades
Persona 5	Fatima	Fatima is serious, attentive and quiet in class and didn't enjoy the MTL sessions
Persona 6	Faaiz	Faaiz is a persistent performer and showed cooperation in traditional class as well as in MTL
Persona 7	Sunny	Sunny is quiet in class but showed good communication skills during his focus group
Persona 8	Fahmi	Fahmi likes to work independently and loves to teach others

5.3.1 Persona1: Zain

Zain joined the Mechanical Engineering program at a local research university with 3.89 GPA after finishing his high school certificate in Foundation of Engineering. He was an odd out of all his class fellows who deliberately mentioned, “I am confident” about his maths performance and “I always score.” Later he admitted that the lack of time management and being “careless” are some of his weaknesses and there is always a need for him to work hard. He only had one bad experience when he once got C grade in his mathematics exam. He used to revise all the concepts before the exam by doing lots of practice and said “practice makes perfect.” He started attending the Engineering Mathematics I course with an expectation of scoring high in the exam but on the other hand, he was “afraid of forgetting”, what he had learned, especially during the exam. He gave himself “grade A” during the self-assessment. He preferred to do examples and exercises during the preparation of exam and asked his friends rather than his teacher in case he has any question.

Zain in the class: He never sits at the front of the class. He used to disturb his friends and class fellows sitting near him with continuous humming during the routine lectures. During the formative assessment in Engineering Mathematics I, Zain gets seventh position in Test 1 but his position is deteriorated during Test 2 to 19^{th}. He surprized his teachers by acquiring the highest marks in the final exam and overall got the first position among all the other students.

Zain in MTL: He had no prior experience of mathematical thinking and can recall a few instances of doing activities related to applying course material in the real world. Although he does not show any particular concern or expectation from the MTL he hopes to do well because of his confidence and adds, “Nothing is impossible.” On the other hand, he does not participate fully in the MTL. He misses some of the problem solving activities and shows lack of interest during the MTL, thus ends up performing low based on his participation in the MTL. Although he does enjoy the new way of learning but he also feels stressed sometimes. He wants to change the pace of learning to a slower level and wants to watch more videos. He also suggests including social media interactions during the MTL.

Zain’s current knowledge state: He never practiced ALEKS to check the discrepancies in his prior knowledge and thus leaving no clues about his current knowledge state in mathematics.

5.3.2 Persona 2: Chen

Chen started his engineering education after completing his matriculation examination with good grades. During the first semester of first year in Mechanical Engineering, he took Engineering Mathematics as a core subject. Although he was aware of his occasional careless attitude towards learning, he was determined to solve any mathematical problem during the course work. He thought that he already had developed good computational skills and his key to success in mathematics was his regular practice, revising the lecture notes and doing past-years questions. He expected to learn through electronic resources and utilize his computational skills in the current mathematics course. On the other hand, he was afraid of doing extensive work in the form of expected assignments and project. He wanted to have free textbooks and electronic resources for this course. Being lazy, careless and sleepy were some of his other concerns during this course.

Chen in Class: Chen stays in his group and mostly busy in doing practice. Chen gets the second position in Test 1 but his position drops to 10^{th} in Test 2. Although if he is on the 15^{th} position in the final exam, he manages to acquire fifth position overall. He self-assessed himself as an “A^{-}” grader. He prefers to take his own notes during lectures and revises the concepts for exam. He never asked question in class.

Chen in MTL: Although Chen was previously involved in doing activities related to applying mathematics in the real world, he only heard about mathematical thinking somewhere and did not have any prior experience to learn how to think mathematically. He was not sure how to succeed in the MTL but he was eager to learn creative thinking as well as mathematical thinking and to utilize those skills in the near future. Although he knew nothing about the MTL, he was excited to gain some knowledge about mathematical thinking. During the MTL, he wants to be more creative and is ready to think more "mathematically”. He is excited and looking forward to enjoy the new experience. Another thing he wants during the MTL, “a quick recipe to solve problems.” He once presented in front of the class by copying the solution of a problem on whiteboard but he was unable to explain that. He participated in every activity during the MTL with full attention and fully cooperated with his team. After doing activities related to “Flaws of averages,” he shared, “I have never realised it before” and “average is NOT the average that we used to know.”

***Chen’s current knowledge state*:** He shows strong conceptual knowledge base in functions during a quiz in the MTL whereas ALEKS shows his moderate current knowledge state after assessing his practice for about four and a half hours.

5.3.3 Persona 3: Abdullah

Abdullah finished his foundation program with 4.0 GPA and got admitted in the Mechanical Engineering Program in a Local Research University (RU). He took engineering mathematics with a goal "to create a new method that can solve any problems" in the future. "Remembering equations" was one of his strengths and he considered "graph plotting" as one of his weaknesses. He "got A+ for additional mathematics in all examinations during secondary school" and was expecting to get "full marks" in the future as well. He has a habit of "practicing past years questions, understanding the concepts and patterns" during the study and "practicing all different kind of questions" during the exams. He was afraid of the "busy life" in the RU and "to forget everything he has learnt earlier." He expected to get "support from everyone including the lecturer" during the semester. He gave himself an "A+" during self-assessment. He preferred to use "books, lecture notes, notes from internet" during his studies and he also consulted his friends during the preparation of final exams.

***Abdullah in Class**:* He was fully attentive in comprehending the new concepts throughout the semester but rarely asked questions in class. He stood 1st in first test in the class but his position went down to 9th in test 2. He did well in the assignments and quizzes but got 18th position in the final exam thus acquiring overall 2nd position in the class. He showed persistence in his performance throughout the semester.

***Abdullah in MTL*:** He has no prior learning experience to think mathematically and never got involved previously in activities related to applying course material in the real world. He was concerned about "what syllabus will he be covering in the MTL." He recorded his reflections about what he hopes to get out of the class and how he is going to achieve it, "Dear me, I know you're good in mathematics and you can do better in this class. Please work harder and put more efforts to achieve what you dream the most. Take this opportunity to gain your knowledge in mathematical thinking, so that you can solve any mathematical problems later and make better choices in the future. Do not be lazy! Remember, whatever you will give, you'll get it back." He attended all the activities during MTL except one quiz but he acknowledged that he did not like the sessions. He wanted to change the syllabus to "real mathematics" like the one he used in traditional class.

Abdullah's Current Knowledge State: His current knowledge state is better than the rest of his class fellows even if he just spent 6 and a half hours on ALEKS doing practice.

5.3.4 Persona 4: Ismail

Ismail joined mechanical engineering after completing his diploma and with three years of industrial experience in an oil and gas company. He took Engineering Mathematics as a core course with a GPA of 3.46. Besides many bad experiences in mathematics, he once "got A for mathematics in foundation." He was not a high achiever and did not expect to score more than a "D" in future as well. Attending classes, doing a lot of exercises, studying in groups, and attending tutorials are some of his study habits whereas during exam he used to revise all the work being done during the course. He was afraid of failing this subject and found it difficult to overcome "being lazy." He said, "he needs lots of practice in the coming days to survive." He preferred the textbook and past papers to prepare for exams.

Ismail in the Class: He always sits in the front row during lectures and tries to capture whatever is delivered whether he understands it or not. He never asked questions during lectures but felt comfortable in asking questions to his friends during and after class. He was unable to score well during the formative assessment in engineering mathematics and in fact, he was the only one who failed in the final exam. In test 1, he got 50^{th} position among 52 students and was on the last position for the rest of the tests.

Ismail in the MTL: He expected to "learn how to solve very complex problems" in the MTL. .He is good in sense making but that is more geared to work in conceptual embodiment whereas he finds it hard to transfer knowledge to the symbolic world of mathematics. He recorded his reflections about his expectation and achievement of goals "Mathematics is not so hard. It is just numbers. What makes it hard is your attitude towards it. Do not be lazy to practice more and more. Yes, you cannot build the Rome Empire in just one night. It takes a lot of hard work and your time to achieve it. Until you don't believe in yourself, you won't go anywhere." He attended all the activities during the MTL except one quiz. He shared that one can use mathematics to "calculate the salary, to buy a property with that salary, calculate how much money you have to save in fixed deposit account for your future use and calculate how much tax you have to pay to the government." He also added, "I am enjoying this class so much because I learn new problem solving method using critical thinking." He enjoyed working in his team and shared mutual respect with all his team members.

Ismail's Current Knowledge State: His conceptual knowledge base is very weak and even he knows that, he is not spending enough time to practice on ALEKS to rebuild those concepts.

5.3.5 Persona 5: Fatima

Fatima was one of the five females in the class who joined Mechanical Engineering after completing her foundation program in engineering with a GPA of 4.0. She has a study habit of focusing during lectures and trying them back at home whereas she used to do past year papers in preparing for exams. She expected to score good in engineering Mathematics with a fear of deriving “lots of equations” and with a difficulty to remember “a lot of formulas”.

Fatima in the Class: She is quiet but focused in class. She talks less, shows more responsible attitude during the lectures. She rarely asks questions in the class but is comfortable asking questions to her friends afterwards. She gets third position in the test 1, 11^{th} in test 2 and 19^{th} in the final test, thus her overall relative position turns out to be 8^{th}.

Fatima in the MTL: She has “participated in a few short mathematical thinking activities” in the past and had an experience of applying the course material in the real world. She is expecting to “think out of the box” with developing “metacognitive” abilities and knowing “applications of maths in real life” during the MTL. She intends to “focus more in the MTL and asks question.” However, she did not participate in all the activities during the MTL and also did not enjoy the sessions and said, “I didn’t really understand the concepts.”

Fatima’s Current Knowledge State:

She did not utilize the ALEKS resource, in fact never opened it but showed poor knowledge base related to functions during a quiz in the MTL.

5.3.6 Persona 6: Faaiz

Faaiz joined Mechanical engineering after completing his foundation program in engineering with a GPA of 3.92. He was not sure about his strengths and weaknesses in maths. His study habits were "doing a lot of exercises and comparing the work with friends" whereas he used to "memorize the formulas and exercises" for the exams. He expected the engineering mathematics to be difficult and at the same time, he wanted "to enjoy and love it." He faced difficulty keeping his focus in the afternoon classes. He gave himself a "B+" during self-assessment.

Faaiz in the Class: He tried to be attentive in class and attended the lecture with full interest. He got the fourth position in Test 1 and jumped to the first in Test 2 whereas in the final exam, got the 12th position, thus acquiring overall 3rd position in class.

Faaiz in the MTL: He has no prior experience with mathematical thinking and never did activities related to applying course material in the real world. He was looking forward "to understand himself better, then to know how to interpret some problem and then to give the best solution." He is also excited to learn how to relax when stuck with problems." He also shows his concern by asking, "How can we think creatively and smartly, enjoying at the same time and becoming more confident to face the real world." He recorded his short reflections during the first session of MTL, as "you're always shy to speak out. You always have an idea but you are afraid that the ideas are funny and irrelevant. You feel jealous when others have many ideas. You are always thinking over and over how to solve a problem but at the end nothing comes out. Make sure in class you improve yourself, do not try to be someone else but be more confident. Try to learn from your friends, lectures and enjoy the moment. Do not be shy to speak out. Beat others. Be a great engineer. Sharpen your thoughts. Give a great effort and we will see the results in the future. Be a gentleman, yeahhh!!!" He participated in all the activities and even did all the quizzes.

Faaiz's Current Knowledge State:

He shows overall good knowledge base compared to others in the class even by practicing for nearly 6 hours on ALEKS.

5.3.7 Persona 7: Sunny

Sunny did matriculation with 3.96 GPA and joined Mechanical Engineering with an expectation to get an overall “A” grade along with the fear of getting bad grades in engineering mathematics. He claimed, “I am lazy to try out problems and need more practice.” He gave himself a grade “B” during self-assessment. He used to prefer “complete notes and formulas in the form of hard copies”, “look at examples” and “revise books and do more and more practice” for the exams.

Sunny in the Class: He never participated in class and never asked any question during lectures. However, he felt comfortable asking questions to his friends outside the classroom. He got the 40th Position in Test 1, improved to 25th in Test 2 and moved down to the 42nd positon in the final exam, thus getting 44th positon overall.

Sunny in the MTL: He was looking forward to “solve everyday problems using mathematics.” He shared his reflections, “you try your best to give the best during classes, but you lose your attention frequently in your class. The environment is very new to you so you must make yourself used to it. There are so many challenges and you must overcome it because everything happening to us makes us better persons. Challenges are very important to make you a perfect person. So enjoy your class and give your best.” He participated in all the activities except for one quiz and a motivational session. He enjoyed the MTL sessions quietly but he proved to be a an effective communicator in focus group.

Sunny’s Current Knowledge State: He never used ALEKS.

5.3.8 Persona 8: Fahmi

Fahmi acquired 4.0 GPA and joined Mechanical Engineering in a local research university after completing his foundation program in engineering. He said, "hopefully, I will use all the formulas I have learned. Because I want to be a great engineer" in the future. He felt great if "called to come in front and explain the solution for the problem given by the prof". He said, "I just need some reading and interactive sources such as books and videos" to study in Mathematics. He liked to prepare for exam by refreshing his memory and then by teaching his friends. He expected to have fun during the new experience but feared of being sleepy during the class. He wanted to have "fast internet connection, to access YouTube." He really loved to watch educational videos because for him "they are very interactive" and always captured his focus. He preferred to use textbook throughout his study and to prepare for exams. He loved to learn so that he could teach others. He considered himself a fast learner in Maths.

Fahmi in the Class: He attended all the lectures with full attention and often interacted with his team members. Although he rarely asked questions in front of the class but he asked so many questions to his friends during lectures. He got 11th position in Test 1 and went down to the 20th in Test 2 and further down to the 29th in the final exam , thus scoring 19th in overall position.

Fahmi in the MTL: He has no prior experience with mathematical thinking and never involved in activities related to applying course material in the real world. He shared, "I would love to learn how to design and invent new things" and recorded his reflections, as "you have been an excellent student throughout. It would be awesome if you could be the most successful engineer, to improve the Muslim engineering world." He participated in all the activities during the MTL, although he did not like the inductive way of teaching in MTL.

Fahmi's Current Knowledge State: He showed strong conceptual knowledge base in functions during a quiz in class and ALEKS reported his current knowledge state "moderate" just by spending about 7 hours of practice.

5.3.9 Situations as Precursors to Scenarios for Problem Analysis and Idea Development

This section is directed towards creating scenarios that describe solutions contributing to Stage 4 (Step 6) of persona development process (Section 3.9). The situations are precursors to scenarios. In this, a number of specific situations are described that could trigger the use of research findings. Every situation or a combination of situations is the basis of a scenario. The performance comparisons of all personas are also highlighted in the respective situations. Their performance during the formative assessment in class, their relative positioning in the classroom performance, the participation-based assessment in the MTL and persona's current knowledge state using ALEKS are shown in Figures 5.1 to 5.4 to describe the situations. The problem analysis and idea development will be discussed in Chapter 7 under student personas.

Table 5.7: Situations for Scenarios

Situation	Description	Data Display	Performance Analysis
A	Personas' performance during the formative assessment in the class	Figure 5.1	Persona 1 scored highest marks whereas Persona 4 scored the lowest. The chart shows the huge disparity between their performances. Persona 3 and Persona 6 are at the same performance level with a slight variation in their test1, test 2, assignments, quizzes, and final test scores. Persona 2 and Persona 5 are again at a slight performance gap of 1 with respect to each other. Persona 8 and 7 are at a noticeable performance gap due to their different marks in final exams.
B	Personas' relative position during the classroom performance	Figure 5.2	Performance variation in the sequence: T1→T2→FE →A→Q→Final Persona 1: 5→5→1→3→2→1 Persona 2:2→3→3→7→4→4 Persona 3: 1→2→4→1→1→2 Persona 4: 8→8→8→8→5→8 Persona 5: 3→4→5→4→7→5 Persona 6: 4→1→2→6→8→3 Persona 7: 7→7→7→5→3→7 Persona 8: 6→6→6→2→6→6
C	Personas' performance during participation-based assessment in the MTL	Figure 5.3	Persona 4 (>90%) has shown the highest performance in the MTL whereas Persona 1 and persona 5 (53%) showed lowest performance. Persona 3 and persona 8 performs equally well and are near the maximum performance (90%). Persona 7 and 2 perform are at (78%) whereas persona 6 is at 85 %.

Table 5.7: Continued

Situation	Description	Data Display	Performance Analysis
D	Personas' current knowledge state using ALEKS	Figure 5.4	Persona 1, 5, 7 showed no activity on ALEKS, Persona 4 showed critically weak current knowledge state, Persona 3 and 6 showed the best knowledge base among others. Persona 2 and 8 are at the second best in knowledge base
E	Personas' in the Class	Section 5.3.1	***Persona 1***: He is the one who never sits at the front of the class. He used to disturb his friends and class fellows sitting near to him with continuous humming during the routine lectures.
		Section 5.3.2	***Persona 2***: Chen stays in his group and mostly found busy in doing practice. He prefers to take his own notes during the lecture and revise the concepts for exam. He never asked question in the class and does not have the habit of asking questions at all.
		Section 5.3.3	***Persona3:*** He was fully attentive in comprehending the new concepts throughout the semester but rarely asked questions in the class.
		Section 5.3.4	***Persona4:*** He always sits in the front row during the lectures and tries to capture whatever is delivered whether he understands it or not. He never asked questions during the lecture but felt comfortable in asking questions to his friends during and after the class.
		Section 5.3.5	***Persona5:*** She was quiet and focused in the class. She talked less, showed more responsible attitude during the lectures. She rarely asked questions in the class but was comfortable asking questions to her friends afterwards.
		Section 5.3..6	***Persona6:*** He tried to be attentive in the class and attended the lecture with full interest.
		Section 5.3.7	***Persona7:*** He never participated in class and never asked any question during the lecture. However, He felt comfortable asking questions to his friends outside the classroom.
		Section 5.3.8	***Persona8:*** He attended all the lectures with full attention and often interacted with his team members. Although he rarely asked questions in front of class but he asked so many questions to his friends during lectures.

Table 5.7: Continued

Situation	Description	Data Display	Performance Analysis
F	Personas' in MTL	Section 5.3.1	***Persona 1***: Zain does not participate fully in MTL. He misses some of the problem solving activities and shows lack of interest during MTL, thus ends up performing low based on his participation in the MTL. Although he does enjoy the new way of learning but he also feels stressed sometimes. He wants to change the pace of learning to a slower level and wants to watch more videos. He also suggests including social media interactions during the MTL.
		Section 5.3.2	***Persona 2***: Chan wants to be more creative and is ready to think more "mathematically". He is excited and looking forward to enjoy the new experience. Another thing he wants during MTL, "a quick recipe to solve the problems.
		Section 5.3.3	***Persona3:*** He attended all the activities during MTL except one quiz but he acknowledged that he did not like the sessions. He wanted to change the syllabus to "real mathematics" like the one he used in traditional class.
		Section 5.3.4	***Persona4:*** He attended all the activities during MTL except one quiz. He shared that one can use mathematics to "calculate the salary, to buy a property with that salary, calculate how much money you have to save in fixed deposit account for your future use and calculate how much tax you have to pay to government." He also added, "I am enjoying this class so much because I learn new problem solving method using critical thinking." He enjoyed to work in his team and shared mutual respect with all his team members
		Section 5.3.5	***Persona5:*** She was intending to "focus more in the MTL and ask question." However, she did not participate in all the activities during MTL and also did not enjoy the sessions and said, "I didn't really understand the concepts."
		Section 5.3..6	***Persona6:*** He participated in all the activities and even took all the quizzes.
		Section 5.3.7	***Persona7:*** He participated in all the activities except one quiz and a motivational session. He enjoyed the MTL sessions quietly but he proved to be an effective communicator in focus group.
		Section 5.3.8	***Persona8:*** He participated in all the activities during the MTL, although he did not like the inductive way of teaching in MTL.

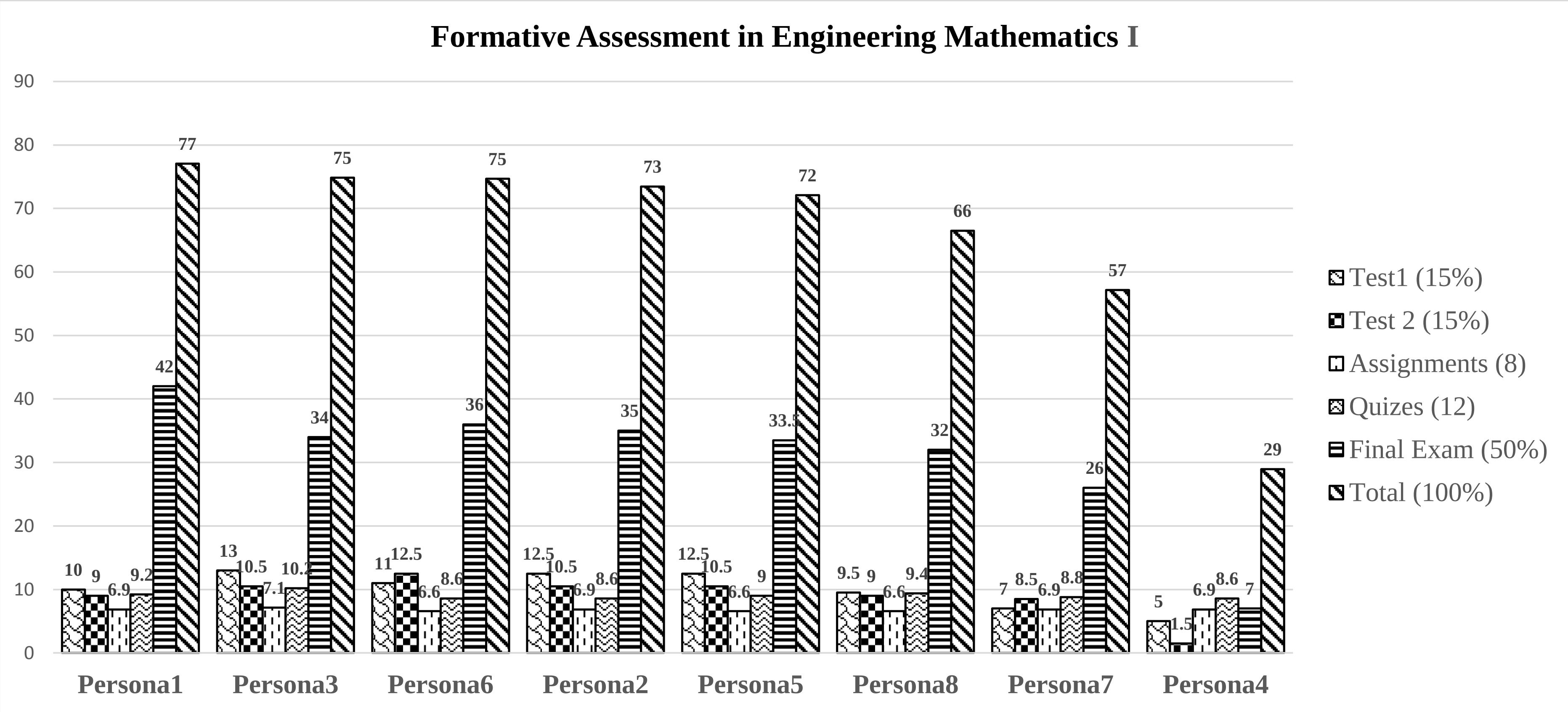


Figure 5.1: Overall performance during the formative assessment in the class

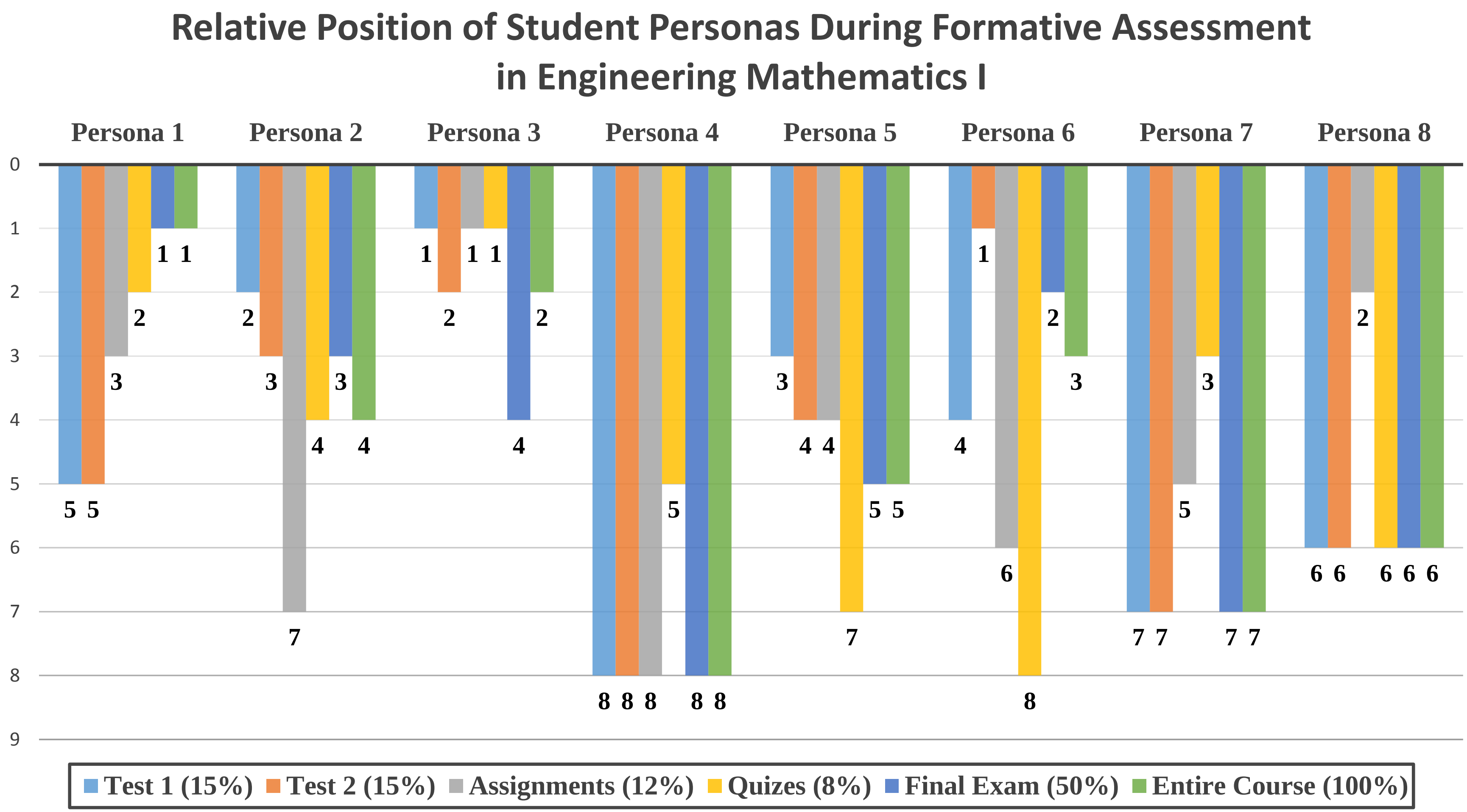


Figure 5.2: Relative Position of Student Persona's During Formative Assessment in Engineering Mathematics I

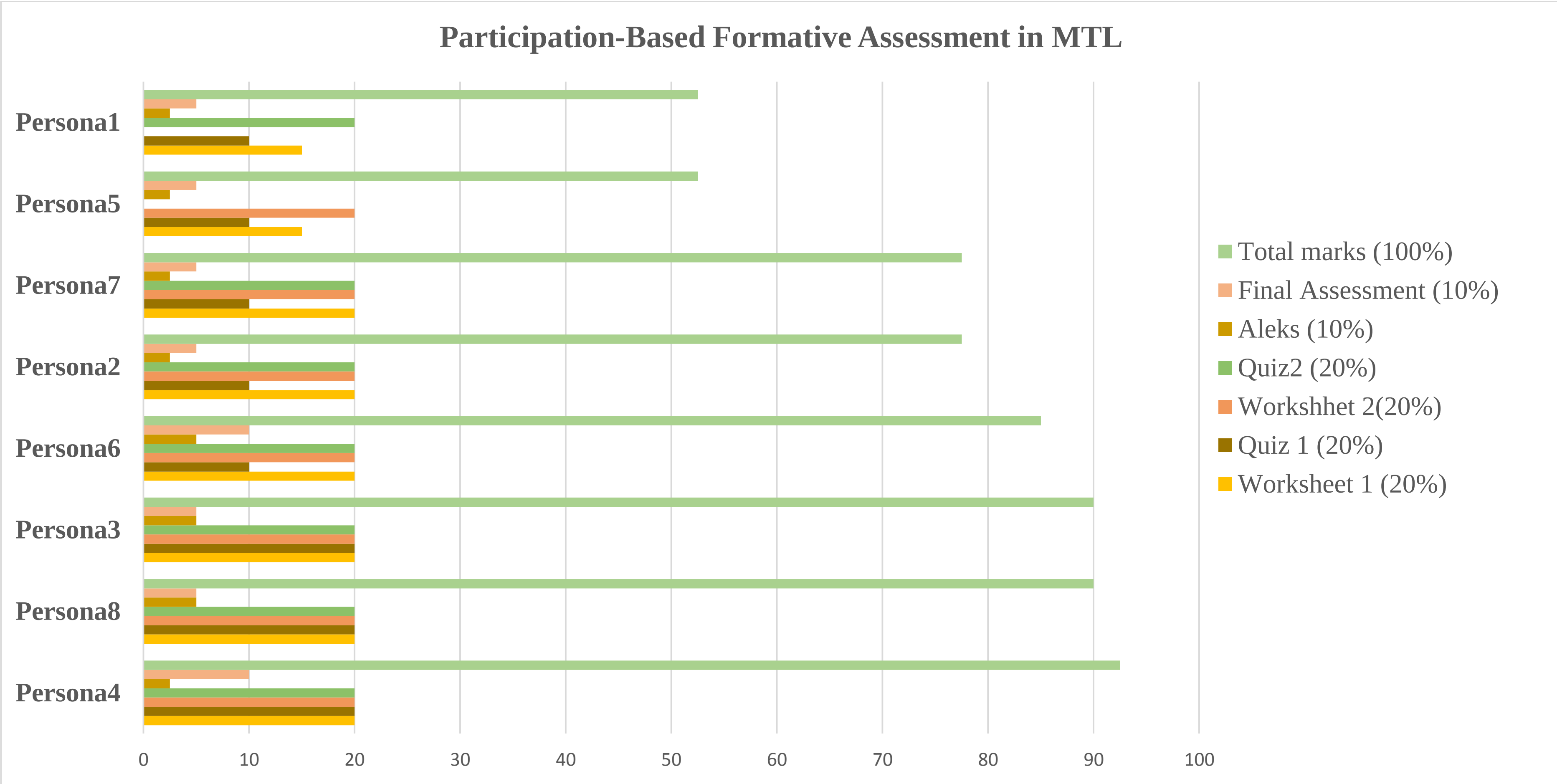


Figure 5.3: Formative assessment for participation in MTL

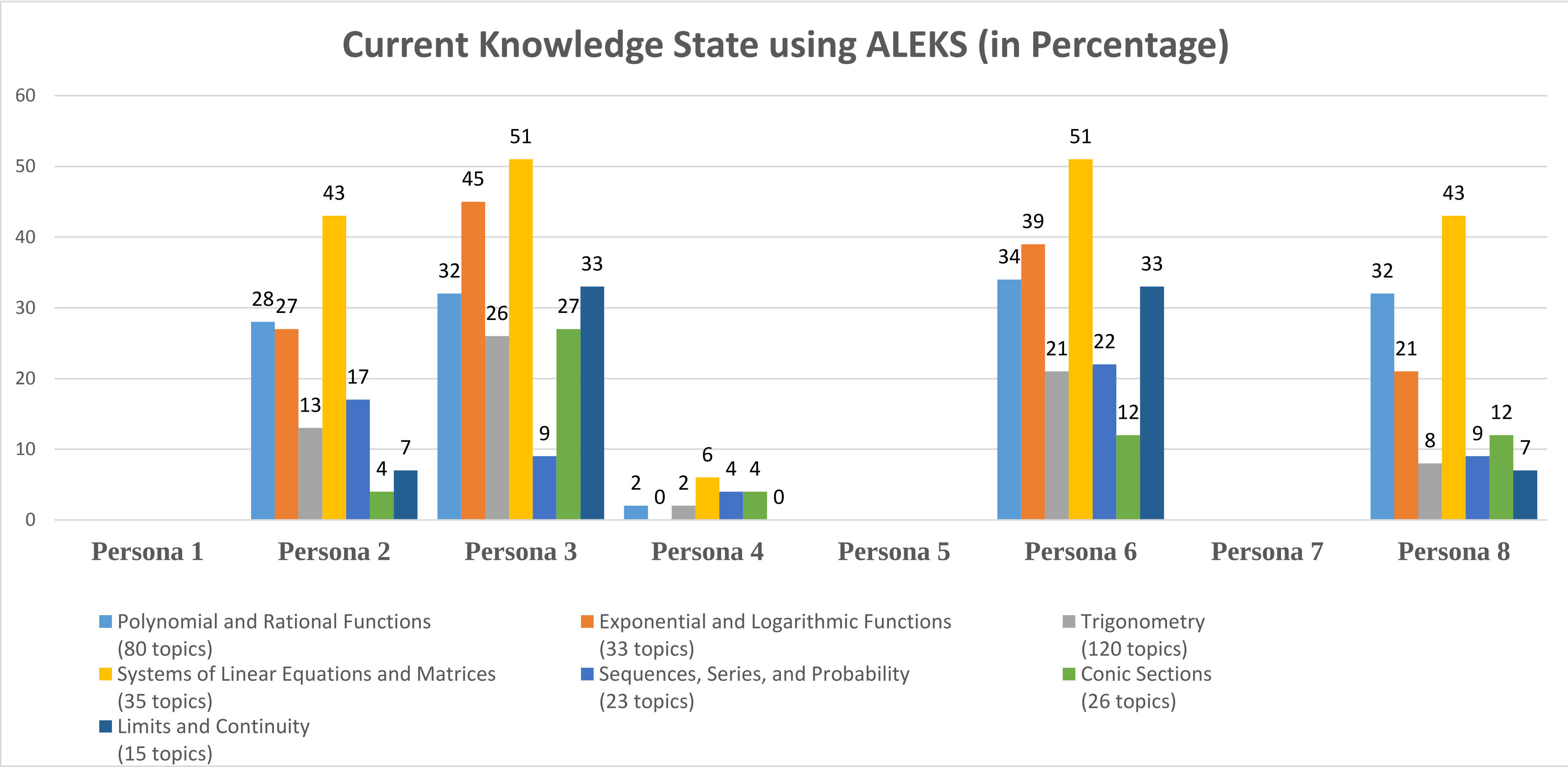


Figure 5.4: Current Knowledge State in ALEKS

5.3.10 Modified Rubric to Assess Mathematical Thinking

The following rubric is adapted and modified after several iterations to assess the activation of mathematical thinking processes during problem solving activities.

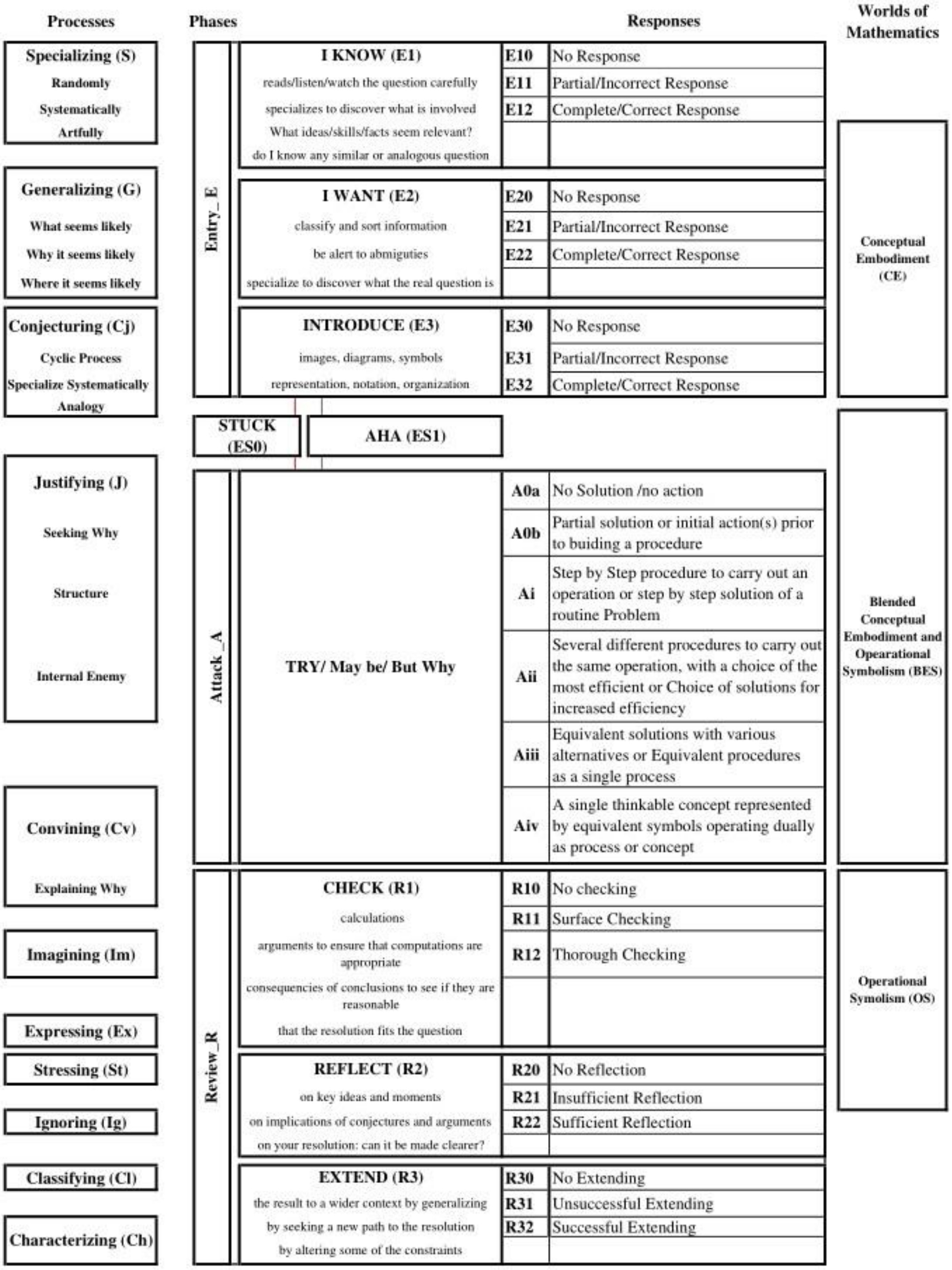


Figure 5.5: Final rubric to assess mathematical thinking adapted from (Mason et al., 2010; Tall, 2013) and refined by researcher

5.3.11 Ismail's Written Activity Response Analysis

The left side of the analysis sheet shows the activation of different processes and right column of the comments shows the interpretation of the particular instances after examining the activity responses. The samples shown in Figure 5.6 describe the whole process in the following steps as shown in the analysis:

1. There is no written response by the student. (Entry Phase: I know)
2. As shown in Figure 5.7, Ismail listed partial information about what is being required by the question. No symbolic expression is introduced at this level. The student is trying to make sense of the problem being addressed while keeping himself in the conceptual embodiment. (Entry Phase: I want).
3. As shown in Figure 5.8, he started extracting what he knows from the question by eliciting conceptual embodiment and at the same time he was doing specializing by recording a partial response on the worksheet. He give partial response of what he can introduce to solve the problem but the way he draws the number of rice in each box shows that he can well understand what the question is all about. (Entry Phase: I introduce).
4. As shown in Figure 5.9, he provided a systematic procedure to carry out an operation by first doing conjecturing and then tried generalizing while shifting his mathematical thinking in the operational symbolism. The resolution shows a step-by-step procedure to find out the value of exponential operation but the student did a mistake by not reaching to a proper generalized form that is 2^n-1. The solution shows that the student partially articulated a sense of the underlying pattern from the entry phase and shows the evidence of conjecture. He then jumped to the number of bags (how did he find?) there is no evidence for that. The solution shows the proceptual-symbolic world that grows out of the embodied world through action (such as making sense of the number of grains on the chess board) and is symbolised as thinkable concepts (such as exponentials) (Attack Phase).

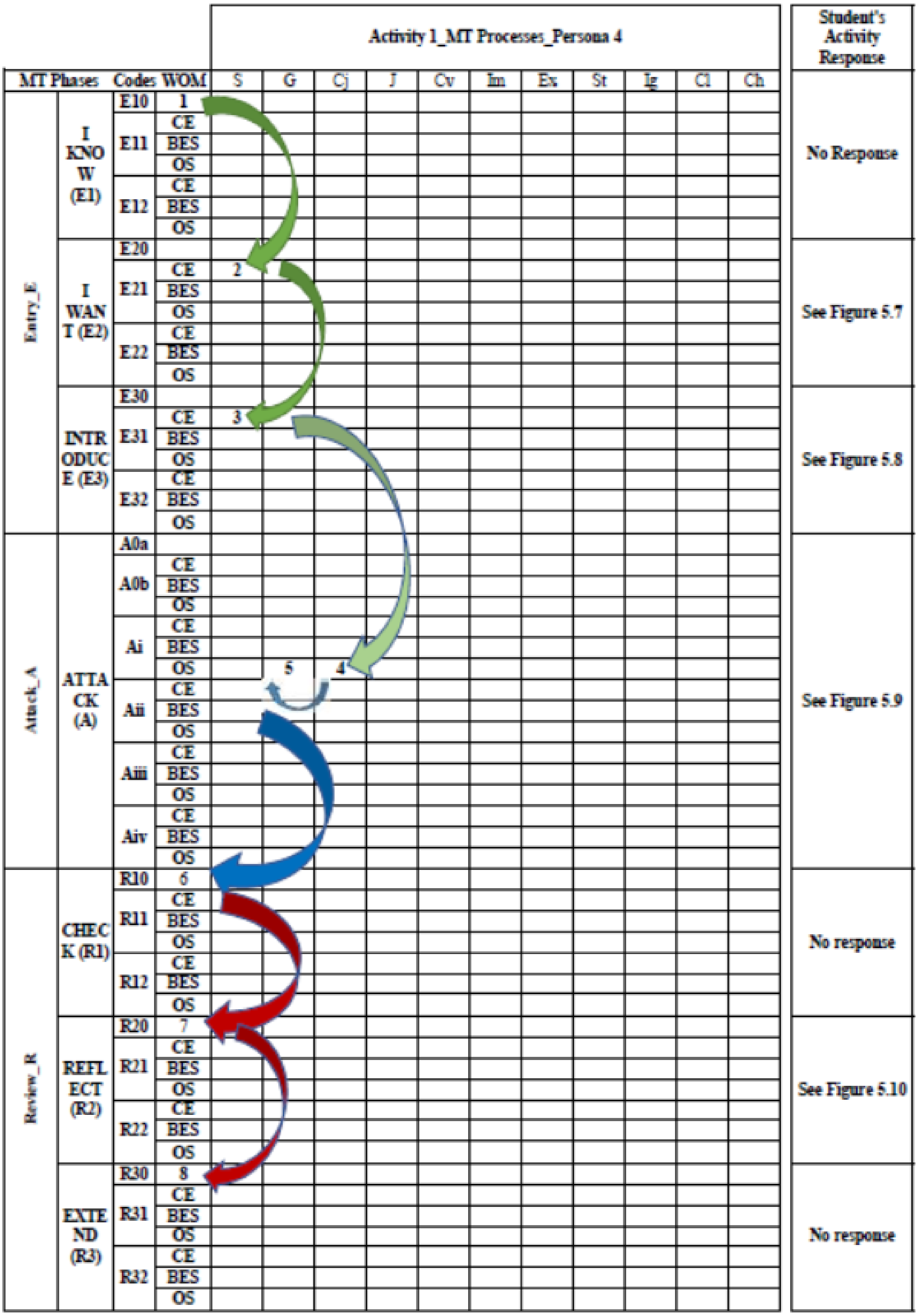


Figure 5.6: Activation of Mathematical Thinking Processes (Module: Exponentials; Persona ID: 4)

5. No evidence of checking the resolution is found means there is no response. The researcher assumed that the students did check their resolutions. In the future, it is recommended to put a check box for students to mark if they have done it. (Review Phase: Check).

6. As shown in Figure 5.10, he was unable to record the reflection after the activity was done in class. The response shows incorrect generalization of the problem cases. The wrong response shows that the student did not utilize the conceptual embodiment and directly tried to map a formula to generalize the situation. (Review Phase: Reflect).

7. Ismail was unable to produce a new example and was unable to replicate the original concept of the module. (Review Phase: Extend).

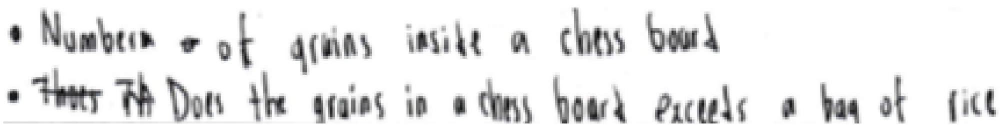


Figure 5.7: Response to I want sub-phase(Module: Exponentials; Persona ID: 4)

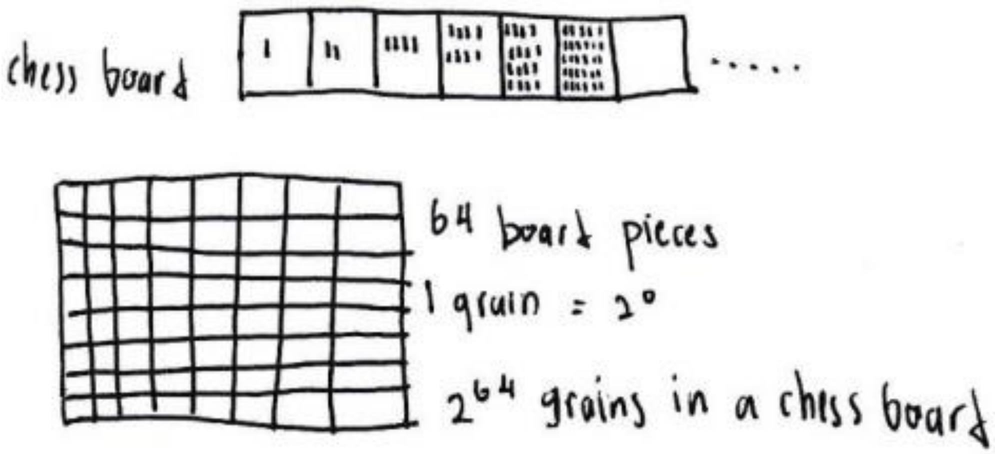


Figure 5.8: Response to Introduce sub-phase(Module: Exponentials; Persona ID: 4)

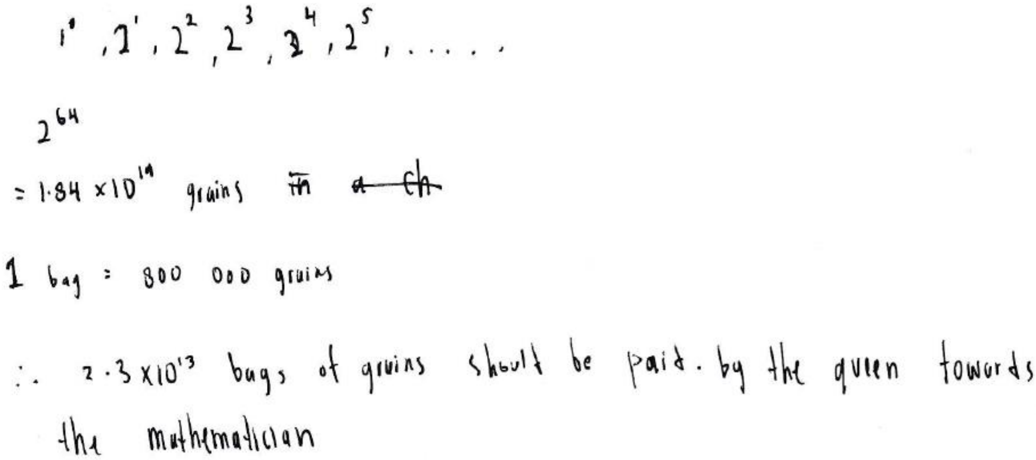


Figure 5.9: Response to Attack Phase (Module: Exponentials; Persona ID: 4)

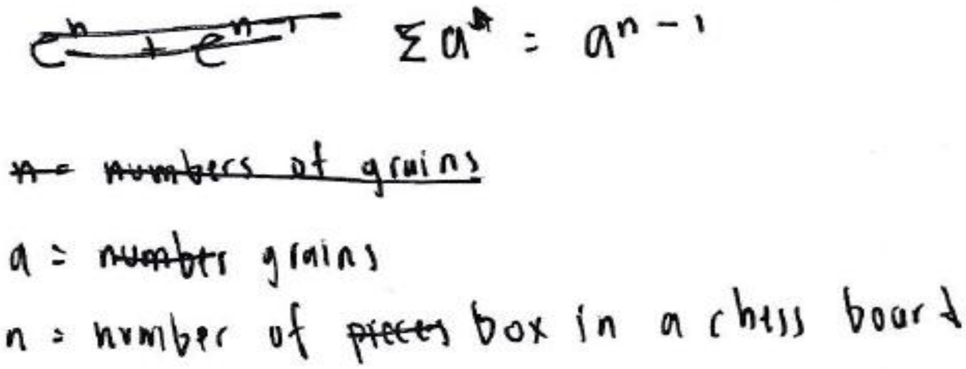


Figure 5.10: Response to Reflect sub-phase (Module: Exponentials; Persona ID: 4)

5.3.12 Zain's Written Activity Response Analysis

The samples shown in Figure 5.11 describe the whole process in the following steps as shown in the analysis:

1. There is no written response by the student. (Entry Phase: I know)

2. The student listed partial information about what is being required by the question. No symbolic expression is introduced at this level. The student tried to make sense of the problem being addressed while keeping himself in the conceptual embodiment. (Entry Phase: I want).

3. The student started extracting with what he knows from the question by eliciting blended embodiment and symbolism by drawing 8 by 8 chess board and filling in the numbers 1,2,4,8,… in the small boxes. He was doing specializing by recording a partial response on the worksheet. He give partial response of what he can introduce to solve the problem but the way he draws the chess box and fill in the numbers does not show his full understanding. (Entry Phase: I introduce).

4. Student now started attacking the problem. He starts by recording " As chess board has 64 squares" and then straight away conjecture that total grain would be 2^63 by using the form 2^(n-1) to find the number of grains. Without even doing generalizing, he jumps to the conclusion that "therefore the number of rice that the queen has to give to the mathematician is > 800,00). (Attack Phase).

5. No evidence of checking the resolution is found means there is no response. The researcher assumed that the student did check his resolutions. (Review Phase: Check).

6. The student was unable to record the reflection after the activity was done in class. The response shows incorrect generalization of the problem cases. The wrong response shows that the student did not utilize the conceptual embodiment and directly tried to map a formula to generalize the situation. (Review Phase: Reflect).

7. The student was unable to produce a new example and to replicate the original concept of the module. (Review Phase: Extend).

MT Phases		Codes	WOM	Activity 1_MT Processes_Persona 1											Student's Activity Response
				S	G	Cj	J	Cv	Im	Ex	St	Ig	Cl	Ch	
Entry_E	I KNOW (E1)	E10	1												No Response
		E11	CE												
			BES												
			OS												
		E12	CE												
			BES												
			OS												
	I WANT (E2)	E20													See Figure 5.12
		E21	CE	2											
			BES												
			OS												
		E22	CE												
			BES												
			OS												
	INTRODUCE (E3)	E30													See Figure 5.13
		E31	CE												
			BES	3											
			OS												
		E32	CE												
			BES												
			OS												
Attack_A	ATTACK (A)	A0a													See Figure 5.14
		A0b	CE												
			BES												
			OS												
		Ai	CE												
			BES												
			OS			4									
		Aii	CE												
			BES												
			OS												
		Aiii	CE												
			BES												
			OS												
		Aiv	CE												
			BES												
			OS												
Review_R	CHECK (R1)	R10	5												No response
		R11	CE												
			BES												
			OS												
		R12	CE												
			BES												
			OS												
	REFLECT (R2)	R20													See Figure 5.15
		R21	CE	6											
			BES												
			OS												
		R22	CE												
			BES												
			OS												
	EXTEND (R3)	R30													No response
		R31	CE	7											
			BES												
			OS												
		R32	CE												
			BES												
			OS												

Figure 5.11: Activation of Mathematical Thinking Processes (Module: Exponentials; Persona ID: 1)

Figure 5.12: Response to I want sub-phase(Module: Exponentials; Persona ID: 1)

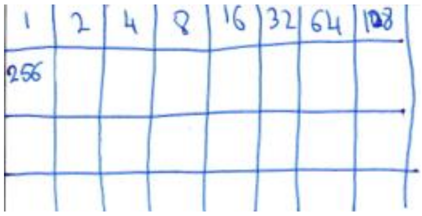


Figure 5.13: Response to Introduce sub-phase(Module: Exponentials; Persona ID: 1)

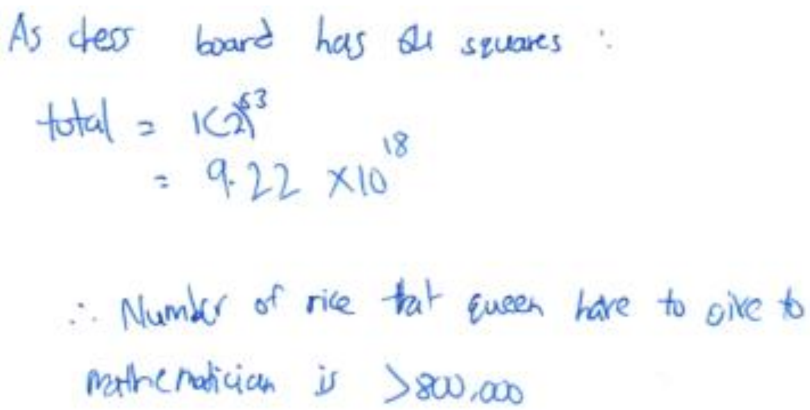


Figure 5.14: Response to Attack Phase (Module: Exponentials; Persona ID: 1)

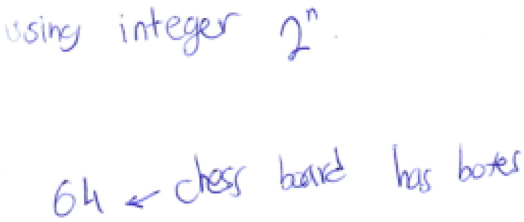
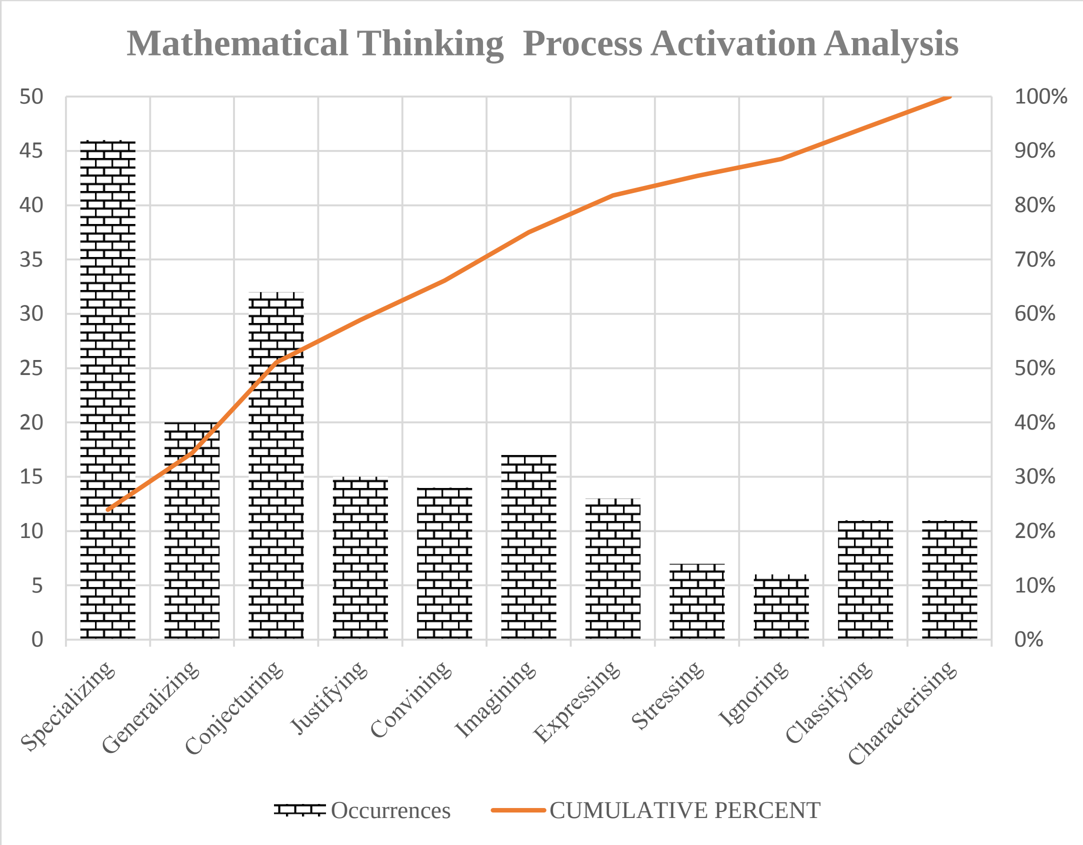


Figure 5.15: Response to Reflect sub-phase (Module: Exponentials; Persona ID: 1)

5.4 Results of Problem Solving Activity Response Analysis

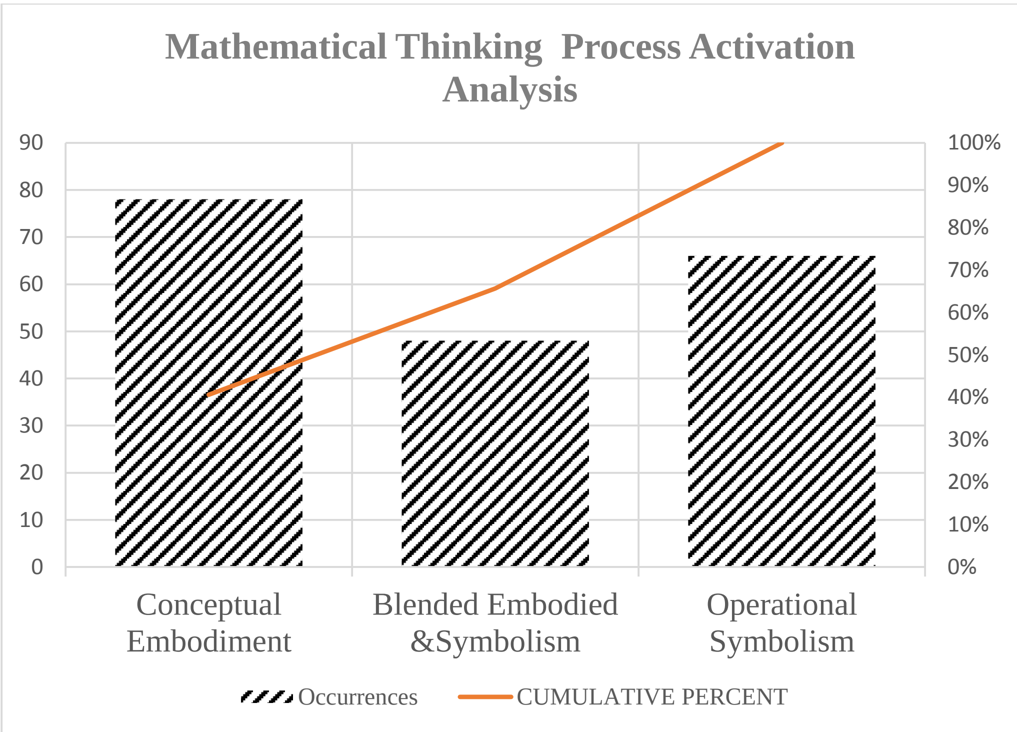


Activation Process	P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	Occurrences	% OF TOTAL	CUMULATIVE PERCENT
Specializing	6	4	6	6	6	6	6	6	46	23.96%	**23.96%**
Generalizing	2	3	3	4	2	4	0	2	20	10.42%	**34.38%**
Conjecturing	3	4	4	6	3	4	2	6	32	16.67%	**51.04%**
Justifying	2	2	2	3	1	2	1	2	15	7.81%	**58.85%**
Convincing	1	1	2	4	1	2	1	2	14	7.29%	**66.15%**
Imagining	2	1	2	4	1	2	2	3	17	8.85%	**75.00%**
Expressing	1	1	2	4	1	1	0	3	13	6.77%	**81.77%**
Stressing	1	0	0	2	0	1	1	2	7	3.65%	**85.42%**
Ignoring	1	0	0	1	0	1	1	2	6	3.13%	**88.54%**
Classifying	1	0	1	2	1	2	2	2	11	5.73%	**94.27%**
Characterizing	1	0	1	2	1	2	2	2	11	5.73%	**100.00%**

Figure 5.16: The graph and findings showing occurrences and cumulative percentage of different processes being activated during the activities

After analyzing the worksheets related to two BLOSSOMS modules with three activities per worksheet for eight personas in the MTL during the main study, the activation of students' thinking in the world of conceptual embodiment was evidenced. The obvious reason was the contextualization of the content presented through BLOSSOMS modules. The summarized activation of different mathematical thinking processes is shown in Figure 5.16.

The total occurrences and their representation of the associated graph shows that specializing and conjecturing are the main processes activated during the implementation of two modules and then come the generalizing, imagining, justifying and convincing. The next active processes can be seen as expressing, classifying, and characterizing whereas there are only 13 occurrences for the pair of stressing and ignoring.



Activation World	P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	Occurrences	PERCENT OF TOTAL	CUMULATIVE PERCENT
CE	3	6	12	21	3	14	7	12	78	40.63%	40.63%
BES	8	4	5	8	7	4	4	8	48	25.00%	65.63%
OS	10	6	6	9	7	9	7	12	66	34.38%	100.00%

Figure 5.17: The graph and findings showing occurrences and cumulative percentage of different processes being activated in Conceptual, Blended, and Symbolic World of Mathematics

The Mathematical worldview analysis shown in Figure 5.17 also elicits that blended learning enables the activation of the mathematical thinking processes in the world of conceptual embodiment in the highest order. The occurrences for the evident processes in the conceptual embodiment are 78; in the operational symbolism are 66 whereas the occurrences in the blended embodied and symbolic are only 48.

5.5 Summary

In this chapter, the introduction is followed by the emergent themes after Stage II data analysis of this research. After describing the emergent themes of student's met-befores, the implications of blended learning as student's met-afters, diligence during mathematical problem solving and student teacher relationships are then discovered and reported. The researcher then described eight evidence-based personas along with their comparative analysis through different charts. Pre-identified deductive coding scheme is discussed followed by a modified rubric to assess the activation of mathematical thinking processes.

Stage II Post data collection analysis of the main study is carried out through data analysis technique as explained in Section 3.7.2. The researcher summarizes the findings guided by the research questions and records her reflective responses in her reflective journal.

CHAPTER 6

DISCUSSIONS

6.1 Introduction

This chapter will give explanations to justify how and why some actions and decisions were made and what happened as a result.

This chapter is all about discussing the results, connecting them back to literature and theories and giving a holistic picture of what has been done and why is that important for the researcher as well as how would it contribute towards the body of knowledge.

6.2 Discussions

In the next paragraphs, the researcher will discuss the development and implementation of blended learning and the development of emergent themes and student personas, to justify the claims to knowledge, validate them and legitimatize this research based on the idea of Lewin (1951, p. 169) “there is nothing so practical as a good theory.”

To further facilitate the discussion, the researcher first compiled the responses of the following three guided questions (Power and Naysmith, 2005, pp. 11–12) for the action research cycles:

What did I do?

- I first developed problem-solving activities by integrated Mason's problem solving strategy with BLOSSOMS modules and then implemented a blended learning environment by utilizing the above three.
- I collected the students' written activity responses during the MTL sessions.
- I explored the students' profiles (extreme/ deviant/ interesting cases) based on classroom and MTL observations during the main study.
- I observed students throughout the engineering mathematics course and the MTL to understand who they are and what do they do and why?
- I adapted and refined a rubric from Mason's book "Thinking Mathematically" to assess the students' written activity responses.
- I explored the students' mathematical thinking processes by assessing their written activity responses with the help of the refined rubric
- I inductively explored student's met-befores related to mathematical thinking, learning, and problem solving.
- I gathered and explored feedbacks from the students on how they learned from MTL by letting them fill online google forms, talking to students in focus groups during assessment week, and video recording their discussions on MT phases and processes.
- I listed down the ideas that might help future students in developing their mathematical thinking skills

What did I discover and what is expected?

- Some students can make sense of MTL session and this often seems to be linked by providing a context so that they can see why it is useful whereas some students are struggling with the new ideas.
- The new content that I teach in the MTL to promote mathematical thinking was a new experience for me as well as for most of the first year students and their conflicting met-befores makes the process of learning a challenge for them.
- Making mathematical thinking phases and processes explicit during problem solving strategy develop the MT skills in a more structured way.

- The students with insufficient prior knowledge were able to survive in the MTL but could not manage to survive in engineering mathematics formative assessment in class.
- Some students have found the MTL useful and relevant while others cannot appreciate its value because of the "grade-based system" in practice and due to their prior "habit of minds."
- Those who failed to respond properly, were predictably those who very quickly showed signs of lack of interest in the MTL.
- Some high graders also show slow learning pace during the new experience of learning.
- I discovered students' conflicting met-befores making learning a challenge for them in a new environment.
- I discovered some contributing issues in the way of developing their mathematical and problem solving skills.
- I discovered emerging personas that have the potential to be used as pedagogical tools to share the research findings with the community of practice (CoP).
- I discovered that student's conflicting met-befores related to mathematical thinking and learning are successfully manipulated into supportive met-afters through blended learning
- I discovered that students' diligence during problem solving and student-teacher relationship has improved through blended learning.
- I discovered that blended learning in this research supports mathematical thinking more in conceptual embodiment than in operational symbolism world of mathematics.

What did I, or will I, change/create as a result of my enquiry?

- Future Recommendations will guide to improve research and practice in the context of mathematical thinking, problem solving, and blended learning.
- Revised worksheets along with revised rubric to assess the activated MT processes during problem solving activities will help future researchers and practitioners to use it in their research/classrooms.

- A common language is established among students, practitioners, and CoP by teaching problem-solving strategy explicitly in the engineering mathematics I course. This not only informed how first year engineering students think mathematically but also provided a positive focus for redirecting future engineers' incongruous attitudes while solving a problem with unknown formula or cook book. The researcher perceived that explicitly teaching mathematical thinking oriented problem solving strategy in a blended learning environment will result in productive classroom.
- Empathy with students helped the researcher to understand pressing issues.
- This has important ramifications for future engineers' mathematical thinking and problem solving skills.

Transformative Learning Experience

This research proved to be a transformative learning experience for the researcher in accordance with Mezirow's explanation:

> "Transformative learning is learning that transforms problematic frames of reference—sets of fixed assumptions and expectations (habits of mind, meaning perspectives, mind-sets)—to make them more inclusive, discriminating, open, reflective, and emotionally able to change."
>
> (Mezirow, 2003)

Inspired by the work of Jack McNiff and Jean Whitehead, the researcher would also like to use Habermas' point "in thinking about the relationship between the generation and evaluation" of her "living theory and the empirical knowledge generated" through this research:

> "...practical reflection which critically appropriates this intuitive knowledge requires a social perspective that goes beyond the first person singular perspective of somebody acting on his preferences."
>
> (Habermas, 2002, p. 26)

6.2.1 Knowledge, Skills and Prior Experiences of Students

This section deals with answering the research question "What knowledge (mathematics), skills (mathematical thinking and problem solving) and prior experiences do students bring along that influence how they learn to think mathematically in a blended learning environment?

The answer to this question started with the quest of knowing the students, knowing what they know, what are their prior experiences related to mathematical thinking and problem solving.

To know the student, the researcher developed an open-ended survey with the help of Google forms. The responses are discussed next:

Knowing my Students (Section 4.8.1)

The main uses of maths in future are for problem solving, calculations and designing purposes. The main strengths of the students are in conceptual areas (e.g. algebra, averages, calculus etc.), computational skills, memory skills, solving problems and attitudinal (that is being confident and loving maths). The main weaknesses are also conceptual and that is prevalent in the findings. Other weaknesses are computational skills, solving problem, lack of time management and again comes out to be attitudinal (e.g. being careless and being lazy to think). Failing in Test is prominent as the emergent bad experience by students followed by failing to meet self-expectations, unable to solve math equations, and bad results. Getting A grade unexpectedly is reported as the first great experience, solving problems is the second and working as a facilitator is the third and answering questions unexpectedly is the fourth. Practice emerged as the major theme regarding "How I study Mathematics" followed by lectures, group study, and reading. Practice again emerged as the major theme of "How I prepare for exams in maths" followed by revision, teaching, memorizing, consulting teachers, and studying hard. Some students expected to get good scores, some found it hard, complicated, and difficult to excel in maths although enjoying and loving it. Exam related fears are dominant among students followed by the workload and negative self-concepts. Practice again emerged as the need of the students followed by the support from others and resources related to the course. Self-negative concepts like being careless, lazy and memory skills, resource management, forgetting the material, lack of focus in class and language barriers are some of the difficulties expected by the students. Nineteen students joined the engineering education with 4.0 GPA, 15 got GPA in the range of 3.90-3.99, 10 in the range of 3.80-3.89, 2 in the range of 3.42-3.46 whereas six did not record their grades. Thirty two students have no experience with MT, 10 students have only heard or read about it, eight have participated in many MT activities and one student has never heard about it and never used it. Thirty-six students have taken course that requires activities related to applying course material in the real world whereas 15 give negative response. More than 50% of the students prefer working in groups whereas the remaining percentage is found in favor of working individually. Their perception about the MTL was not very clear showing that MTL was a new learning experience for them. The major themes that emerged were "practice based system", "grade-based system," "fear of failing in exam", "procedural skills". The themes would also help the researcher to regroup them under the emergent theme of students' met-befores related to problem solving and mathematical thinking.

Knowing the current knowledge state (Section 4.8.2)

Only 56% (29 out of 52) students used the electronic resource of ALEKS and out of those only 7% (2 out of 29) were able to show sufficient knowledge base whereas the rest of the students possessed insufficient knowledge base at the time of joining the engineering education. This was one of the reason why simple concepts like exponentials and averages were selected for the MTL.

Initial Diagnosis (Section 4.8.3)

Building upon the data collected relating to the students' prior experience with mathematics, the researcher inferred that very few students were able to think mathematically, even doing a simple task related to function. Only two groups out of 13 managed to attain level six according to Table 4.29. They were able to select the appropriate functions as well as describe the relationship between the independent and dependent variables. They were also able to identify the domain and range of the function under investigation through symbolic manipulation. Five groups managed to select the appropriate functions but were unable to identify the domain and range of the functions. So they are ranked at Level 4. The rest of the six groups were at level zero and have shown very low performance showing the need to intervene with their conceptual accommodation.

After knowing the students, their knowledge base and prior experiences relating to mathematical thinking, learning and problems solving helped the researcher to develop and implement a blended learning environment.

6.2.2 Development and Implementation of Blended Learning Environment

This section deals with answering the research question "What would be the process to develop, and implement a blended learning environment that incorporates a well-practiced problem solving strategy and a pedagogical tool supporting engineering students' mathematical thinking, learning, and problem solving skills?" This research question is thoroughly addressed in Chapters 4. Figure 4.1 showed the relationship of the development and implementation of a blended learning environment during the preliminary and pilot study. Here the researcher will evaluate the implemented blended learning environment.

During the quest of discovering the theoretical and conceptual underpinning of the findings of this study, the researcher tried to make sense of the emergent themes first by using "How People Learn" meta-framework, followed by making sense of the pre-identified themes based on "How humans learn to think mathematically" by David Tall (2013). As "we are dealing with complex and voluminous data, diagrams can help the researcher disentangle the threads of our analysis and present results in a coherent and intelligible form, it is not just a way of decorating our conclusions, they also provide a way of reaching them" (Dey, 1993, p. 201).

To start with, the underlying theories are reviewed during the process of mapping this research to the How People Learn (HPL) model known as meta-framework for instructional design defined by Bransford et al., (1999) and its four main components are shown in Figure 6.1. The theories related to student-centered, knowledge-centered, assessment-centered and community-centered frameworks (Marilla, 2008) are shown in Figures 6.2 to 6.5. The shaded nodes with grey color show the relevance of theories with the research findings. The findings are at the bottom level, and are traced back to get them in place with higher order theories.

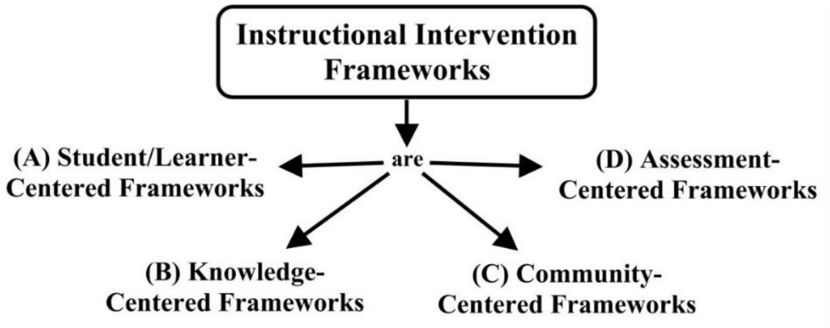


Figure 6.1: Meta Framework of HPL showing the related theories used for all types of instruction intervention frameworks

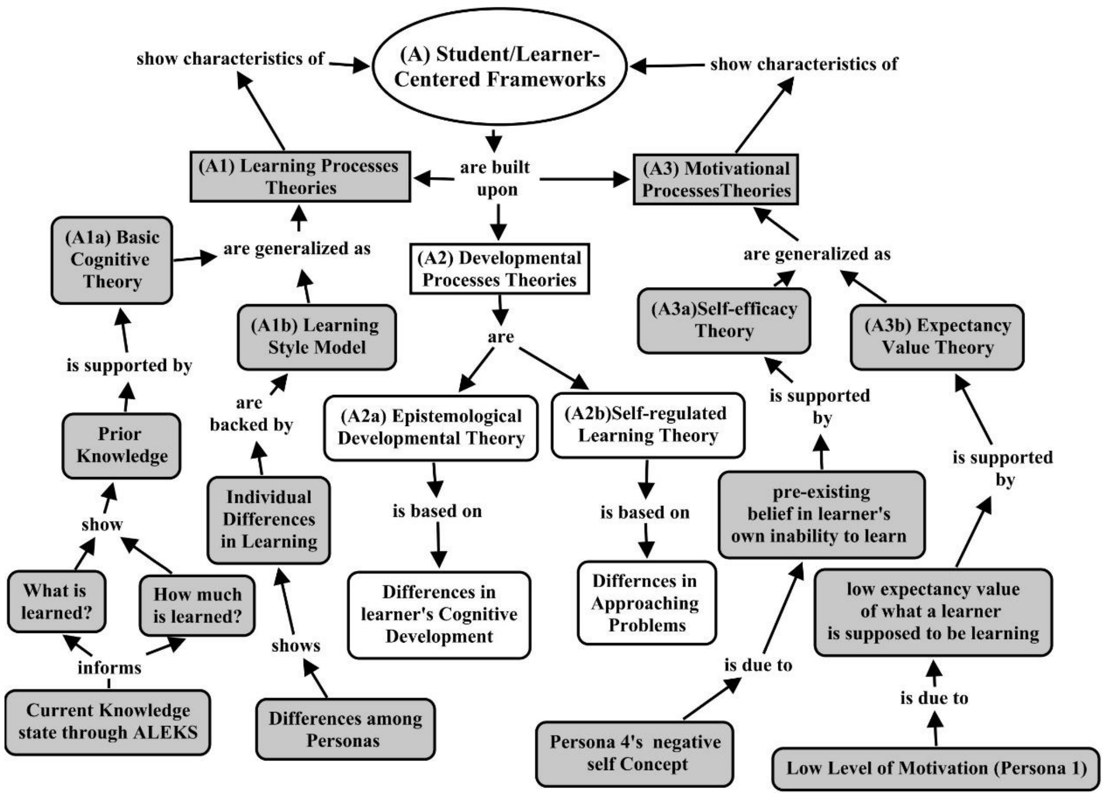


Figure 6.2: Relevance with Student/Learner-Centered Frameworks and related theories

To meet the first condition of HPL model of "learner-centeredness," the researcher tried to select activities and pre-conditions in a way to focus on the needs, objectives, interests and goals of the learners. The researcher concentrated on understanding what students know already, how they are different in their learning approaches, and provided a strategic guidance for them to understand their learning challenges. To achieve the quality in the intervention, the researcher put our maximum effort in understanding who the students are, learning what they enjoy to learn, learning about their skills, goals, and trying to align our practice accordingly. Blended learning also provided a room to conduct problem solving activities in such a way that provoked the students' responsibility, their understanding of their struggles in a new learning environment and their sense of ownership. Most importantly, the researcher discovered that the less-represented and mostly wrongly-labelled students like Ismail got a chance to express their knowledge in the world of sense making. This study helped those students to unleash their hidden strengths and bring them in front of others to give value to their sense making skills.

The concept map in Figure 6.2 shows that how knowing the current knowledge state of students through ALEKS and knowing the differences among different personas are supported by "Basic Cognitive Theory" and "Learning Style Model" respectively. In addition, knowing Ismail's negative self-concept that "He is not good in maths and cannot do very well" is supported by self-efficacy theory whereas low level of motivation of Zain can be linked back to "Expectancy Value Theory" as he was unable to give value to the new way of learning.

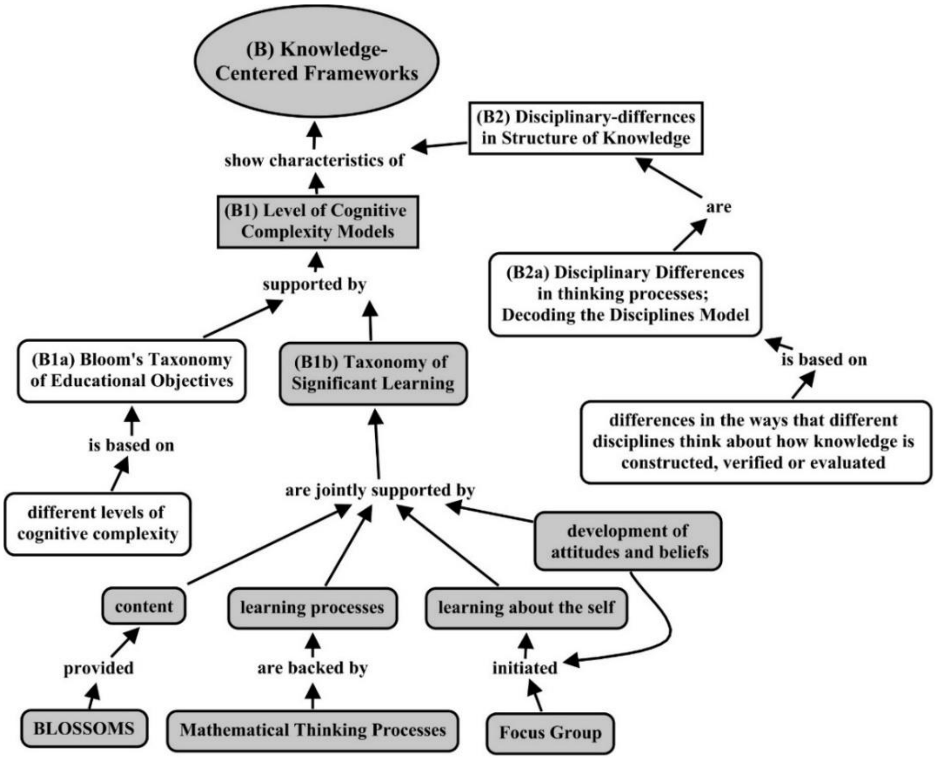


Figure 6.3: Relevance with Knowledge Centered Frameworks and related theories

To meet the condition of "Knowledge-Centeredness," the blended learning in this study was designed to get the maximum benefit of BLOSSOMS as online component as well as Masons' problem solving approach to be used in the face-to-face setting with the help of worksheets. The design of blended learning was guided by what knowledge first year engineering students bring along into the new learning environment. It also fulfils the need of knowledge-centeredness by focusing on mathematical thinking and learning processes rather than just memorizing formulas, equations and doing procedural mathematics. The activities were aligned with the need to engage students with knowledge building rather than just getting good grades. During focus groups, the researcher tried to link the idea of mathematical thinking and problem solving in relation with their future workplace needs and demands (Bransford et al., 2000, p. 139). This study also tries to create a sense of meaningfulness for first year engineering students and to promote mathematical thinking and learning by understanding. The concept map in Figure 6.3 shows how "Taxonomy of Significant Learning" jointly supports BLOSSOMS modules, teaching mathematical thinking processes, and creating meaningfulness through focus group.

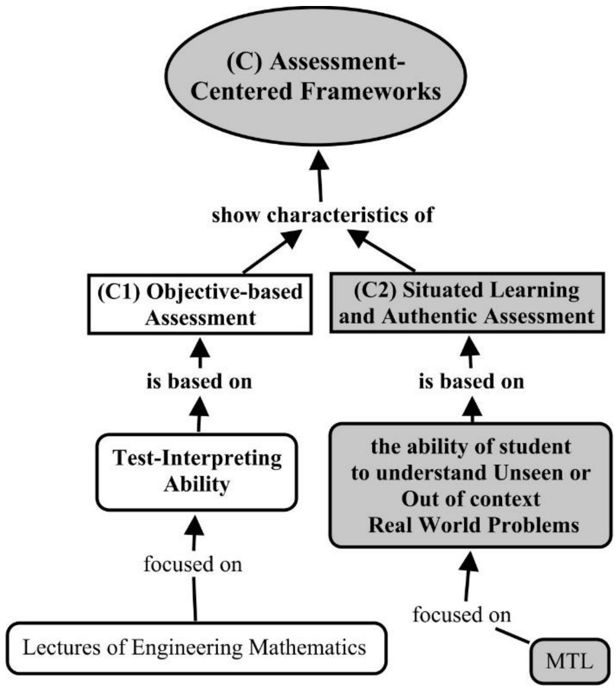


Figure 6.4: Relevance with Assessment Centered Framework and related theories

To meet the goal of "Assessment-Centeredness", this study helped students make their thinking visible by verbalizing and recording their written responses on the worksheets so that the researcher can provide feedback and review her own practice of teaching in return. This study also used ALEKS as an online component to assess the current knowledge state of the students and it turned out to be an appropriate decision for gauging the current knowledge state of students. The verbal responses during the problem solving sessions also contributed towards assessing the students' level of participation and interest. Although this research was unable to internalize a mechanism of peer-assessment and self-assessment, that would serve as a potential future research venue to further explore this aspect. The concept map in Figure 6.4 shows that the Mathematical Thinking Lab (MTL) focused on the ability of the students to understand unseen and out of context real world problem and that in turn is supported by "Situated Learning and Authentic Assessment" theory.

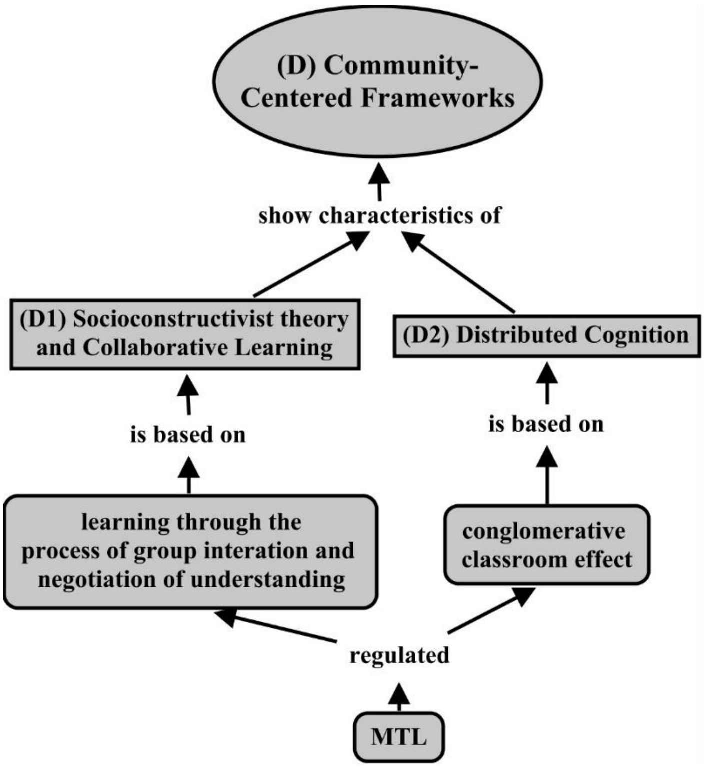


Figure 6.5: Relevance with Community Centered Framework and related theories

To meet the goal of "Community Centered," this research tried to promote the development of a sense of collaboration, connectedness and safety by integrating BLOSSOMS modules with Mason's problem solving strategy. The concept map in Figure 7.5 shows that the MTL successfully regulated the learning through the process of group interaction and negotiation of understanding supported by "Socioconstructivist theory and collaborative theory" and the "conglomerative" classroom effect supported by "Distributed Cognition" respectively.

The HPL meta-framework affirms that the blended learning environment in this research is simultaneously learner-centered, knowledge-centered, assessment-centered, and community-centered.

This interpretive research shows that blended learning also supports the need to emphasize on methodical problem solving (Polya, 1962, pp. xi–xii). The constructive teaching in blended learning matches the inquiry-based inductive teaching and learning (Prince and Felder, 2006). Situated Cognitive Theory (SCT) (Moreno, 2010, p. 273) legitimates the use of blended learning because it supports the students' context of learning and plays a fundamental role in activating mathematical thinking processes.

6.2.3 Emergent Themes

This section will try to answer the research question "What are the emergent themes translating into the implications of blended learning on students' mathematical thinking and learning during problem solving activities?" The first two themes that emerged as a result in categorization of emerging factors is "student met-befores" and "challenges whilst problem solving." Based on the description of data display (Sections 5.2.1), the next paragraphs will discuss them based on Tall's framework of "How humans learn to think mathematically" and exploring the conceptual and symbolic worlds of mathematics.

Students' met-befores

The development of mathematical thinking of first year engineering students depends profoundly on three set-befores "recognition", "repetition" and "language" (Tall, 2008). Building up knowledge based on these three set-befores, the students' met-befores were inconsistent with a newly met situation of blended learning conducive to mathematical thinking and problem solving. All the inconsistent met-befores cause mental confusion that hinders the learning process of thinking mathematically (Tall, 2008).

The students' met-befores in terms of their "habits of mind" are based on their prior experiences. Their passive attitudes towards asking questions in class even if they do not understand the concepts restrict their ability to build new knowledge. Their habit of doing individual assignment was also inconsistent with the new way of collaborative group work. Their habit of drill and practice was strongly associated with mathematics and thus turns out be inconsistent with learning the process of mathematical thinking. The didactic teaching and teacher-centered learning without focusing on different learning style with a few instances of exposing students to mathematical thinking and problem solving strategies restrict their in-born ability to think mathematically in class. Grade-based system is prevalent reinforcing the fear of failures in exam and focusing on short-term goals like getting good grades in exam thus promoting the "surface learning" and do not go along with the meaningfulness of what they are doing in mathematics. Students rarely get constructive feedback on their wrong or incomplete solutions and that contributed towards the misconceptions related to mathematical concepts. The classroom conversations were mostly carried out in their national language (Malay) and they did not feel comfortable to communicate in English language and this was again served as an inconsistent met-before and thus shake their confidence to participate in classroom discussions and their interaction with an international facilitator.

Student's met-afters in comparison with their met-befores

In this section, the researcher will try to discuss the students' met-afters during the blended learning in comparison with their met-befores.

In this research, the researcher investigated the key met-befores related to mathematical thinking, learning, and problem solving as discussed in the previous sub-section. Most of the met-befores were found to be conflicting whereas a few were found to be supportive especially in the case of diploma students. In the following table, the researcher illustrated the impact of blended learning on the students' met-befores and transformed them into met-afters as listed in Table 6.1.

The researcher also found that students with matriculation and foundation in engineering background struggled more in making sense of new learning experience during the MTL and it was difficult for them to change their “habits of mind”, meaning perspectives and mind-sets (Mezirow, 2003), whereas the students with diploma with some work experience enjoyed the sessions informing that the sense-making experience during the MTL was not totally strange for them.

Table 6.1: Students’ met-befores and Student’s met-afters

Student's Met-befores	**Student's Met-afters**
Passive in Asking Questions	Active in asking Questions
Individual Task Assignments	Group Activities
Drill and Practice	Sense-Making
Teacher-Centered Learning	Student-Centered Learning
Didactic Teaching	Constructivist Teaching
Traditional Learning Styles	Duet Teaching and multiple Learning Styles
Minimal Exposure to Mathematical Thinking	Enhanced Exposure to Mathematical Thinking
Grade-Based System	Participation-Based System
Rare Feedback on Work	Frequent Feedback on Work
Problem Solving Strategies	Problem Solving Strategies
Pressure of examination failure	Enjoying the activities
Focusing Short Term Goals	Focusing long Term Goals
Language Barrier	Language Freedom

Students’ diligence during problem solving

Students’ diligence during problem solving activities through blended learning was enhanced as reported in Table 5.3. The higher level of attentiveness is coupled with the students’ high engagement with BLOSSOMS videos and collaborative teamwork. The researcher encouraged them to participate in the MTL by asking question, and supporting their persistent efforts in problem solving contributed toward increasing diligence during problem solving. The main constituent of the diligence is sensed to be the students' engagement so it will be discussed in more detail in the following paragraph.

Student's Engagement

"To teach is to engage students in learning."

(Christensen and Garvin, 1991)

The students were engaged with BLOSSOMS modules serving as pedagogical tools by showing diligence during mathematical problem solving (Section 5.2.3.2). That accompanies the student attentiveness, classroom engagement, teamwork, asking questions, persistence, and ability to think mathematically. Out of five benchmarks of pedagogies of engagement during the classroom-based practices that are active and collaborative learning, student-faculty interaction, enriching educational experiences, level of academic challenge, and supportive campus environment (Smith et al., 2005), this study successfully internalizes the first two as discussed below.

Active and collaborative learning helped students to improve their diligence during problem solving activities (Section 5.2.3.2), which was enhanced by student attentiveness, classroom engagement, teamwork, asking questions, persistence and ability to think mathematically.

Student-Teacher Relationship

Student-teacher relationship (Section 5.2.3.3) is improved by friendly relationships, democratic classrooms, empowering students, scaffolding, acknowledgements, empathizing, providing support, and listening and thus fulfils one of the five benchmarks of pedagogies of engagement that is "student-faculty interaction" (Smith et al., 2005) during the MTL.

Challenges Whilst Problem Solving

Challenges whilst problem solving are concluded after analyzing the data for 52 students. Based on the data listed in Table 5.5, the emergent challenges whilst problem solving were "lack of confidence", "lack of intrinsic motivation", "lack of perseverance", "lack of retention", "different levels of participation", "aversion to word problems", "eagerness for formula", "different speeds in computations and thinking", "disparity in prior knowledge", "lack of resource management". Knowing these challenges led the researcher to address those challenges by narrow downing our focus on some interesting and unusual student cases. The next section will discuss the development of student personas aligned with the same purpose as above.

6.2.4 Development of Students' Personas

This section will address the research question "What would be the process to develop students' personas to describe archetype students in different scenarios (the Classroom and the MTL) and illustrate the activation of their mathematical thinking processes in embodied and symbolic world of mathematics?"

Student Personas

All the four stages of persona development process adapted from Lene Nielsen (2013) were described in Section 5.3. The outcome of the first two stages of the persona development process are reported in Sections 5.3.1 to 5.3.8, the third stage is discussed in Section 5.3.9 by providing different situations for creating scenarios. The situation-based scenarios are created followed by the identification of the challenges and the idea development for their solutions in this section. Stage 4 is also partially employed during this research by involving the participants of this research for member checking through emails. The researcher planned to conduct a series of workshops to introduce personas to peer researchers in the near future and showed that this research opened up new venues for potential future research.

Personas are primarily developed to

- provide channels for conveying a broad range of research data related to classroom innovation to foster mathematical thinking during problem solving.

- help educators teaching Engineering Mathematics to identify the implications of this research on students for their own students.
- help educators overcome/avoid student stereotypes or assumptions in mathematics.
- help educators develop more empathy for student experiences during classroom and MTL sessions.

We discovered one of the issues related to student persona is similar to what is described below:

> "At this stage, students with very weak backgrounds typically do much better than those with more mathematical studies behind them. It's easier to learn when you start with a blank slate than when you first have to unlearn -- or at the very least adjust -- prior learning that was incomplete. The problem for most people is undoing educational damage done earlier. Those who were "good at math" at school often have the hardest time of all."
>
> (Keith Devlin, MOOC Communication, October 21, 2014)

However, this research went into depth of the issues and discovered that it is not true for all the good graders.

Future Research Direction related to using Personas as Pedagogical tools

The researcher planned to replicate the online workshop "Identifying Implications: Using Personas to Bridge the Gap between Research Findings and the Design of Educational Experiences" in her RU by using the related material with the permission of Jim Borgford-Parnell, Jennifer Turns, & Toni Ferro (Jim Borgford-Parnell, Personal communication, September 29, 2014). The participants will either be post-graduate students or educators. The researcher will use evidence-based personas developed in this research for the workshops and ask the participants to respond to the followings:

- Highlight pertinent (relevant) characteristics of your persona
- Identify pertinent (relevant) challenges of your persona

- Identify who, in engineering education, might benefit from knowing your persona’s situation
- Identify a decision of one of those people that would help to address your persona’s challenge (s)

To develop a baseline for the above future work, the researcher tabulated her own suggested decisions to address the pertinent challenges of the personas in specific scenarios. That compilation is provided in Table 6.2 and its further refinement through future research will help practitioners to address the similar persona challenge(s) in their own classrooms. This future research will unfold the potential of student personas as a pedagogical tool to communicate the research findings to the Community of Practice (CoP). It would also help the future research to answer, “How can the design of learning environments and curricula capitalize on diverse learning styles and mathematical thinking skills?

Table 6.2: Situations, Scenarios, Pertinent Challenges and Decisions to address the Pertinent Challenges

Persona	Situations	Scenarios	Pertinent Challenges	Decisions to Address the Pertinent Challenges
Zain	A, B, C,D	Although Zain scored highest in the final exam, he found it difficult to make sense of problem solving during the MTL sessions, thus ended up showing the lowest performance in the MTL.	Lack of Self Actualization Cognitive rigidity No evidence of Knowledge base	Cueing Using Differentiated Instruction Apprenticeship
	Classroom	Zain never sits at the front. He use to distract his friends by making noises (humming). It seems like he is not interested in his study	Inactive listening	Differential Reinforcement
	MTL	He found the activities difficult and asked the instructor to slow down the pace of teaching.	Suppose he has strong procedural mathematical skills but those did not work in mathematical thinking Cognitive rigidity Inconsistent "Habit of Mind"	Inconsistent "Habit of Mind" Conceptual Change
Chen	A, B, C,D	Chen adopted practice-based approach in mathematics. He is always looking for ready-made solutions. He became prominent when he tried to provide an online solution of a problem solving activity but when the instructor asked him to explain, he was unable to do so	Strong Procedural Knowledge Base worked in class but is not working in MTL Well-practice Schema for solving mathematical problems	Conceptual Change
Abdullah	A, B, C,D	Abdullah proved to be a persistent performer throughout the semester even if he did not manage to get the highest marks in the final exam. Although he participated in the MTL,did not enjoy attending it.	Struggling with Conceptual accommodation during MTL Cognitive rigidity	Conceptual Change
Ismail	A, B, C,D	Ismail is gravitated towards making sense of the world in conceptual embodiment during MTL but he failed in Engineering Mathematics with alarmingly low grades	Conceptual Assimilation worked in MTL but not in tradition Classroom, Conflicting Met-befores (Negative Transfer) in Class but Supportive Met-befores (Positive transfer) in MTL, Negative Self-concept Low self-esteem(Self-worth)	Apprenticeship Meaningful learning in the classroom Relatedness Self-paced Learning Scaffolding

Table 6.2: continued

Persona	Situations	Scenario	Pertinent Challenges	Decisions to Address the Pertinent Challenges
Fatima	A, B, C,D	Fatima is serious, attentive and quiet in class but did not enjoy the MTL sessions	Schema Activation Failed	Continuous reinforcement
Faaiz	A, B, C,D	Faaiz is a persistent performer and showed cooperation in traditional class as well as in MTL	Conceptual Flexibility (Positive)	Enhancing Dispositional Interest
Sunny	A, B, C,D	Sunny is quiet in class but showed good communication skills during focus group	Suffering from Cognitive Overload Conflicting Met-befores (Negative Transfer)	Continuous reinforcement Apprenticeship Meaningful learning in the classroom Relatedness Self-paced Learning Scaffolding
Fahmi	A, B, C,D	Fahmi likes to work independently and loves to teach others	Cognitive rigidity Struggling with Conceptual accommodation during MTL	Conceptual Change

6.2.5 Activation of Mathematical thinking Processes during Problem Solving

In comprehending the summary of activated mathematical thinking processes as shown in Figures 5.16 and 5.17, the researcher claims that blended learning supports the activation of mathematical thinking processes in conceptual embodiment, blended embodiment and symbolism and operational symbolism. The activation of different processes however supports only “horizontal mathematization that leads from the world of life to the world of symbols. In the world of life one lives, acts (and suffers)” (Freudenthal, 2002, p. 41) and blended learning in this study satisfies Freudenthal’s principle of “choosing learning situations within the learner’s current reality, appropriate for horizontal mathematising” (Freudenthal, 2002, p. 56).

6.3 Making Sense of Researcher’s Reflective Practice

A major contribution in reflection practice of the researcher was dependent on her reflective thinking.

> “Reflective thinking is always more or less troublesome because it involves overcoming the inertia that inclines one to accept suggestions at their face value; it involves willingness to endure a condition of mental unrest.”
>
> (Dewey, 1910, p. 13)

The researcher’s thinking was the result of observing her students and the consequences of the students’ behaviors both in the classroom and in the MTL. That learning is supported by Sociocognitive Theory (Moreno, 2010, p. 592).

Some of the sense-making experiences of the researcher’s reflective practice are described below:

> “Fifteen years ago, I started teaching as a positivist practitioner who tried to figure out the problems that students face, fix or eliminate those problems. Now I realize as an engineering educator that I do not want the students' problems to simply go away. I want students to grapple

with problems that are worth grappling with, and I want the rest of them maybe disappearing into the background. So we can still appreciate them provided we can turn them into learning opportunities for our students.”

(Excerpt from researcher’s reflective journal on January 18, 2014)

“During my quest to find resolutions of all my teaching issues and the associated tensions, I myself struggled with my own perception about teaching, learning, and thinking. My focus needed to have a shift from “becoming a good teacher” without understanding “why my students are doing what they are doing” to “improve my practice” by learning about my students’ learning and researching about all the associated “whys.” As the time passes by, I realized that I need to rethink critically about my philosophical assumptions and to reengineer the way I have been practicing in the past.”

(Excerpt from researcher’s reflective journal on June 24, 2013)

Before going into a deeper discussion, the researcher shared her initial expectations from her reflective practice and then the later experiences she had:

After reading the book “You and your action research project” written by Jack and Jean (2010) and knowing that “action research has become a recognised practice internationally” and the “interest has exploded around the world in the idea of practitioners studying their own practices”, I felt confident about the way I planned my research. My reflective practice during this entire study should help me to address the need for practitioners “to give explanations for their practices, which is a process of theorising, and show how they hold themselves accountable for what they are doing.”

(Excerpt from researcher’s reflective journal on August 12, 2013)

“Adhering to my earlier stance on reflective practice, I have experienced that action research offered me a way forward, by accentuating the need for lifelong learning about how to improve my own practice, how to refine my capabilities, and how to realize my

> potential contributions to the emergent needs and demands of the engineering education. In this study, the action research is not only cased in the mould of professional development but also for theory generation by presenting my own cogent explanations for my practice (McNiff and Whitehead, 2010)."
>
> (Excerpt from researcher's reflective journal on December 29, 2013)

During this research, the researcher also developed a habit of asking herself "why?" every time she heard the word "should" for example, you should take a situated approach, "why?" You should use this particular methodology, "why?" You should keep a reflexivity journal when you are doing qualitative research, "why?" You should have to do this step in this particular way "why?" After getting out of the perplexing situation and having got all the answers of "why?" she used to resume her attention towards "should". She described her experience during the main study as:

> "It is interesting watching the first year engineering students in MTL who were actually having problems that were very useful for their learning. Although I am trying to provide support for them to think through those problems or to understand or appreciate what they were learning from those problems. I also tried to make them realize that "being stuck" is not a bad state during problem solving. It becomes a sticky issue in my head trying to figure out how to navigate between this paradigm of finding student's problems and fixing them or students having problems being a chance to learn more."
>
> (Excerpt from researcher's reflective journal on October 1, 2013)

The sense making of reflective practice during this research is further refined after getting involved with an online project. Through that project, the researcher got a chance to interview a pioneer in engineering education research who helped the researcher to comprehend that reflective practice is incomplete without making sense of it and for that you need someone from the community of practice (CoP) to listen to your reflections, to rephrase whatever you have said and to help you in uncovering the hidden meanings of what you have experienced. After that exposure, the researcher understood what reflection really means and how to reflect on experiences, it is about this intentional going back and looking at what you've done and making sense and meaning out of it. The interview turned into an interesting conversation and the expert started asking questions to the researcher about her research and teaching practice. Some of the excerpts from the conversation between EER Expert (E) and the researcher (R) on April 4, 2014 are recorded in Appendix N to illustrate how the sense making phenomena occurred in this research.

In the end, the researcher is still left with new questions ignited from her quest and are supported by the experiences of Jack and Jean as below:

> "It does not matter if the social situation does not reach successful closure; it probably will not because any solution allows new questions to emerge. What matters is that you show your own processes of learning and explain how your learning has helped you develop your work within the situation."
>
> (McNiff and Whitehead, 2010, p. 19)

During the journey from a positivist practitioner to become an interpretive, reflective engineering educator, the researcher has realized that it is difficult to compartmentalize the two stances strictly. The already drawn boundary lines between both the worldviews are just lines on the sand and can easily be washed away and redrawn based upon the researcher's perceptions of what is "valuable" and "useful". The researcher thus, felt gravitated more towards becoming "Critical Praxis Researcher" who breathe in deeply in her world of academia and learn to survive as a researcher without the aid of positivist contrivance (Kress, 2011b, p. 61).

The development of the researcher's self-efficacy through this research was accompanied by her perplexing experiences throughout the journey. She gained her efficacy perceptions through her pattern of past successes and failures in practice (personal performance and accomplishments), by comparing herself to the effective peer practitioners (vicarious learning), through receiving encouragement or discouragement from others (social persuasion), by dealing with pleasant and unpleasant physical and emotional reactions (physiological states and reactions). The above four sources acted interactively and simultaneously in the development of her perception of self-efficacy as believed by Bandura resulting in three behavioral consequences backed by Bandura (1977) as described by Fantz, Siller, & Demiranda (2011):

1. What she is willing to try and from what she stays away (Approach versus avoidance)
2. The ability demonstrated to translate her previously learned skills into new situation of engineering education (Performance)
3. Not giving up to make a positive change in engineering education (Persistence)

6.4 Challenges faced during the Study

Some of the challenges faced by the researcher during this entire research are tabulated below:

Challenges Faced	**Description**
Time	25% of total time was allocated for MTL that was not enough to complete a module in one session
Space	Although computer labs were better than traditional classrooms, both classrooms and computer lab were not the ideal places to conduct group activities.
Language barrier	Researcher is non-native Malay speaker
Balancing between Online and f2f	Researcher has to struggle to maintain a balance to deliver the best of both online and f2f component of blended Learning
Low contribution in grading (5%)	Final grades of engineering mathematics do not reflect the mathematical thinking skills of students
Uneven workload during the semester	some weeks were non-productive due to the heavy work-load of other subjects taken by the students

6.5 Limitations and Delimitations of the Study

The limitations of this study are listed next:

1. The research was limited to first year engineering students for which permission to conduct research could be readily obtained at the time of the study.

2. The timing of the mathematical thinking lab sessions for problem solving activities and interviews was constrained by considerations for the engineering program, parallel course workload, and day- to-day considerations of the students' needs.
3. The use of personal achievement data was limited by considerations of confidentiality.

Some of the delimitations of the study are listed below:

1. The research was delimited to the researcher's own university having support from her supervisors, some teaching staff, and research assistants who were interested in introducing innovative methods in the classroom. This was intended to provide a rich situational context for the study.
2. The study was restricted to a research and engineering university in the local area to reduce the time involved in travelling for the researcher who was anticipating prolonged engagement in the field for the current study and out of consideration for the electronic equipment involved, which had to be transported daily to and from the sites in tropical climatic conditions.

The following section will describe the future recommendations based on this research.

6.6 Summary of this chapter

This concluding chapter consists of discussions about the research questions and their investigated answers so that the researcher's contribution can be located in relation to the research gaps (Section 1.3). After justifying the claims to knowledge, validating them, and legitimatizing this research, the researcher explains the sense making process of her own reflective practice. The researcher then concluded the thesis along with research implications and future recommendations in the next chapter.

CHAPTER 7

CONCLUSIONS AND FUTURE RECOMMENDATIONS

In this final chapter, the conclusions will be drawn followed by the implications, and future recommendations based on this study.

7.1 Conclusions

The descriptive style of the thesis and the reduced data displays (using tables and concept maps), the researcher's effort to capture insurmountable research situation within surmountable research interpretations affirms her declarative knowledge during this study. Her reflections showed how she has drawn inspirations from social, psychological, and practical perspectives in her practice to generate her own living theories. She elaborated the reasons for her actions and their significance as part of the explanations in terms of her values. Her values are reflected in all the discussions, conclusions, and future recommendations in this concluding chapter of her thesis.

The researcher has drawn all the conclusions by completing the two staged analysis as outlined in Figure 3.9. Stage II data analysis and results showed how blended learning supports the activation of mathematical thinking processes during context-rich problem solving activities. It contains features like student's met-befores related to learning mathematics, student's met-afters through blended learning, and student personas to give voice to the unheard stories, impoved problem solving diligence and student-teacher relationship. This research will influence future researchers in two ways, first to inform the over-looked factors in the design and implementation of a blended learning environment supporting mathematical thinking and second to prepare practitioners for the potential challenges and hindrances while conducting action research.

It is evident from the data analysis, findings and discussions that the integration of BLOSSOMS modules with Mason's problem solving strategy in the form of activity-based worksheets helped the researcher to show the activated mathematical thinking processes during problem solving activities through blended learning. The adapted and reviewed rubric to analyze the activity responses would not only serve as a need for the response analysis but can also be used as a great resource to develop new activities. A generalized template of the worksheet is developed so that researchers can transform it to be used for any problem solving activity. The flexibility of the worksheet, rubric and coding guide would serve as a supporting toolkit to effectively implement the BLOSSOMS modules or other Open Educational Resources (OERs) in first year engineering mathematics classroom. Online MTL sessions can also be developed by using the worksheet's template and by embedding the rubric within each activity.

Mason's problem solving strategy activates the mathematical thinking processes whereas this activation is also dependent on the problems being posed through the BLOSSOMS module. The context is presented in the form of stories to pose mathematical problem to students. This also served as the online (asynchronous) component of blended learning. The pedagogy of duet teaching facilitated the transfer of control to the class teacher to scaffold the problem solving process through prompts and questions and to make sense of the problem presented. Worksheets played a role of medium or a tool to capture the students' mathematical thinking and problem solving process and are the cheap alternative to other means of capturing mathematical thinking and problem solving processes. The improvement in the development of worksheets was recursive and iterative after every action research cycle and the physical form can also vary form hard copy to web based worksheets.

To understand how engineering students learn to think mathematically through blended learning, emerging personas helped the researcher as tools to improve the design and implementation of action research. This rigorous study of investigating pragmatic implications of blended learning for mathematical thinking will help to transfer important research findings into the minds and hands of engineering and mathematics educators and action researchers. This research has transformative impact on the researcher in shifting her epistemological worldview from positivist to interpretivist. This research also gives voice to the unheard stories about the experiences and lack of support system for under-represented students.

Evidence-based student personas are developed as potential pedagogical tools to provide channels for conveying a broad range of research data of this study, help educators identify the implications for research on students to make decisions for their own students, help educators overcome/avoid student stereotypes or assumptions, help educators develop more empathy for student experiences.

This research is about improving practice through

- improving practitioner's learning of focused area of research (mathematical thinking processes, problem solving strategies, blended learning environment),
- improving contextual knowledge (first year, first semester engineering students entering the engineering program),
- improving pedagogical knowledge (how to use blended learning by manipulating its multiple dimensions),
- improving knowledge about learners (who they are, what is their current knowledge state, what are their met-befores related to mathematical thinking and problem solving, their believes and attitudes towards learning in new ways)
- articulating the reasons of change employed and
- developing student personas as potential pedagogical tools to highlight the potential significance of this research and its contribution towards CoP to help them find better ways to make academic change happen.
- making sense of her own reflective practice.

The outcome of this research are successful manipulation of conflicting met-befores into supportive met-afters, improved student retention, student-teacher relationship, improved pedagogy, evidences of active “horizontal mathematization” to improve students’ mathematical thinking skills, improved students’ problem solving skills and the obvious change in habit of mind of the researcher in becoming an influencer rather than a transmitter of knowledge for her students. Another outcome is the evidence based student personas serving as a potential pedagogical tool to help the researcher communicate implications of this research on first year engineering students to the engineering faculty, mathematics faculty, and other engineering students.

This research also informs future action researcher to consider the following insights before designing the classroom instruction for engineering students to thinking mathematically:

- Learning to think mathematically is longitudinal and time consuming
- Learning to think mathematically is context-based.
- Learning to think mathematically is subjective.
- Learning to think mathematically is impacted by space where teaching and learning occurs.
- Learning to think mathematically is based on sociological needs of the students.
- Learning to think mathematically is based on psychological needs of the students
- Learning to think mathematically can be made effective through motivating the students intrinsically.

Hence, learning to think mathematically is based on the factors like where, what, whom, for how long, and how we teach.

The next section will describe the implications of this study.

7.2 Implications of the Study

Keeping in view the gap between the research of how classroom innovation helps students to think mathematically and current practices in engineering mathematics education, this action research focused on teaching context-rich problem-solving strategy and making mathematical thinking explicit in a blended learning environment to promote mathematical thinking skills among future engineers. The researcher kept her focus on capturing the students' experiences related to learning how to think mathematically in a new learning environment. To address the issue of handling big data and reporting the findings, she used emergent personas. She anticipated that these personas can serve as potential pedagogical tools to present the research findings to the Community of Practice (CoP) in an effective way but that requires further research. This rigorous qualitative study of promoting, cultivating, and integrating mathematical thinking in engineering mathematics will help to transfer important research findings into the minds and hands of those who can make the best use of them – the engineering as well as mathematics faculty, staff, and students. She used an interpretive action research performing multiple iterations (Preliminary Cycle, Pilot Cycles 1 and 2, Main Cycles 1 and 2), ongoing interpretive analysis guided by her reflective journals. She proved that action research can be used as a potential methodology for doctorate study and thus can have a transformational impact on future engineering educators. This research also has the potential to be transformative by informing how learning and teaching trajectories of both teacher and student influence current practices. The outcome of this research is the thick descriptions of the learning and teaching experiences that can bridge the gap between research and practice in the respective domain. The transferability of this qualitative research has the potential to be adapted by local and global engineering educators who want to improve their practices by exploring the emergent themes and by constructing knowledge within their respective domains. This research also has the potential to influence P-12 (secondary) engineering education by reporting the current state of insufficient mathematical thinking and problem solving skills of the students exiting this pipeline and facing difficulties during the transition to engineering education. This research would also provide a constructive feedback that would help the P12 (secondary) engineering educators to revamp the mathematics instruction ensuring students to enter in engineering programs with adequate mathematical thinking skill set.

The next section will describe the future recommendations based on this study.

7.3 Future Recommendations

Based on the prolonged involvement on the research setting, the context of this research and the findings of this research, the researcher proposes the following recommendations to further improve the practice and research in the future:

7.3.1 Redesign the mathematical thinking experience

The results of this action research demand to redesign the mathematical thinking experience for first year engineering students before or during the learning of advanced concepts in Engineering Mathematics I.

7.3.2 Make small changes to see high impacts

We can change the overall quality of the instruction by making small changes in the classroom like changing sitting arrangements, improving classroom atmosphere, redesigning the white boards for students' responses in the class, using microphone facility for the instructor, using tablet pc to display the teacher's work and encouraging students to ask questions.

7.3.3 Cooperative learning

Mathematical Thinking based tasks are usually time consuming and demand brainstorming. Cooperative learning would help to cover the intended content by keeping the students involved in constructing sense for newly met concepts and experiences in small groups. Supporting attitudes towards working in teams and mathematical thinking based problem solving activities should be part of the teaching and learning culture.

7.3.4 Learning by Doing

Mathematical thinking can be fostered by incorporating experiential learning with the help of manipulatives for the designed activities.

7.3.5 Different Students and Different Motivations

People learn differently and get their motivation differently in different conditions so introducing multiple teaching styles and blending conditions can cover a broader range of students.

7.3.6 Conceptual Accommodation

Before starting the advanced topics, the teaching team needs to devise some methods for the students' conceptual accommodation. Online adaptive educational resources like ALEKS can be used to allow the students to rebuild their concepts where as teachers can review the performance and get guidance for redesigning their instruction using the online tracking of students' progress.

7.3.7 Asking probing questions, but in a supportive way

During the teaching practices, try to understand what interests your students and then ask a lot of questions to make connections of their interests to their learning experiences. (Jennifer Turns, Personal Communication, April 4, 2014)

7.3.8 Helping students to make sense of their reflections

Reflective practices can be effective in mathematics classes if followed by sense-making of students' reflections. Hence, the instructor should also contribute as a mentor to help students make sense of their reflective practices during and after the mathematical thinking and learning activities. (Jennifer Turns, Personal Communication, April 4, 2014)

7.3.9 Encouraging students to build portfolios

Encourage students to build portfolios and teach them how to integrate their knowledge in them. When they build portfolios, they will grapple with issues of identity and will be surprised that certain knowledge like team working or being empathetic could actually be externalized and made available to others. (Personal Communication, Jennifer Turns, April 4, 2014)

7.3.10 Engaging students

Engagement of fresh engineering students can be improved by involving them in different activities like letting them record their ideas on white boards, scaffolding them to do tasks and construct new examples, teaching them how to make sense mathematically, getting them involved in real world problem solving, helping them aware of their own mathematical thinking powers, letting them persevere with the contextualized problems, allowing them to reflect on their emotions and attitudes, appreciating the mathematical structures, educating them to show willingness of being misunderstood, encouraging them to ask questions in the class and additionally helping them to be aware of false labeling and its impact on their attitudes towards learning new concepts. Small moments of self-empowerment, positive messages regarding their learning styles and small timely conversations can make a big difference throughout the journey of learning advanced mathematical thinking.

REFERENCES

Abdul Rahman, R., 2007. Translating Learning Theories into Practise in Enhancing Students' Mathematical Learning at University. In: Third International Conference on Research and Education on Mathematics.

Abdul Rahman, R., 2008. Changing my own and my Students' Attitudes to Calculus through working on Mathematical Thinking. Open University, United Kingdom.

Abdul Rahman, R., Baharun, S., Mohd. Yusof, Y., 2007. Enchancing Thinking through Active Learning in Engineering Mathematics. In: Fourth Regional Conference on Engineering Education (RCEE 2007). Johor Bahru, Malaysia.

Abdul Rahman, R., Mason, J.H., Mohamad Yusof, Y., 2010. Factors Affecting Students ' Change of Learning Behaviour. In: RCEE & RHEd 2010. Kuching,Sarawak, Malaysia.

Abdul Rahman, R., Mohammad Yusof, Y., 2008. Implementing strategies to invoke student's use of mathematical thinking powers in engineering mathematics. In: International Congress on Mathematics Education. Mexican Mathematical Society, Monterrey, Mexico, pp. 6–13.

Abdul Rahman, R., Mohd Yusof, Y., 2002. Invoking Students ' Mathematical Thinking in the Classroom : The Teaching Of Differentiation. In: Persidangan Kebangsaan Pendidikan Matematik. Kuala Lumpur, Malaysia.

Abdul Rahman, R., Mohd. Yusof, Y., Mason, J.H., 2005. Mathematical Knowledge Construction: Recognizing Students' Struggle. In: PME29. Melbourne, Australia.

Adams, R., Evangelou, D., English, L., De Figueiredo, A.D., Mousoulides, N., Pawley, A.L., Schifellite, C., Stevens, R., Svinicki, M., Trenor, J.M., Wilson, D.M., 2011. Multiple Perspectives on Engaging Future Engineers. Journal of Engineering Education 100, 48–88.

Adlin, T., Pruitt, J., 2010. The Essential Persona Lifecycle: Your Guide to Building and Using Personas. Elsevier Inc., Amsterdam, Boston, Heidelberg, london, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sydney, Tokoyo.

Airey, J., Linder, C., 2009. A disciplinary discourse perspective on university science learning: Achieving fluency in a critical constellation of modes. Journal of Research in Science Teaching 46, 27–49.

Aldrich, C., 2006. Simulations and the future of learning: An innovative (and perhaps revolutionary) approach to e-learning. Pfeiffer, San Francisco.

ALEKS Corporation, 2014. Overview of ALEKS [WWW Document]. URL http://www.aleks.com/about_aleks/overview (accessed 1.20.14).

Allen, I.E., Seaman, J., Garrett, R., 2007. Blending in: The extent and promise of blended education in the United States. Sloan-C™.

Allie, S., Armien, M.N., Burgoyne, N., Case, J.M., Collier-Reed, B.I., Craig, T.S., Deacon, A., Fraser, D.M., Geyer, Z., Jacobs, C., Jawitz, J., Kloot, B., Kotta, L., Langdon, G., le Roux, K., Marshall, D., Mogashana, D., Shaw, C., Sheridan, G., Wolmarans, N., 2009. Learning as acquiring a discursive identity through participation in a community: improving student learning in engineering education. European Journal of Engineering Education 34, 359–367.

Alpers, B., 2010. Studies on the Mathematical Expertise of Mechanical Engineers. Journal of Mathematical Modelling and Application 1, 2–17.

Alsina, C., 2002. Why the professor must be a stimulating teacher. In: The Teaching and Learning of Mathematics at University Level. Springer, pp. 3–12.

Anderson, J.R., Greeno, J.G., Reder, L.M., Simon, H.A., 2000. Perspectives on Learning, Thinking, and Activity. Educational Researcher 29, 11–13.

Anvari, F., Tran, H.M.T., 2013. Persona ontology for user centred design professionals. In: Proceedings of The 4th International Conference on Information Systems Management and Evaluation ICIME 2013. p. 35.

Argyle, S., 2012. Mathematical thinking: From cacophony to consensus. Kent State University.

Argyris, C., Putnam, R., Smith, D.M., 1985. Action Science: Concepts, Methods, and Skills for Research and Intervention. Jossey-Bass Publisher, San Francisco.

Baharun, S., Mohammad Yusof, Y., Abdul Rahman, R., 2008. Facilitating Thinking and Communications in Mathematics. In: ICME 11th Mexico. Mexico.

Baharun, S., Mohd. Yusof, Y., Abdul Rahman, R., 2007. Enhancing Engineering Student’s Mathematical Competency through Mathematical Thinking. In: Third

International Conference on Research and Education on Mathematics 2007. pp. 1–6.

Baillie, C., Douglas, E.P., 2014. Confusions and Conventions: Qualitative Research in Engineering Education. Journal of Engineering Education 103, 1–7.

Bajpai, A.C., 1985. The role of mathematics in engineering education: a mathematician's view. International Journal of Mathematical Education in Science and Technology 16, 417–430.

Baskerville, R., Myers, M.D., 2004. Special issue on action research in information systems: making is research relevant to practice - foreword. MIS Quarterly 28, 329–335.

Bassey, M., 1995. Creating education through research: A global perspective of educational research for the 21st century. Kirklington Moor Press, together with the British Educational Research Association, Kirklington, UK.

Becker, F.S., 2010. Why don't young people want to become engineers? Rational reasons for disappointing decisions. European Journal of Engineering Education 35, 349–366.

Bergsten, C., 2007. Investigating quality of undergraduate mathematics lectures. Mathematics Education Research Journal 19, 48–72.

Bersin, J., 2004. The Blended Learning Book Best Practices, Proven Methodologies and Lessons Learned. John Wiley & Sons, Inc., San Francisco, CA.

Boaler, J., 2013. Stanford University MOOC "EDUC115N How to learn maths" [WWW Document]. URL https://class.stanford.edu/courses/Education/EDUC115N/How_to_Learn_Math/

Bonk, C.J., Charles, R.G., 2006. The Handbook of Blended Learning : Global Perspectives, Local Designs. John Wiley & Sons, Inc., San Francisco, CA.

Borovik, A. V., Gardiner, T., 2006. Mathematical abilities and mathematical skills. In: World Federation of National Mathematics Competitions. Cambridge, England.

Bourne, J., Harris, D., Mayadas, F., 2005. Online engineering education: Learning anywhere, anytime. Journal of Engineering Education.

Bransford, J., 2007. Preparing People for Rapidly Changing Environments. Journal of Engineering Education 96, 1–3.

Bransford, J.D., Brown, A.L., Cocking, R.R. (Eds.), 2000. How People Learn: Brain, Mind, Experience, and School, Committee on learning research and educational practice. National Academy Press, Washington D.C.

Breen, S., O'Shea, A., 2011. The use of mathematical tasks to develop mathematical thinking skills in undergraduate calculus courses–a pilot study. Proceedings of the British Society for Research into Learning Mathematics 31, 43–48.

Broadbridge, P., Henderson, S., 2008. Mathematics education for 21st century engineering students. Australian Learning and Teaching ….

Bruce, C., Pham, B., Stoodley, I., 2004. Constituting the significance and value of research: views from information technology academics and industry professionals. Studies in Higher Education 29, 219–238.

Bruner, J., 1986. Actual minds, possible worlds. Harvard University Press, Cambridge, MA.

Bruner, J.S., 1961. The Act of Discovery. Harvard Educational Review 31.

Bryman, A., 2001. Introduction. In: Bryman, A. (Ed.), Ethnography. Sage Publications, London, UK.

Burns, R.B., 2000. Introduction to research methods. Sage Publications, London.

Burton, L., 2002. Methodology and Methods in Mathematics Education Research: Where Is "The Why"? In: Goodchild, S., English, L. (Eds.), Researching Mathematics Classroom: A Critical Examination of Methodology. Praeger, Westport, CT, pp. 1–10.

Cardella, M.E., 2006. Engineering Mathematics: an Investigation of Students' Mathematical Thinking from a Cognitive Engineering Perspective. University of Washington.

Cardella, M.E., 2007a. What your engineering students might be learning from their mathematics pre-reqs (Beyond Integrals and Derivatives). In: 2007 37th Annual Frontiers in Education Conference - Global Engineering: Knowledge without Borders, Opportunities without Passports. IEEE, pp. S4F–1–S4F–6.

Cardella, M.E., 2007b. Which Mathematics Should We Teach Engineering Students? An Empirically-Grounded Case for Mathematical Thinking.

Cardella, M.E., Atman, C., 2007. Engineering Students' Mathematical Thinking: In the Wild and with a Lab-based Task. In: American Society for Engineering Education.

Carlson, M., Bloom, I., 2005. The cyclic nature of problem solving: An emergent problem-solving framework. Educational Studies in Mathematics 58, 45–75.

Carr, W., Kemmis, S., 1986. Becoming Critical: Education, Knowledge and Action Research. Routledge, Falmer with Taylor & Francis Group, London and New York.

Carstensen, A.-K., Bernhard, J., 2009. Student learning in an electric circuit theory course: critical aspects and task design. European Journal of Engineering Education 34, 393–408.

Case, J.M., Marshall, D., 2008. The "no problem" Discourse model: Exploring an alternative way of researching student learning. International Journal of Educational Research 47, 200–207.

Cherry, N., 1999. Action Research: A Pathway to Action, Knowledge and Learning. RMIT University Press, Melbourne.

Chism, N.V.N., Douglas, E., Wayne J. Hilson, J., 2008. Qualitative Research Basics : A Guide for Engineering Educators.

Cho, J., Trent, A., 2006. Validity in qualitative research revisited. Qualitative Research 6, 319–340.

Christensen, C.R., Garvin, D.A., 1991. Education for Judgment: The Artistry of Discussion Leadership. Harvard Business School, Cambridge, Massachusettes.

Chua, W., 1986. Radical Developments in Accounting Thought. The Accounting Review 601–632.

Chubin, D., Donaldson, K., Olds, B., Fleming, L., 2008. Educating Generation Net—Can U.S. Engineering Woo and Win the Competition for Talent? Journal of Engineering Education 97, 245–257.

Clandinin, D.J., Connelly, F.M., 1998. Personal Experience Methods. In: Denzin, N.K., Lincoln, Y. (Eds.), Collecting and Interpreting Qualitative Materials. Sage Publications, Thousand Oaks, CA, pp. 413–427.

Clark, J., Linder, C., 2006. Changing teaching, changing times: Lessons from a South African township science classroom. Sense Publishers, Rotterdam, Netherlands.

Cohen, L., Manion, L., Morrison, K., 2007. Research Methods in Education, Sixth Edit. ed. Routledge, Taylor & Francis Group, London and New York.

Common Core State Standards for Mathematics, 2010.

Cousin, G., 2009. Researching Learning in Higher Education : an Introduction to Contemporary Methods and Approaches. Taylor & Francis Routledge, New York and London.

Creswell, J.W., 2007. Qualitative Inquiry and Research Design Choosing Among Five Traditions, Second Ed. ed. Sage Publications, Inc., California, New Delhi, London.

Creswell, J.W., 2012. Educational Research Planning, Conducting and Evaluating Quantitative and Qualitative Research, Fourth. ed. Pearson Education, Inc.

Creswell, J.W., Miller, D.L., 2000. Determining Validity in Qualitative Inquiry. Theory Into Practice 39, 124–130.

Croft, T., Grove, M., 2006. Mathematics Support: Support for the Specialist Mathematician and the More Able Student. MSOR Connections 6, 1–6.

Crotty, M., 1998. The foundations of social research: Meaning and perspective in the research process. Allen & Unwin, Sydney, Australia.

Curriculum Development Centre, Ministry of Education Malaysia, 2004. Integrated Curriculum for Secondary Schools (Syllabus Mathematics).

Dahlstrom, E., Walker, J.D., Dziuban, C., 2013. ECAR study of undergraduate students and information technology, 2013. Louisville, CO: EDUCAUSE Center for Analysis and Research. 2012.

Daiute, C., Lightfoot, C. (Eds.), 2004. Narrative Analysis: Studying the Development of Individuals in Society. SAGE, Thousand Oaks.

Davis, B., Sumara, D., 2006. Complexity and Education: Inquiries into Learning, Teaching, and Research. Lawrence Erlbaum Associates, Mahwah, NJ.

Devlin, K., 2012. Coursera MOOC "Introduction to Mathematical Thinking" [WWW Document]. URL https://www.coursera.org/course/maththink

Dewey, J., 1910. How we think, 1991 editi. ed. Prometheus Books, Amherst, NY.

Dewey, J., 1997. How We Think. Dover, Mineola, New York.

Dey, I., 1993. Qualitative data analysis: A user-friendly guide for social scientists. Routledge, London.

Dignen, B., 2014. Five reasons why feedback may be the most important skill [WWW Document]. URL http://www.cambridge.org/elt/blog/2014/03/five-reasons-feedback-mayimportant-skill/ (accessed 12.2.14).

Downey, G., Lucena, J.C., Moskal, B.M., Parkhurst, R., Bigley, T., Hays, C., Jesiek, B.K., Kelly, L., Miller, J., Ruff, S., Lehr, J.L., Nichols-Belo, A., 2006. The globally competent engineer: Working effectively with people who define problems differently. Journal of Engineering Education 107–122.

Doyle, T., 2008. Helping students learn in a learner-centered environment: A guide to facilitating learning in higher education. Stylus Publishing, LLC., Sterling, Virginia.

Dubinsky, E., McDonald, M.A., 2001. APOS: A constructivist theory of learning in undergraduate mathematics education research. The teaching and learning of mathematics at university level: An ICMI study 273–280.

Duderstadt, J.J., 2008. Engineering for a Changing World: A Roadmap to the Future of Engineering Practice, Research, and Education. The Millennium Project, The University of Michigan, Ann Arbor, MI.

Duderstadt, J.J., 2010. Engineering for a Changing World: A Roadmap to the Future of American Engineering Practice, Research, and Education. In: Grasso, D., Burkins, M.B. (Eds.), Holistic Engineering Education: Beyond Technology. Springer, New York, NY, pp. 17–35.

Ebenezer, J. V., Fraser, D.M., 2001. year chemical engineering students' conceptions of energy in solution processes: Phenomenographic categories for common knowledge construction. Science Education 85, 509–535.

edX, 2015. BlendedX Blended Learning with edX [WWW Document]. URL https://courses.edx.org/courses/course-v1:edX+BlendedX+1T2015/info

EERC, 2006. The Research Agenda for the New Discipline of Engineering Education, Journal of Engineering Education. Blackwell Publishing Ltd.

Elliott, J., 2005. Using Narrative in Social Research: Qualitative and Quantitative Approaches. Sage Publications, London, Thousand Oaks, New Dehli.

Elliott, L., 2009. UK "must find 600,000 new engineers in seven years", The Observer [WWW Document]. URL http://www.theguardian.com/technology/2009/nov/29/manufacturing-engineering-recession-recovery

English, L., 2002. Priority Themes and Issues in International Research in Mathematics Education. In: English, L.D. (Ed.), Handbook of International Research in Mathematics Education. Lawrence Erlbaum Associates, Inc., Mahwah, NewJersey, London.

Eysenck, H.J., 1976. Introduction. In: Eysenck, H.J. (Ed.), Case Studies in Behaviour Therapy. Routledge, London, UK, pp. 1–15.

Faily, S., Flechais, I., 2011. Persona Cases : A Technique for grounding Personas. CHI 2011 1–4.

Fairclough, N., 2003. Analysing discourse: Textual analysis for social research. Routledge, London, UK.

Fantz, T.D., Siller, T.J., Demiranda, M. a., 2011. Pre-Collegiate Factors Influencing the Self-Efficacy of Engineering Students. Journal of Engineering Education 100, 604–623.

Fereday, J., Muir-cochrane, E., 2006. Demonstrating Rigor Using Thematic Analysis : A Hybrid Approach of Inductive and Deductive Coding and Theme Development. International Journal of Qualitative Methods 5 (1), 1–11.

Ferri, R.B., 2012. Mathematical thinking styles and their influence on teaching and learning mathematics. In: 12th International Congress on Mathematical Education. COEX, Seoul, Korea.

Finelli, C.J., Daly, S.R., Richardson, K.M., 2014. Bridging the Research-to-Practice Gap: Designing an Institutional Change Plan Using Local Evidence. Journal of Engineering Education 103, 331–361.

Fink, L.D., Ambrose, S., Wheeler, D., 2005. Becoming a professional engineering educator: A new role for a new era. Journal of Engineering Education 185–194.

Forfás, 2009. The Expert Group on Future Skills Needs Statement of Activity 2008. Dublin.

Fortenberry, N., 2006. An extensive agenda for engineering education research. Journal of Engineering Education 3–5.

Freudenthal, H., 2002. Revisiting mathematics education: China lectures. Kluwer Academic Publishers, Dordrecht / Boston / London.

Fritze, Y., Nordkvelle, Y.T., 2003. Comparing lectures: Effects of the technological context of the studio. Education and Information Technologies 8, 327–343.

Gainsburg, J., 2006. The Mathematical Modeling of Structural Engineers. Mathematical Thinking and Learning 8, 3–36.

Garrison, D.R., Vaughan, N.D., 2008. Blended Learning in Higher Education: Framework, Principles, and Guidelines. John Wiley & Sons, Inc, San Francisco, CA.

Gee, J.P., 2011. An Introduction to Discourse Analysis. Routledge, Taylor & Francis Group, New York and London.

Gergen, K.J., 1994. Realities and relationships: Soundings in social construction. Harvard University Press, Cambridge, MA.

Ginsburg, Herbert, Jacobs, S., Lopez, L.S., 1998. The Teacher's Guide to Flexible Interviewing in the Classroom : Learning What Children Know About Math. Allyn and Bacon.

Given, L.M. (Ed.), 2008. The SAGE encyclopedia of qualitative research methods. SAGE Publications, Inc., Los Angeles, London, New Dehli, Singapore.

Glaser, B.G., Strauss, A.L., 1967. The Discovery of Grounded Theory: Strategies for Qualitative Research. AldineTransaction, New Brunswick (USA) and London (UK).

Goodwin, K., 2008. Getting from research to personas: harnessing the power of data [WWW Document]. Cooper. URL http://www.cooper.com/journal/2002/11/getting_from_research_to_perso.html

Goold, E., Devitt, F., 2013. Mathematics in engineering practice: Tacit trumps tangible. In: Williams, B., Figueiredo, J., Trevelyan, J. (Eds.), Engineering Practice in a Global Context: Understanding the Technical and the Social. CRC Press Taylor and Francis Group, pp. 245–264.

Goos, M., Galbraith, P., Renshaw, P., 2002. Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. Educational studies in Mathematics 49, 193–223.

Graham, C., Dziuban, C., 2008. Blended learning environments. Handbook of research on educational communications … 269–276.

Grudin, J., Pruitt, J., 2003. Personas : Practice and Theory. In: Proceedings of DUX03: Designing for User Experiences. ACM Press, pp. 1–15.

Guba, E.G., Lincoln, Y.S., 2005. Paradigmatic Controversies, Contradictions, and Emerging Confluences. In: Denzin, N.K., Lincoln, Y.S. (Eds.), The Sage Handbook of Qualitative Research. SAGE Publications, Inc., Thousand Oaks, CA, pp. 191–216.

Gupta, A., Elby, A., 2011. Beyond Epistemological Deficits: Dynamic explanations of engineering students' difficulties with mathematical sense-making. International Journal of Science Education 33, 2463–2488.

Güzer, B., Caner, H., 2014. The Past, Present and Future of Blended Learning: An in Depth Analysis of Literature. Procedia - Social and Behavioral Sciences 116, 4596–4603.

Habermas, J., 1976. Communication and the Evolution of Society. Heinemann, London.

Habermas, J., 2002. The Inclusion of the Other: Studies in Political Theory. Polity, Oxford.

Hammersley, M., Atkinson, P., 2007. Ethnography : Principles in Practice, Third. ed. Taylor & Francis e-Library.

Hamuy, E., Galaz, M., 2010. Information versus communication in course management system participation. Computers & Education 54, 169–177.

Han, S., 2010. Cultural Differences in Thinking Styles. In: Glatzeder, B.M., Müller, A. von, Goel, V. (Eds.), Towards a Theory of Thinking: Building Blocks for a Conceptual Framework. Springer, Heidelberg, pp. 279–288.

Harun, S.N.F.B., 2012. Comapring the Expected, Intended and Implemented Mathematical Skills for Engineering Education in a Selected Public University. Universiti teknologi Malaysia.

Heinze, A., 2008. Blended Learning: An Interpretive Action Research Study. University of Salford, Salford, UK.

Hicks, D., 1995. Chapter 2 Discourse, Learning, and Teaching. Review of Research in Education 21, 49–95.

Hinchman, L.P., Hinchman, S., 1997. Memory, identity, community: The idea of narrative in the human sciences. University of New York Press, Albany, NY.

Homer, M., 2012. What is Mathematical Engineering? [WWW Document]. URL http://www.postgraduate-courses.net/articles/mathematical_engineering.htm

Hopkins, D., 1985. A teacher's guide to classroom research. Open University Press, Bukingham.

Hull, M.M., Kuo, E., Gupta, A., Elby, A., 2013. Problem-solving rubrics revisited: Attending to the blending of informal conceptual and formal mathematical reasoning. Physical Review Special Topics - Physics Education Research 9, 10105.

Hunt, D., Davis, Ki., Richardson, D., Hammock, G., Akins, M., Russ, L., 2014. It is (More) About the Students: Faculty Motivations and Concerns Regarding Teaching Online. Online Journal of Distance Learning Administration 17, 62–71.

Inoue, Y., 2010. Cases on Online and Blended Learning Technologies in Higher Education: Concepts and Practices. IGI Global, Hershey, New York.

Ismail, Z., Kasmin, M.K., 2008. Using cognitive tools to enhance understanding in differential equations. Jurnal Teknologi 48, 161–168.

Iversen, J.H., Mathiassen, L., Nielsen, P.A., 2004. Managing risk in software process improvement: an action research approach. MIS Quarterly 23, 395–433.

Jayarajah, K., Mohd Saat, R., Abdul Rauf, R.A., 2012. An Alternative Perspective for Malaysian Engineering Education : A Review from Year 2000-2012 1, 1–23.

Kashefi, H., 2012. Mathematical Thinking in Multivariable Calculus through Blended Learning. Universiti Teknologi Malaysia.

Kashefi, H., Ismail, Z., Mohammadyusof, Y., Mirzaei, F., 2013. Generic skills in engineering mathematics through blended learning: A mathematical thinking approach. International Journal of Engineering Education 29, 1222–1237.

Kashefi, H., Ismail, Z., Yusof, Y.M., 2012. Overcoming Students Obstacles in Multivariable Calculus through Blended Learning: A Mathematical Thinking Approach. Procedia - Social and Behavioral Sciences 56, 579–586.

Kaur, M., 2013. Blended Learning - Its Challenges and Future. Procedia - Social and Behavioral Sciences 93, 612–617.

Kemmis, S., 1982. Action research. In: Husén, T., Postlethwaite, T.N. (Eds.), International Encyclopaedia of Education: Research & Studies. Pergamon Press, Oxford.

Kemmis, S., McTaggart, R., Nixon, R., 2014. The Action Research Planner: Doing Critical Participatory Action Research. Springer, Singapore.

King, R., 2008. Addressing the Supply and Quality of Engineering Graduates for the new Century.

Koro-Ljungberg, M., Douglas, E.P., 2008. State of Qualitative Research in Engineering Education: Meta-Analysis of JEE Articles, 2005-2006. Journal of Engineering Education 163–175.

Kress, T.M., 2011a. The Interplay of Identity, Context, and Purpose in a Study of Mathematics Teaching and Learning. In: Critical Praxis Research Breathing: Breathing New Life into Research Methods for Teachers, Explorations of Educational Purpose. Springer Science+Business Media, Inc., pp. 193–208.

Kress, T.M., 2011b. Critical Praxis Research Breathing: Breathing New Life into Research Methods for Teachers. Springer Science+Business Media, Inc.

Kuhn, T.S., 1962. The Structure of Scientific Revolutions. University of Chicago Press, Chicago.

Lakatos, I., 1976. Proofs and Refutations. Cambridge, CUP.

Larson, R., Murray, M., 2008. Open educational resources for blended learning in high schools: Overcoming impediments in developing countries. Journal of Asynchronous Learning Networks 12, 85–103.

Leron, U., 2003. Origins of mathematical thinking: A synthesis. In: European Research in Mathematics Education III. pp. 1–8.

Leron, U., Dubinsky, E., 1995. An abstract algebra story. American Mathematical Monthly 102, 227–242.

Lesh, R., Harel, G., 2003. Problem Solving, Modeling, and Local Conceptual Development. Mathematical Thinking and Learning 5, 157–189.

Lewin, K., 1951. Field theory in social science; selected theoretical papers. Harper & Row, New York.

Light, G., Case, J.M., 2011. Emerging Methodologies in Engineering Education Research. Journal of Engineering Education 100, 186–210.

Lindgren, R., Henfridsson, O., Schultze, U., 2004. Design principles for competence management systems: a synthesis of an action research study. MIS Quarterly 28, 435–472.

Lynn, L., Salzman, H., 2009. The “new” Globalization of Engineering: How the Offshoring of Advanced Engineering Affects Competitiveness and Development. Economics, Management, and Financial Markets 4, 11–46.

Macdonald, D.G., 2008. Constructivism Cmap.

Madsen, A., McKagan, S.B., Sayre, E.C., Martinuk, M., Bell, A., 2014. Personas as a Powerful Methodology to Design Targeted Professional Development Resources. arXiv preprint arXiv:1408.1125.

Mahmood, A., Othman, M.F. Bin, Yusof, Y.M., 2013. Activity Theory Model to Cultivate Higher Order Mathematical Thinking. In: Research in Engineering Education Symposium (REES2013) 4-6 July , 2013, ISBN 978-967-0194-35-6. Putrajaya, Malaysia, pp. 263–275.

Mahmood, A., Othman, M.F., Yusof, Y.M., 2012. A Conceptual Framework for Mathematical Ability Analysis through the Lens of Cultural Neuroscience. Procedia - Social and Behavioral Sciences 56, 175–182.

Mahmood, A., Othman, M.F.B., Yusof, Y.M., 2013. Activity theory model to cultivate higher order mathematical thinking. In: 2013 Research in Engineering Education Symposium, REES 2013. Department of Engineering Education, School of Graduate Studies, Universiti Teknologi Malaysia, 81310, Malaysia, pp. 263–275.

Malaysia Education Blueprint 2013 - 2025, 2013.

Mamona-Downs, J., Downs, M., 2002. Advanced Mathematical ThinkingWith a Special Reference to Reflection on Mathematical Structure. In: Handbook of International Research in Mathematics Education. Lawrence Erlbaum Associates, Inc., Mahwah, NewJersey, London.

Manen, M. Van, 1990. Researching lived experience: Human science for an action sensitive pedagogy. State University of New York Press, New York, NY.

Mann, L., Chang, R.L., 2010. Creating an Engineering Education Community of Practice within an Institutional Setting : A Blueprint for Action. In: AaeE. Sydney, Australia, pp. 656–663.

Marshall, D., Case, J., 2010. Rethinking “disadvantage” in higher education: a paradigmatic case study using narrative analysis. Studies in Higher Education 35, 491–504.

Mårtensson, P., Lee, A.S., 2004. Special Issue on Action Research in Information Systems. MIS Quarterly 28, 507–536.

Marton, F., 1989. Towards a pedagogy of content. Educational Psychologist 24, 1–23.

Marton, F., Booth, S., 1997. Learning and awareness. Lawrence Erlbaum Associates, Mahwah, NJ.

Mason, J., Burton, L., Stacey, K., 2010. Thinking Mathematically, Second Edi. ed. Pearson Education Limited.

Mason, J., Johnston-Wilder, S., 2004. Fundamental constructs in mathematics education. The Open University in association with Taylor and Francis Group, London.

Mathematical Sciences Education Board and the Board on Mathematical Sciences National Research Council, National Research Council, 2000. Everybody counts: a report to the nation on the future of mathematics education. National Academy Press, Washington, D.C.

Maydonik, C., 2010. Constructivism CMap.

McGinn, M.K., Boote, D.N., 2003. A First-Person Perspective on Problem Solving in a History of Mathematics Course. Mathematical Thinking and Learning 5, 71–107.

McKinsey, 2011. Growth and renewal in the United States : Retooling America ' s economic engine.

McNiff, J., Whitehead, J., 2006. All you need to know about Action Research: An Introduction. Sage Publications, London, Thousand Oaks, New Delhi.

McNiff, J., Whitehead, J., 2010. You and your action research project, 3rd ed. Routledge, Taylor & Francis Group, London and New York.

Mezirow, J., 2003. Transformative Learning as Discourse. Journal of Transformative Education 1, 58–63.

Micari, M., Light, G., Calkins, S., Streitwieser, B., 2007. Assessment beyond performance: Phenomenography in educational evaluation. American Journal of Evaluation 28, 458.

Miles, M.B., Huberman, A.M., 1994. An Expanded Sourcebook: Qualitative Data Analysis, Second. ed. Sage Publications, Thousand Oaks, London, New Delhi.

MIT-LINC, 2013. MIT BLOSSOMS : An initiative of MIT's Leading International Network Consortium [WWW Document]. URL http://blossoms.mit.edu/ (accessed 4.3.13).

Mohammad Yusof, Y., Abdul Rahman, R., 2001. Students' Difficulties with Multiple Integration : A Preliminary Study. In: 3rd Southern Hemisphere Symposium on Undergrad. Mathematics Teaching.

Mohammad Yusof, Y., Abdul Rahman, R., 2004. Teaching Engineering Students to Think Mathematically. In: CEE04.

Mohammad Yusof, Y., Abdul Rahman, R., Mason, J.H., 1999. Being Mathematical: New Questions for Old.

Mohammad Yusof, Y., Aliah Phang, F., Kashefi, H., Ismail, Z., Yusof, Y.M., 2012. Supporting Engineering Students' Thinking and Creative Problem Solving through Blended Learning, Procedia - Social and Behavioral Sciences.

Mohammad Yusof, Y., Tall, D., 1999. Changing attitudes to University Mathematics Through Problem Solving. Educational Studies in Mathematics 67–82.

Moreno, R., 2010. Educational Psychology. John Wiley & Sons, Inc.

Moss, P.A., Phillips, D.C., Erickson, F.D., Floden, R.E., Lather, P.A., Schneider, B.L., 2009. Learning From Our Differences: A Dialogue Across Perspectives on Quality in Education Research. Educational Researcher 38, 501–517.

Mouyabi, J.S.M., 2011. Higher education in the wake of new ICT: Reaping benefits or creating more problems through e-learning? South African Journal of Higher Education 25, 1178–1189.

Moyer, P., Milewicz, E., 2002. Learning to Question: Categories of Questioning Used by Preservice Teachers During Diagnostic Mathematics Interviews. Journal of Mathematics Teacher Education 5, 293–315.

National Academy of Engineering, 2005. Educating the engineer of 2020: Adapting engineering education to the new century. The National Academies Press, Washington D.C.

National Science Foundation, 1995. Restructuring Engineering Education: A Focus on Change. Division of Undergraduate Education, National Science Foundation, Arlington, VA.

NCTM, 2009. Focus in High School Mathematics: Reasoning and Sense Making, The Mathematics Teacher. National Council of Teachers of Mathematics, Reston, VA.

Nelson, C., Treichler, P.A., Grossberg, L., 1992. Cultural studies: An intrnduction. In: Nelson, C., Treichler, P.A., Grossberg, L. (Eds.), Cultural Studies. Routledge, New York, pp. 1–16.

Nielsen, L., 2013. Personas - User Focused Design. Springer, London, Heidelberg, Newyork, Dordrecht.

Northedge, A., 2003. Enabling Participation in Academic Discourse. Teaching in Higher Education 8, 169–180.

Oates, B.J., 2006. Researching Information Systems and Computing. Sage Publications, London, Thousand Oaks, New Dehli.

Oliveira, M.C., 2005. Using narrative in social research: Qualitative and quantitative approaches by Jane Elliott. Narrative Inquiry 15, 421–429.

Organisation for Economic Co-Operation and Development, 2010. The High Cost of Low Educational Performance: The Long-Run Economic Impact of Improving PISA Outcomes. Paris.

Patton, M.Q., 2002. Qualitative evaluation and research methods, 3rd ed. Sage Publications, Inc., Thousands Oaks, CA.

Pegg, J., Tall, D., 2005. The fundamental cycle of concept construction underlying various theoretical frameworks. International Reviews on Mathematical Education (Zentralblatt für Didaktik der Mathematik) 37, 468–475.

Piaget, J., 1972. The Psychology of the Child. Basic Books, New York.

Picciano, A.G., 2007. Blended Learning: Research Perspectives. The Sloan Consortium, United States of America.

Polkinghorne, D.E., 1995. Narrative configuration in qualitative analysis. International Journal of Qualitative Studies in Education 8, 5–23.

Polya, G., 1957. How to Solve it: A New Aspect of Mathematical Method, Second Edi. ed. Princeton University Press, Princeton and Oxford.

Polya, G., 1962. Mathematical Discovery: On Understanding, Learning and Teaching Problem Solving, Combined e. ed. Wiley, New York.

Power, R., Naysmith, J., 2005. Action Research: A Guide for Associate Lecturers.

Prince, M., Felder, R., 2006. Inductive Teaching and Learning Methods: Definitions, Comparisons, and Research Bases. Journal of Engineering Education.

Rahman, R.A., Mason, J.H., Yusof, Y.M., 2012a. Factors affecting students’ change of learning behaviour. Procedia-Social and Behavioral Sciences 56, 213–222.

Rahman, R.A., Yusof, Y.M., Baharun, S., 2012b. Realizing Desired Learning Outcomes in Undergraduate Mathematics. Outcome-Based Science, Technology, Engineering, and Mathematics Education: Innovative Practices: Innovative Practices 182.

Ratey, J.J., 2001. A user’s guide to the brain: Perception, attention, and the four theatres of the brain. Vintage.

Richardson, L., 2000. Evaluating Ethnography. Qualitative Inquiry 6, 253–255.

Rittle-Johnson, B., Koedinger, K.R., 2005. Designing Knowledge Scaffolds to Support Mathematical Problem Solving. Cognition and Instruction 23, 313–349.

Rover, D.T., 2008. Engineering Education in a Global Context. Journal of Engineering Education 105–108.

Sam, H.K., Ngiik, T.L., Usop, H.H., 2009. Status of mathematics teaching and learning in Malaysia. International Journal of Mathematical Education in Science and Technology 40, 59–72.

Sam, L.I.M.C., Yong, H.W.A.T., 2006. Promoting mathematical thinking in the Malaysian classroom: issues and challenges. In: Meeting of the APEC-Tsukuba International Conference. meeting of the APEC-Tsukuba International Conference, Japan.

Sandberg, J., 2000. Understanding Human Competence at Work: An Interpretative Approach. Academy of Management Journal 43, 9–25.

Scanlan, J.O., 1985. The role of mathematics in engineering education: an engineer's view. International Journal of Mathematical Education in Science and Technology 16, 445–451.

Schoenfeld, A. (Interviewee), 2013. Alan Schoenfeld presenterar olika matematikproblem.

Schoenfeld, A.H., 1985. Mathematical Problem Solving. Academic Press, Inc., Orlando, San Diego, New York, London, Toronto, Montreal, Sydney, Tokyo.

Schoenfeld, A.H., 1992. Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In: D. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning. MacMillan, New York, pp. 334–370.

Schunk, D.H., 2012. Learning Theories: An Educational Perspective, Sixth. ed. Pearson Education, Inc., Boston, Massachusetts.

Seymour, E., 2002. Tracking the processes of change in US undergraduate education in science, mathematics, engineering, and technology. Science Education 86, 79–105.

Sfard, A., 2001. There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. Educational Studies in Mathematics 46, 13–57.

Sheppard, S.D., Pellegrino, J.W., Olds, B.M., 2008. On Becoming a 21st Century Engineer. Journal of Engineering Education 97, 231–234.

Siddiqui, J.A.W., 2014. Transformation of Engineering Education : Taking a Perspective for the Challenges of Change. Purdue University.

Smith, K.A., Sheppard, S.D., Johnson, D.W., Johnson, R.T., 2005. Pedagogies of Engagement: Classroom-Based Practices. Journal of Engineering Education 94, 87–101.

Smith, T., 2000. Bridging the Research-Practice Gap: Developing a Pedagogical Framework that Promotes Mathematical Thinking and Understanding. Mathematics Teacher Education and Development 2, 4–16.

Sriraman, B., 2009. Educational Psychology and Mathematics Education: Potential for a New Symbiosis?, Mathematical Thinking and Learning.

Stake, R.E., 2005. Qualitative case studies. In: Denzin, N.K., Lincoln, Y.S. (Eds.), The Sage Handbook of Qualitative Research. SAGE Publications, Inc, Thousand Oaks, CA, pp. 443–466.

Sterling, S., 2004. Higher Education, Sustainability, and the Role of Systemic Learning. In: Corcoran, P.B., Wals, A.E.J. (Eds.), Higher Education and the Challenge of Sustainability: Problematics, Promise, and Practice. Springer, Dordrecht, Netherlands, pp. 49–70.

Stocking, G.W., 1983. Observers Observed: Essays on Ethnographic Fieldwork. The University of Wisconsin Press, Madison, WI.

Strauss, A., Corbin, J., 1994. Grounded theory methodology: An overview. In: Denzin, N.K., Lincoln, Y.S. (Eds.), Handbook of Qualitative Research. Sage, Thousand Oaks, CA, pp. 273–285.

Street, C.T., Meister, D.B., 2004. Small business growth and internal transparency: the role of information systems. MIS Quarterly 28, 473–506.

Swarat, S., Light, G., Park, E.J., Drane, D., 2011. A typology of undergraduate students' conceptions of size and scale: Identifying and characterizing conceptual variation. Journal of Research in Science Teaching 48, 512–533.

Tall, D., 1998. Changing attitudes to university mathematics through problem solving. Educational Studies in Mathematics 37, 67–82.

Tall, D., 2002. Advanced Mathematical Thinking. Kluwer Academic Publishers.

Tall, D., 2004. Introducing Three Worlds of Mathematics. Learning of Mathematics.

Tall, D., 2008. The transition to formal thinking in mathematics. Mathematics Education Research Journal 20, 5–24.

Tall, D., 2013. How Humans Learn to Think Mathematically: Exploring the Three Worlds of Mathematics (Learning in Doing: Social, Cognitive and Computational Perspectives). Cambridge University Press, New York , USA.

Thorne, K., 2003. Blended Learning : How to Integrate Online and Traditional Learning. Kogan Page Limited, Great Britain and the United States.

Tolbert, D.A., Cardella, M.E., 2013. Early work for the Mathematics as a Gatekeeper to Engineering Project : A Review of Informal Learning , Engineering and Design Thinking Literature. In: 120th ASEE Annual Conference and Exposition. Atlanta.

Tomal, D.R., 2010. Action Research for Educators, Second. ed. Rowman & Littlefield Publishers, Inc.

Tracy, S.J., 2010. Qualitative Quality: Eight "Big-Tent" Criteria for Excellent Qualitative Research. Qualitative Inquiry 16, 837–851.

Trevelyan, J., 2009. Steps Toward a Better Model of Engineering Practice. In: Research in Engineering Education Symposium Palm Cove, REES 2009, 20–23 July 2009. Queensland, Australia, pp. 1–9.

Turns, J., Borgford-Parnell, J., Ferro, T., 2013. Identifying Implications: Using Personas to Bridge the Gap Between Research Findings and the Design of Educational Experiences. Poster presentation in 2013 University of Washington Teaching and Learning Symposium.

UTM, 2013. MIT-UTM Blossoms Project [WWW Document]. NewsHub – UTM News Portal Published by alin. URL http://www.news.utm.my/2013/05/mit-utm-blossoms-project-8jan2013/

Volkmann, M.J., Zgagacz, M., 2004. Learning to teach physics through inquiry: The lived experience of a graduate teaching assistant. Journal of Research in Science Teaching 41, 584–602.

Vygotsky, L.S., 1978. Mind in Society. Harvard University Press, Cambridge, Massachusettes.

Ward, J.L., 2010. Persona development and use, or, how to make imaginary people work for you.

Weber, K., 2004. Traditional instruction in advanced mathematics courses: A case study of one professor's lectures and proofs in an introductory real analysis course. The Journal of Mathematical Behavior 23, 115–133.

Wenger, E., McDermott, R., Snyder, W.M., 2002. Cultivating Communities of Practice. Harvard Business School Press, Boston, Massachusetts.

Wetherell, M., Yates, S., Taylor, S., 2001. Discourse theory and practice: A reader. Sage Publications, London.

Whitehead, J., 2009. Justifying the use of a living theory methodology in the creation of your living educational theory: Responding to Cresswell [WWW Document]. URL http://www.actionresearch.net/writings/arsup/Cresswellqualitativemethods.htm (accessed 9.15.14).

Whitehead, J., 2014. A Living Educational Theory (Living Theory) Approach to Research and Life [WWW Document]. URL http://www.actionresearch.net/

Whitehead, J., McNiff, J., 2006. Action Research: Living Theory. Sage Publications, London, Thousand Oaks, New Dehli.

Whyte, W.F., Greenwood, D.J., Lazes, P., 1991. Participatory Action Research: Through practice to science in social research. In: Whyte, W.F. (Ed.), Participatory Action Research. Sage Publications, London, pp. 19–55.

Wikberg Nilsson, Å., Fältholm, Y., Abrahamsson, L., 2010. Reframing practice through the use of Personas. Reflective Practice 11, 285–298.

Winkelman, P., 2009. Perceptions of mathematics in engineering. European Journal of Engineering Education 34, 305–316.

Wood, L.N., Mather, G., Petocz, P., Reid, A., Engelbrecht, J., Harding, A., Houston, K., Smith, G.H., Perrett, G., 2011. University Students' Views of the Role of Mathematics in Their Future. International Journal of Science and Mathematics Education 10, 99–119.

Woods, D.R., 2000. An Evidence-Based Strategy for Problem Solving. Journal of Engineering Education 443–459.

Yusof, Y.M., Othman, M.F. Bin, Mahmood, A., 2014. Making Students' Thinking Explicit: Learning What They Know about Functions. In: International Conference on Teaching and Learning in Computing and Engineering. IEEE Computer Society Press, Kuching, Malaysia, pp. 256–261.

Zanzali, N.A.A., 2012. Trends of Mathematics Education in Malaysia: The continuing Issues. In: ISSME.

Zeynivandnezhad, F., 2014. Mathematical Thinking in Differential Equations through a Computer Algebra System. Universiti Teknologi Malaysia.

APPENDIX A

First page of worksheet "The Power of Exponentials, Big and Small" used in main cycle 1

Student's Name____________________________________Matrik card no.____________

Group Name__________________ Role________________Date:______________________

Mathematical and Design Thinking Lab Session II

BLOSSOMS MODULE_The Power of Exponentials, Big and Small

(Worksheet)

Greetings and Introduction to the Module

Assalmualikum and welcome to our module on "The power of exponentials, big and small.

Module Entry (required to raise the curiosity about the topic)_watch video segment#1

Activity 1:

Reward:

1 grain of rice for the 1st square of the chessboard, two for the second, four for the third, and so on. Every square must have double the number of grains as the previous one.

Questions:

Do you think the award was very small? Was the treasurer able to pay off the mathematician with a single bag of rice? Was the queen able to pay off the mathematician at all?

Assumption:

- Number of grains of the rice in a bag = 800,000

Instructions

- Please record your responses in the space provided under each question.
- Prompts will help you to come out of any STUCK state.

Entry Phase:

Entry 1: What do you KNOW from the question and also from your past experience?

• 64 squares on a chess board (example)

Entry 2: What do I WANT?

Entry 3: What can I INTRODUCE? (e.g. images, diagrams, symbols)

APPENDIX B

First page of worksheet "Flaws of Averages" used in main cycle 2

Group Name_____________________________Date:___________________

Coordinator____________________Motivator_________________________

Reflector_________________________Presenter______________________

Coordinator is responsible for the flow of activity while keeping all members working together within given time limit.
Motivator is responsible for helping group member to get out of stuck state or discouraged state.
Reflector is responsible to review and record the collective efforts and behaviors.
Presenter is responsible to present the results and findings of the group.

****** If any team member is absent then kindly mention who is taking the responsibility as the replacement*****

BLOSSOMS MODULE_FLAW OF AVERAGES_Worksheet_Session 9
Greetings and Introduction to the Module
Module Entry (required to raise the curiosity about the topic)_watch video
Activity 1: Flaw of Average #1
"the average is not always a good description of the actual situation."

Modeling:
Let's imagine that you're at the edge of a river that you want to cross. But, there is a sign. The sign says, "**Average river depth: one meter**." Now, given this sign, would you cross the river?

- Please record your responses in the space provided under each question.
- Prompts will help you to come out of any STUCK state.

Entry Phase:

Entry 1: What do you KNOW from the question and also from your past experience?
Entry 2: What do I WANT?
Entry 3: What can I INTRODUCE? (e.g images, diagrams, symbols)

Attack Phase:

- Do the necessary calculations
- Justify your answers.
- Think of examples where you would be able to keep your head above the water.
- Think of examples where you would not be able to keep your head above the water.
- Try different shapes of the riverbed with an average depth of 1 meter.
- Draw different shapes of the riverbed
- Check the Resolution

APPENDIX C

Quiz # 2_Mathematical Thinking Lab

Group Name ________________________________Date:______________________

Question 1: Paper Strip

Imagine a long thin strip of paper stretched out in front of you, left to right. Imagine taking the ends in your hands and placing the right hand end on top of the left. Now press the strip flat so that it is folded in half and has a crease. Repeat the whole operation on the new strip two more times.

a) How many creases are there?
b) How many creases will there be if the operation is repeated 10 times in total?
c) What pattern have you noticed in the creases?
d) mathematical thinking processes are required to solve this problem?

Entry Phase:

Entry 1: What do you KNOW from the question and also from your past experience?
Entry 2: What do I WANT?
Entry 3: What can I INTRODUCE? (e.g. images, diagrams, symbols)

Attack Phase:

STUCK? TRY IT NOW

➢ Specialize mentally by counting the creases after two folds.
➢ Perhaps a diagram will steady your mental image.
➢ Specialize by trying it on a strip of paper.
➢ Try three folds and four folds. Look for a pattern.
➢ What do you want to find? Be clear and precise.
➢ Is there something related to the creases that you can count more easily?
➢ Check any conjectures on new examples!
➢ Justify your answer

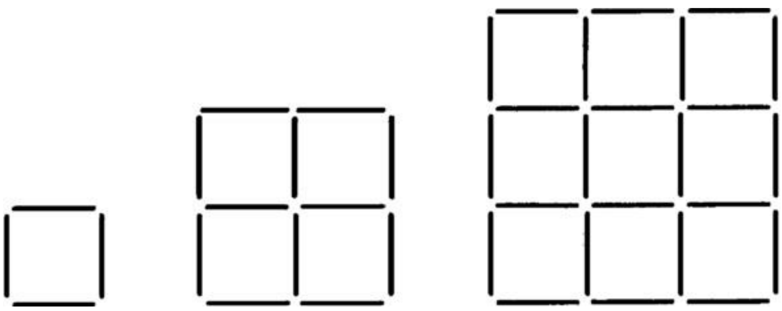
Review Phase:

Review 1: CHECK the resolution
Review 2: REFLECT on the key ideas and key moments/findings

Question 2: Squares of Matches

How many matches are required to make N^2 unit squares in a square array as in the following sequence?

Phase:
Entry
you
the
also
past
Entry



Entry
1: What do
KNOW from
question and
from your
experience?
2: What do I
WANT?

Entry 3: What can I INTRODUCE? (e.g. images, diagrams, symbols)

Attack Phase:

STUCK? TRY IT NOW

➢ Specialize systematically.
➢ Count the matches!
➢ Do the next example. Look for a pattern.
➢ How did you count the matches? Generalize!

APPENDIX C

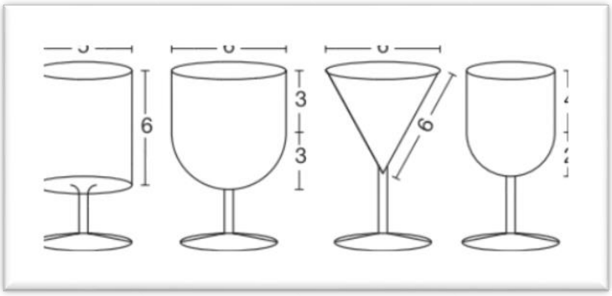
➤ Try other systematic methods of counting.

Review Phase:

Review 1: CHECK the resolution
Review 2: REFLECT on the key ideas and key moments/findings

Question 3- Glasses

Figure below shows four glasses (not drawn to scale).



The second and fourth are cylinders with a hemispherical bottom.

a) Find the vertical height of the third and the diameter of the fourth. Then put them in order from largest capacity to smallest by estimation.
b) Now estimate the water levels if each glass was half-full.
c) Draw the approximate functions for the filling of first and third glasses if both are filled with liquids at the same rate.

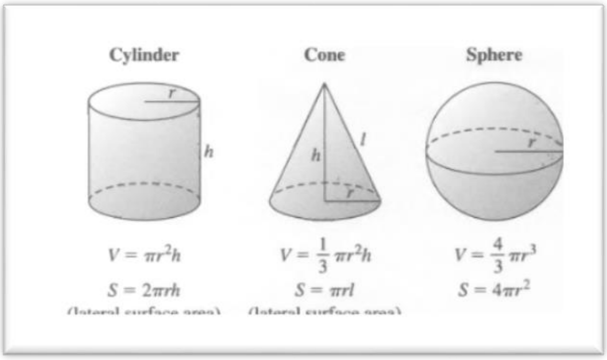
Entry Phase:

Entry 1: What do you KNOW from the question and also from your past experience?
Entry 2: What do I WANT?
Entry 3: What can I INTRODUCE? (e.g. images, diagrams, symbols)

Attack Phase:

STUCK? TRY IT NOW

- Check your conjecture by calculating the volumes.
- Check your estimates by calculating the levels. Use the help box below to use the formulas for volumes.



- The approximate function i.e.height of liquid in the glass vs time can be drawn

Review Phase:

Review 1: CHECK the resolution
Review 2: REFLECT on the key ideas and key moments/findings

APPENDIX D

Sample of Reduced Data for Persona Development (Persona ID# 1)

Educational Background	
Gender	Male
Major or field of interest	Mechanical Engineering (pure)
Pre-university mathematics classes already taken	Engineering mathematics
Background Institution (high school, community college, matriculation center, etc.)	Centre of Foundation in Science University of Malaya
List the topic already learned	Algebra & Geometry, Calculus, Engineering Mathematics, Vector
Current GPA	3.89

Strengths and weaknesses in Maths	
Strengths	Confident
Weaknesses	Lack of time management
Prior experiences in Maths	
One example of a great experience	always scored good
One example of a bad experience	got C in exam
Habits of studying and preparing for Exam	
Studying	Revision
Exam Preparation	Practice
Expectations, fears, difficulties, and needs from Engineering Mathematics	
Expectations	I can score provided I do all the course
Fears	Forget what I have learned especially in exam
Difficulties	Carelessness
Needs	Hardworking
Responses related to Mathematical Thinking Lab	
Previous experience with mathematical thinking	no experience
Any experience of doing activities related to applying course material in the real world	Yes
Expectations from MTL	No response
Reservations or concerns	Nothing because I do not familiar about that
Suggestions for success criteria to determine how useful this MTL	hustle, royalty, respect :P
Self- reflection about what he hopes to get out of the class and how he is going to achieve his goals.	Dear me, just do it and nothing is impossible.

Observed attitude in the Class and in MTL	
Attitude in Class	Humming, disturbing friends, lower level of attention in the class
Attitude in MTL	Low interest, low level of motivation, and low attention

Relative Position is Class during Formative Assessment	
Test 1 (15%)	7th
Test 2 (15%)	19th
Assignments (12%)	21st
Quizzes (8%)	9th
Final Exam (50%)	1st
Overall Position	1st

APPENDIX E

Deductive Coding Scheme

Code	Long Name	Description
MTPSP	Mathematical Thinking Problem Solving Phases	The process problem solving is divided loosely into three phases, called Entry, Attack and Review. Moving from one phase to another corresponds to a change in your feelings about the question and reflects the progress which is or is not being made. Learning to identify these phases in your own thinking will enable you to recognize appropriate activities.
E	Entry Phase	This first phase of tackling a question begins when someone first encounter the question, and ends when someone has become involved in attempting to resolve it. Work in the Entry phase often begins with specializing in order to get to grips with the question. The Entry phase work is largely in formulating the question precisely and in deciding exactly what I want to do. The other activity, which often takes place during Entry, is to make some technical preparations for the main attack, such as deciding on a notation or a means of recording the results of specializing.
E1	Entry 1_I Know	What do I KNOW? What do you KNOW from the question? From the past experience (Prior Knowledge)
E10	Know_No Response	No written response on the activity worksheet during the Entry 1 sub-phase
E11	Know_Partial/Incorrect Response	Partial Response or Incorrect response during the Entry 1 sub-phase
E12	Know_Complete/Correct Response	Complete response and correct response during the Entry 1 sub-phase
E2	Entry 2_I Want	What do I WANT? What do you WANT from the problem?
E20	Want_No Response	No written response on the activity worksheet during the Entry 2 sub-phase
E21	Want_Partial/Incorrect Response	Partial Response or Incorrect response during the Entry 2 sub-phase
E22	Want_Complete/Correct Response	Complete response and correct response during the Entry 2 sub-phase
E3	Entry 3_I Introduce	What can I INTRODUCE? What could you INTRODUCE to be able to express what you WANT succinctly?
E30	Introduce_No Response	No written response on the activity worksheet during the Entry 3 sub-phase
E31	Introduce_Partial/Incorrect Response	Partial Response or Incorrect response during the Entry 3 sub-phase
E32	Introduce_Complete/Correct Response	Complete response and correct response during the Entry 3 sub-phase
A	Attack Phase	The major effort to resolve a question occurs in the Attack phase. This may lead ultimately to a complete resolution, or it may terminate in an incomplete resolution consisting of conjectures and unresolved questions. In either case, activity should not cease until after a final phase of Review. The states which are particularly associated with Attack are STUCK! and AHA! and the fundamental mathematical processes called upon are conjecturing and justifying convincingly. These in turn depend on specializing and generalizing. Attempts to resolve difficulties may stay within Attack or may lead back to Entry. Before leaving a question it is essential to carry out a third phase, Review. Discovery of an error or inadequacy may lead back to Entry or to Attack, and if an interesting new question is uncovered, perhaps through generalizing the resolution, the whole process begins again. During Attack several different approaches may be taken and several plans may be formulated and tried out. When a new plan is being implemented, work may progress at a great rate. On the other hand, when all ideas have been tried, long periods of waiting and mulling for new insight or for a new approach may characterize the phase.

APPENDIX E

A0	Pre-Procedure	Before an operational procedure has been developed for a particular problems there may be no solution or a partial solution
A0a	No solution	No written response on the activity worksheet during the attack phase
A0b	Partial solution or initial action(s)	Partial solution or initial action(s) prior to building a procedure
Ai	Procedure	It is the step-by-step procedure to carry out an operation or step-by-step solution of a routine Problem. Procedural stage is sufficient to solve a routine problem using a specific procedure.
Aii	Multi-Procedure	It represents several different procedures (or representations) to carry out the same operation, with a choice of the most efficient or Choice of solutions for increased efficiency. Multi-procedural stage has more than one procedure, offering the possibility of selecting a more efficient procedure for a routine problem.
Aiii	Process	It represents equivalent solutions with various alternatives or Equivalent procedures as a single process. Process stage sees the process as input-output operation, where the various procedures that can try out the process are seen to be equivalent.
Aiv	Procept	A single thinkable concept represented by equivalent symbols operating dually as process or concept. Procept stage is a flexible stage when it is possible to think about the symbols in the problem either as process or concept, and write them in different ways that are conceived as alternative ways of writing a single crystalline concept
R	Review Phase	It involves both looking back, to CHECK what you have done and to REFLECT on key events, processes and difficulties, and looking forward to EXTEND the processes and the results to a wider context. Discovery of an error or inadequacy may lead back to Entry or to Attack, and if an interesting new question is uncovered, perhaps through generalizing the resolution, the whole process begins again.
R1	Check	Checking the calculations and arguments to ensure that computations are appropriate, consequences of conclusions to see if they are reasonable and the resolution fits the question
R10	No checking/no tracking	No checking or tracking evidence on the activity worksheet during the Check sub-phase
R11	Partial Checking	Partial Checking evidence on the activity worksheet during the Check sub-phase
R12	Complete Checking	Complete Checking evidence on the activity worksheet during the Check sub-phase
R2	Reflect	Reflecting on key ideas and moments, on implications of conjectures and arguments, on your resolution: can it be made clearer?
R20	No Reflection	No Reflection on the activity worksheet during the Reflect sub-phase
R21	Partial Reflection	Partial Reflection on the activity worksheet during the Reflect sub-phase
R22	Complete Reflection	Complete Reflection on the activity worksheet during the Reflect sub-phase
R3	Extend	Extending the result to a wider context by generalizing, by seeking a new path to the resolution, and by altering some of the constraints
R30	No Extending	No Extending on the activity worksheet during the Extend sub-phase
R31	Partial Extending	Partial Extending on the activity worksheet during the Extend sub-phase
R32	Complete Extending	Complete Extending on the activity worksheet during the Extend sub-phase

APPENDIX E

MTW	Mathematical Thinking World	Thinking mathematically involves movements between different worlds of experience. These three worlds provide a background structure for the process of building mathematical models of situations arising in the material or in the mathematical world: perceiving a situation in mathematical terms through recognizing relationships and conceiving of them as properties that can hold in many situations, and expressing these properties in some form, usually but not always algebraic.
CE	Conceptual Embodiment	the conceptual-embodied world, based on perception of and reflection on properties of objects, initially seen and sensed in the real world but then imagined in the mind;
BES	Blended Embodied and Symbolic	Blending the Conceptual Embodiment and Operational Symbolism
OS	Operational Symbolism	the proceptual-symbolic world that grows out of the embodied world through action (such as counting) and is symbolized as thinkable concepts (such as number) that function both as processes to do and concepts to think about (procepts);
AF	*** Not used*** Axiomatic Formalism	the axiomatic-formal world (based on formal definitions and proof), which reverses the sequence of construction of meaning from definitions based on known objects to formal concepts based on set- theoretic
SB	Set-Befores	genetic facilities that we all share that are the basis of mathematical thinking and operate in history are SET-Befores
MB	Met-Befores	individual experiences we have in life that lead to the personal development of mathematical thinking are MET-Befores
MTP	Mathematical Thinking Process	We use the processes of Specializing, generalizing, Conjecturing, Justifying etc. to think mathematically.
S	Specializing	The process in which examples are used to learn about the question or problem. The examples chosen are special in the sense that they are particular instances of a more general situation in the question. It is advised to be done systematically.
G	Generalizing	The process of trying to articulate a sense of some underlying pattern is called generalizing. The process of moving from a few instances to making guesses about a wide class of cases. *It starts when you sense an underlying pattern, or relationship even if you cannot articulate it.* Types of generalizing: empirical and structural. *Empirical Generalization* comes about when you look at several, sometimes many, cases or instances and ask yourself what is the same about them all. By stressing the sameness (and consequently ignoring differences), you effectively generalize. When you articulate the sameness, you produce a conjectured general property, which then has to be justified by reference to structure. *Structural Generalization* arises when you recognize relationships in one or very few cases. Perceiving these relationships as properties, your articulation is again a conjectured generalization which then has to be justified by reference to underlying structure
Cj	Conjecturing	It is the process of sensing or guessing that something might be true and investigating its truth. The process of conjecturing hinges on being able to recognize a pattern or an analogy, in other words, on being able to make a generalization. Part of the art of conjecturing is to be open to new interpretations, which arise unexpectedly in what otherwise, might seem a familiar context. Conjecturing on a small scale lies at the heart of mathematical thinking. A conjecture is a statement which appears reasonable, but whose truth has not been established. In other words, it has not been convincingly justified and yet it is not known to be contradicted by any examples, nor is it known to have any consequences which are false. *Articulating, testing and modifying conjectures form the backbone of a resolution.*

APPENDIX E

J	Justifying (Seeking Why)	Seeking why involves getting a sense of some underlying reason for the truth of your conjecture. The process of justifying the conjecture involves more generalizing, with a shift in emphasis from guessing what may be true to seeing why it may be true. justifying has to do with revealing an underlying structure or relationship that links I KNOW with I WANT
Cv	Convincing (Explaining Why)	Explaining why involves convincing yourself and, more importantly, convincing others that you can justify your arguments. Explaining why is largely based on the idea of mathematical structure, an important notion that lies behind attempts to explain why something might be true, and it is a development of conjecturing.
Im	Imagining	By imagining we include all forms of mental imagery, not only images like 'pictures in the mind', but also any recalled sense-based experience.
Ex	Expressing	The process to describing/showing in different forms what is being imagined . Learning to express what you are imagining, to capture and 'tie down' distinctions and relationships, to articulate or express perceived properties is to bring imagery to expression. It is possible to use material objects, diagrams and pictures, voice tones and gestures, words and symbols to express discerned objects, recognized relationships and perceived properties.
St	Stressing	In mathematics, the action of either extending or restricting meaning is a manifestation of amplifying and diminishing, of stressing and ignoring.
Ig	Ignoring	In mathematics, the action of either extending or restricting meaning is a manifestation of amplifying and diminishing, of stressing and ignoring.
Cl	Classifying	To classify something is thus to perceive it as an instance of a property, having first discerned 'it' from its surroundings. To characterize it is to produce an alternative collection of properties so that anything belonging to the classification satisfies those properties, and anything satisfying those properties belongs to the classification. A pervasive mathematical theme is to classify objects by properties, and then to characterize those properties by means of other proper- ties
Ch	Characterizing	To characterize it is to produce an alternative collection of properties so that anything belonging to the classification satisfies those properties, and anything satisfying those properties belongs to the classification. A pervasive mathematical theme is to classify objects by properties, and then to characterize those properties by means of other proper- ties

APPENDIX F

Sample Entries in database for BLOSSOMS video

No	Title	Summary	Form 3	Form 4	Form 5	Pre-Uni	Subject
1	**Amazing Problems: Arithmetic and Geometric Sequences**	This lesson aims to introduce arithmetic sequences and geometric sequences to secondary school students. It uses two wonderful problems to do that and it explains the laws of those two kinds of sequences. All of the information upon which this lesson relies should have been introduced to students previously in intermediate school. These include equations and their operations, so the lesson has no prerequisites of secondary school lessons. The video lesson is 20 minutes long with approximately 20 to 25 minutes of activities. There are no additional materials for this lesson, but it is recommended that teachers have a chessboard with 1KG of wheat to show to students and ask them to try to put wheat seeds on the 64 chess squares as mentioned in the video!	**Yes**	**Yes**	**Yes**	**Yes**	Mathematics
2	**Are Random Triangles Acute or Obtuse?**	This learning video deals with a question of geometrical probability. A key idea presented is the fact that a linear equation in three dimensions produces a plane. The video focuses on random triangles that are defined by their three respective angles. These angles are chosen randomly subject to a constraint that they must sum to 180 degrees. One class period is required to complete this learning video, and the only prerequisites are a familiarity with geometry and an understanding of the equation for a plane, which is presented in the module. Materials needed for this lesson include blackboard and chalk. Optional materials include a cardboard box and colored paper. An example of the types of in-class activities for between segments of the video is: Ask six students for numbers and make those numbers the coordinates x,y of three points. Then have the class try to figure out how to decide if the triangle with those corners is acute or obtuse.	**No**	**YES**	**Yes**	**Yes**	Mathematics

APPENDIX G

Problem Solving Activity and Activity Response Analysis

BLOSSOMS MODULE_The Power of Exponentials, Big and Small
Module Integration and Implementation

Watch Video Segment # 1
A situation is modelled to raise the curiosity about the topic and to introduce the problem related to exponentials in the form of a story.
Story Problem embedded in Segment # 1:
Scene 1*: [Two students Nataly and Swati talking to each other while struggling with homework related to the concept of exponentials]*
Swati: Nataly, I just hate doing this homework.
Nataly: I know. Exponentials are a huge drag.
Swati: Yeah, well, now that you mentioned it, let me tell you a story my grandmother once told me about exponentials.
Scene 2*: [A queen and a mathematician is shown playing the game of chess]*
Background narration: There used to be a queen in India, and she got really bored playing the routine games. And so she asked all the mathematicians in the country to come up with a new game to amuse her.
And there was a poor mathematician who after years of carefully working through different ideas came up with the game of chess. And the queen, she was so pleased, and asked the mathematician to name her price.
Queen: Tell me, my math wizard, any reward you feel worthy of. I love your game of chess.
Mathematician: My lady, I'm a poor woman. All that I need is to have enough grains of rice to feed my family. I would like to have one grain of rice for the first square of the chessboard, two for the second, four for the third, and so on. Every square must have double the number of grains as the previous one.
Queen: Ha. A chessboard has only 64 squares. That's a triviality for a mighty queen like me.
Scene3: *[Queen ordering his treasurer where as treasurer is nodding the head]*
Queen: Treasurer, please give this young mathematician all the grains she's asked for.
Background narration: The queen seemed to think that the award was very small, a triviality, and that she would be able to pay the mathematician with a single bag of rice. Do you think the award was very small?
Scene4*: [Showing 2 students again and asking audience (who are in the actual classroom) some questions related to problem posed in the above scenario]*
Nataly: Hm. I think I need some help here. Do you think the award was very small? Was the treasurer able to pay off the mathematician with a single bag of rice? Was the queen able to pay off the mathematician at all? Discuss with your neighbours and with your teacher, and we'll be back in a few minutes.
After the first video segment, the face 2 face instruction starts and control is transferred to class teacher. Class teacher instructs the students to start recording their responses on the worksheets provided at the start of the session.

Do Activity #1:
Activity 1 related details are shown on the projector as well as printed on the worksheets already given to students.

Reward:
1 grain of rice for the 1st square of the chessboard, two for the second, four for the third, and so on. Every square must have double the number of grains as the previous one.

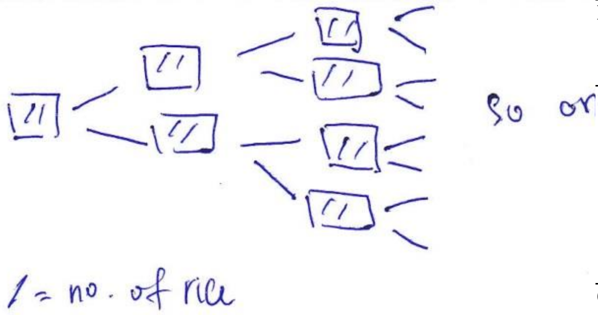
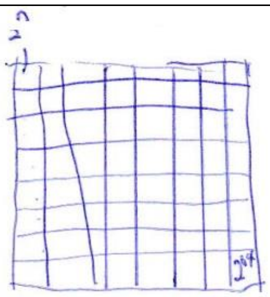
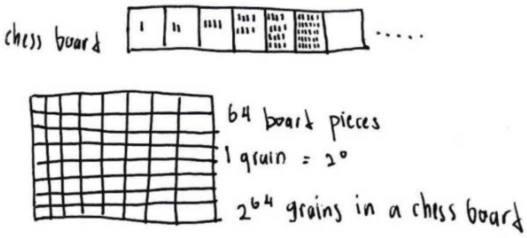
Questions:
Do you think the award was very small? [require speculation]
Was the treasurer able to pay off the mathematician with a single bag of rice? [need conceptual embodiment and operational symbolism to answer the question]
Was the queen able to pay off the mathematician at all? [need specializing, generalizing, conjecturing, Justifying to answer the question]

Assumption:
- Number of grains of the rice in a bag = 800,000

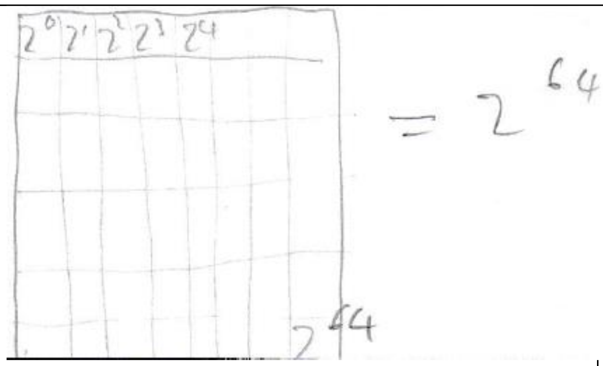
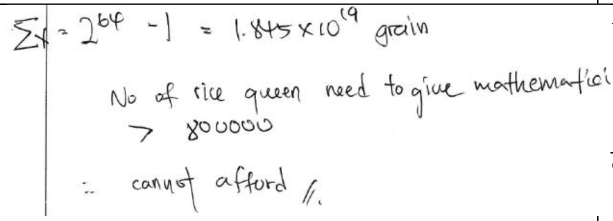
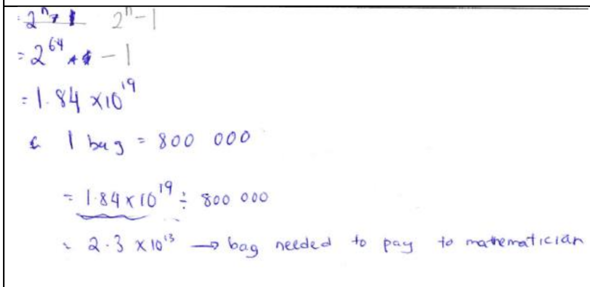
Instructions: [Instructions are printed on the worksheets as well as verbally conveyed to the students during the session]
- Please record your responses in the space provided under each question.
- Question will guide how to respond to different Phases.
- Prompts will help you to come out of any STUCK state.

Activity Item	Activity Response Examples (4examples/item)	Coding	Reason for coding
Entry Phase: Entry 1: What do you KNOW from the question and also from your past experience?	**No response**	E10	There is no written response
	No response	E10	There is no written response
	No response	E10	There is no written response
	1 bag consist of 800,000 number of grain	E11	Partial Response
Entry 2: What do I WANT?	To find the amount of rice awarde	E21	Partial Response
	• to calculate the amount of ~~rice~~ grain that queen needs to pay to mathematician	E21	Partial Response
	• Number of grains • Number ~~o~~ of grains insite a chess board • ~~Thots~~ ~~TA~~ Does the grains in a chess board exceeds a bag of rice	E21	Partial Response
	Summation of all grain in each squares - either it exceed the bag of rice.	E22	Correct Response

APPENDIX G

Entry 3: What can I INTRODUCE ? (e.g images, diagrams, symbols)		E32	Correct Response
		S	The response shows that student tries to do specializing by randomly getting the feel of the question and systematically preparing the ground for generalizing
		CE	The diagram shows the conceptual-embodied world, based on perception of and reflection on properties of objects, initially seen and sensed in the real world but then imagined in the mind and then drawn on the worksheet
		E32	Correct Response
		S	The response shows that student tries to do specializing by randomly getting the feel of the question and systematically preparing the ground for generalizing
		BES	The response shows the blending of the Conceptual Embodiment and Operational Symolism
		E32	Complete as well as Correct Response
		S	The response shows that student tries to do specializing by randomly getting the feel of the question and systematically preparing the ground for generalizing and artfully, moving towards the generalization
		CE	The diagram shows the conceptual-embodied world, based on perception of and reflection on properties of objects, initially seen and sensed in the real world but then imagined in the mind and then drawn on the worksheet
		E32	Correct Response

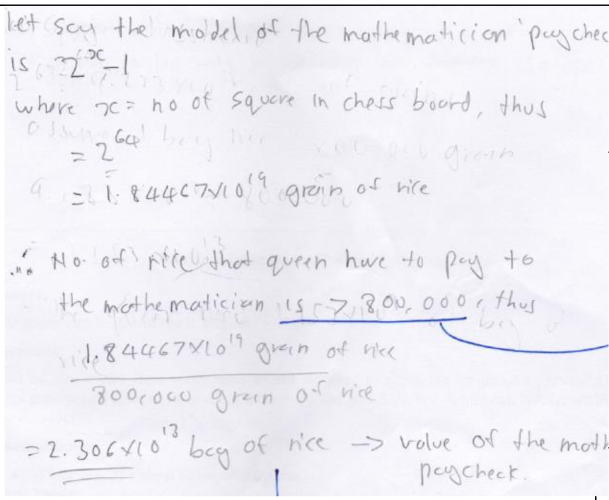
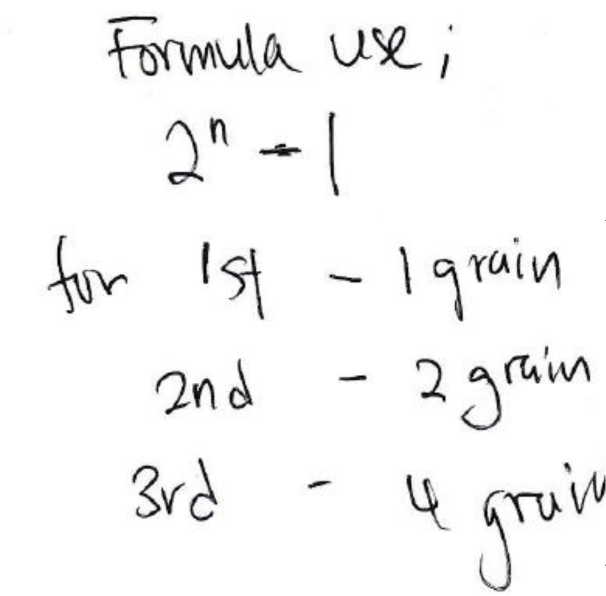
APPENDIX G

		S	The response shows that student tries to do specializing by randomly getting the feel of the question and systematically preparing the ground for generalizing
		BES	The response shows the blending of the Conceptual Embodiment and Operational Symbolism
Attack Phase: ➢ Do the necessary calculations ➢ Justify your answers.		Ai	The resolution shows the step by sep procedure to find out the value of exponential operation
		G→J	The solution shows that the student successfully articulated a sense of the underlying pattern from the entry phase although he did not elaborate the transition in detail to give evidence of conjecture. He then tried to justify why queen will not be able to buy the mathematician an award.
		OS	The solution shows the proceptual-symbolic world that grows out of the embodied world through action (such as making sense of the number of grains on the chess board) and is symbolized as thinkable concepts (such as exponentials)
		Ai	The resolution shows the step by step procedure to find out the value of exponential operation
		G→J	The solution shows that the student successfully articulated a sense of the underlying pattern from the entry phase although he did not elaborate the transition in detail to give evidence of

APPENDIX G

			conjecture. He then tried to justify the huge number of bags after dividing the total number of grains from the number of grains per bag.
		OS	The solution shows the proceptual-symbolic world that grows out of the embodied world through action (such as making sense of the number of grains on the chess board) and is symbolized as thinkable concepts (such as exponentials)
	$1^0, 2^1, 2^2, 2^3, 2^4, 2^5, \ldots\ldots$ 2^{64} $= 1.84 \times 10^{19}$ grains ~~in a ch~~ 1 bag = 800 000 grains $\therefore$ 2.3×10^{13} bags of grains shoult be paid. by the queen towa the mathematician	Ai	The resolution shows the step by step procedure to find out the value of exponential operation but the student did a mistake by not reaching to a proper generalized form that is 2^n-1
		G→Cj→J	The solution shows that the student partially articulated a sense of the underlying pattern from the entry phase and shows the evidence of conjecture. He then jumped to the number of bags (how did he find?)
		OS	The solution shows the proceptual-symbolic world that grows out of the embodied world through action (such as making sense of the number of grains on the chess board) and is symbolized as thinkable concepts (such as exponentials)
		Ai	The resolution shows the step by step procedure to find out the value of exponential operation but the student did a mistake by not using the generalized form that is 2^n-1 in the actual calculation that shows the regularities are missing for the concept exponentials. In a broader sense the value is very big thus nullifying the

APPENDIX G

	 let say the model of the mathematician 'paycheck' is $2^{x}-1$ where x = no of square in chess board, thus $= 2^{64}$ $= 1.84467\times10^{19}$ grain of rice ∴ No. of rice that queen have to pay to the mathematician is 7,800,000, thus $\frac{1.84467\times10^{19} \text{ grain of rice}}{800,000 \text{ grain of rice}}$ $= 2.306\times10^{13}$ bag of rice → value of the math paycheck.		effect of 1 being subtracted
		G→J	The solution shows that the student partially articulated a sense of the underlying pattern from the entry phase and did not show the evidence of conjecture. He then jumped to the number of bags (how did he find?)
		OS	The solution shows the proceptual-symbolic world that grows out of the embodied world through action (such as making sense of the number of grains on the chess board) and is symbolized as thinkable concepts (such as exponentials)
Review Phase: Review 1: CHECK the resolution	We suppose that students check their resolutions. In future, we recommend putting a check box for students to mark if they have done it.		
Review 2: REFLECT on the key ideas and key moments/findings	 Formula use; $2^{n}-1$ for 1st − 1 grain 2nd − 2 grain 3rd − 4 grain	R21	The response shows the partial reflection to make clear what and how the concept is used but the response does not show any reflection on the key moments during the resolution
		G→S	The response shows that student tries to go back from generalizing to specializing by rethinking about the initial problem posed. But he was unable to relate the sum of grains has the above formula not the number of grains on every square
		OS	The solution shows the response in symbolic world even if the individual cases does not match with the actual general form. This would be counted as a careless mistake rather than a conceptual mistake.
	No response	R20	No reflection
		R21	Partial Reflection

APPENDIX G

	~~$e^n + e^{n-1}$~~ $\Sigma a^n : a^{n-1}$ ~~n = numbers of grains~~ a : ~~number~~ grains n : number of ~~pieces~~ box in a chess board	G	The response shows incorrect generalization of the problem cases
		OS	The wrong response shows that the student did not utilize the conceptual embodiment and directly tried to map a formula to generalize the situation.
	formula use Total = $2^n - 1$ 1st square of chess board – 1 grain 2nd square of chess board – 2 grain 3rd square of chess board – 4 grai	R21	The response shows the partial reflection to make clear what and how the concept is used but the response does not show any reflection on the key moments during the resolution
		G→S	The response shows that student tries to go back from generalizing to specializing by rethinking about the initial problem posed. But he was unable to relate the sum of grains has the above formula not the number of grains on every square
		OS	The solution shows the response in symbolic world even if the individual cases does not match with the actual general form. This would be counted as a careless mistake rather than a conceptual mistake.
Review 3: EXTEND to a wider context by introducing examples showing the power of exponentials.	Exponent are applied to measure the strength of earthquak Earthquake level 1 is 1x10 while second level is 10x1 x1 and so on.	R32	Correct example to map the exponentials
	No response	R30	No example given
	No response	R30	No example given
	Exponential of nuclear deccaying c) To calculate the rate of decay of ~~Nuclear~~ 150to	R31	Partial extending

Watch video segment#2
Scene5: *[Treasurer explaining the queen mathematically that they are unable to pay the reward]*
Treasurer: My lady, I think there is a problem.
Queen: What is it?
Treasurer: Well, we've used hundreds of bags of rice, and we have not even covered half of the squares of the chessboard.
Queen: How can that be? We just started with one grain of rice.
Treasurer: Well, as you can see, at the beginning, we just needed one grain of rice. And then three and seven. Each time we added the rice for a new square, we added one more grain than the amount on all of the previous squares combined. On some of the early squares, I noticed that the total we had paid out up to that square was 2 to the n minus 1 for the square n. For instance, we paid 2 to the third minus 1 or seven grains up through square three. Since we added eight more on square four, we had paid a total of 2 to the fourth minus 1, or 15 grains through that square.

APPENDIX G

Through a technique called mathematical induction, we can show that we paid 2 to the n minus 1 grains through the nth square no matter the value of n.
Queen: But what caused us to pay so much rice. Even though we doubled from the previous square, we have only 15 grains of rice till now.
Treasurer: Well, the problem, Your Majesty, is that after the first few squares, our payouts grow very rapidly. While at the end of the first row, we have only placed down a few hundred grains, and by the second, we have still placed down less than a full bag.
We reached a million grains paid by the third row, and this is when the payments started getting big quickly. We had already placed down a billion grains of rice, more than 1,000 sacks by the fourth row when we still had more than half of the board to go.
Had we been able to keep bringing the rice, we would have paid a trillion grains by the fifth row, a quadrillion by the seventh row, and according to my calculations, more than 18 quintillion grains of rice in total. That heap of rice would have been larger than the largest mountain in the world.
Queen: Curse you, wise mathematician, you tricked me. I have gone from the richest queen in the world to the poorest woman.
Scene 6: *[Showing Swati and Nataly again making sense of the mathematical concept of exponentials and student 3 named John jumps in to introduce the next activity]*
Swati: See, ignore exponentials, and you could lose a fortune.
Nataly: Well, that make sense. But I am not a queen, and I have no mathematicians in my employ, so I am still stuck with my homework.
John: Guys, guys, oh my, let me tell you. I just made a fabulous discovery.
Swati: Hey, John, what's going on?
John: Nataly, let me tell you. So I found when you take a stack of paper and you tear it in half and put one half on top of the other, it becomes twice as tall with each tear, which means in no time flat this pile is going to be enormous, very tall. So let me tell you my friends, today, I am going to the moon. See you.
Nataly: OH geez, John must have had too much coffee today.
Swati: It seems strangely similar to the one our queen faced in the story. There with every new square, the amount of grain doubled, and here with every new tear, the height of the stack doubles. So since we know exponentials grow so fast, you never know, starting with a 0.1 millimeter thick sheet, you might actually reach the moon.
Nataly:: I don't buy that. The moon is too far away. Let's think of a height of a person first.
What do you think? How many tears would we need to reach the height of a person? How many tears would John need to reach the moon? Do you think we should call NASA with our new discovery? Think about it a little bit, and we will come back shortly.
[Control now is again passed to the class teacher and class teacher used worksheet already designed to record students' responses]

Do Activity 2

Process:
Tear paper into half, and stack. Repeat [Instead of tearing paper, class teacher suggested to fold the paper in half and then repeat the process

Questions:
What do you think? How many tears would we need to reach the height of a person? How many tears would John need to reach the moon? Do you think we should call NASA with our new discovery?

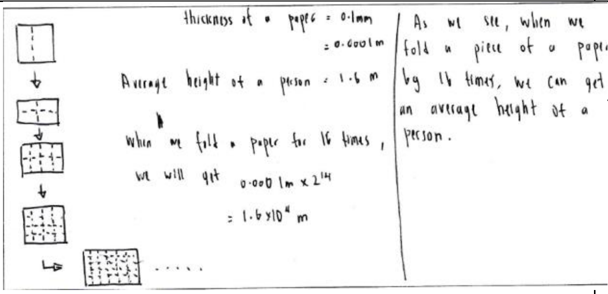
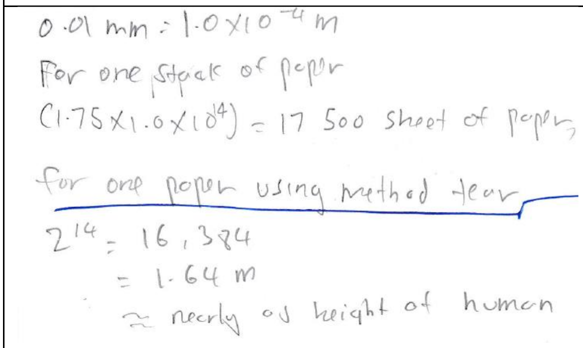
Assumption:
Thickness of a sheet of paper = 0.1mm

Activity Item	Activity Response Example	Coding	Reason for coding
Entry Phase: 1) What do you KNOW?	Max. no of tear to reach a person's height is thickness of sheet of paper = 0.1 mm A stack of paper = 12.6mm	E12	Correct response
	•Thickness of a shee paper = 0.1mm	E11	Partial response
	Height of A Person = 1.55m Thickness of a sheet of paper = 0.1mm	E12	Correct response
	The thickness of a sheet of paper = 0.1mm = 0.0001(My height is 1.74m need (174,000 sheet of paper to reach my height)	E12	Correct response
2) What do you WANT to KNOW?	height of person 1.55cm	E21	Incorrect Response
	How many tears needed to reach the height of a person = 155	E21	Incomplete Response
	How many papers used to reach the height of a person How many tears used used to reach the height of a person?	E21	Incomplete Response

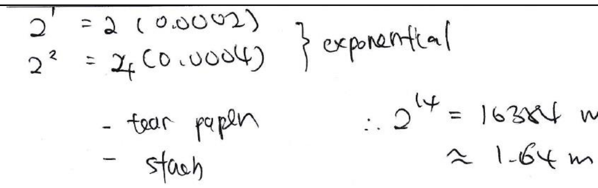
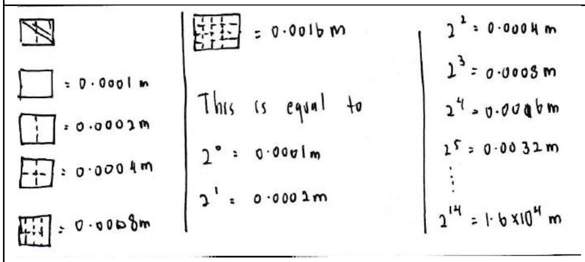
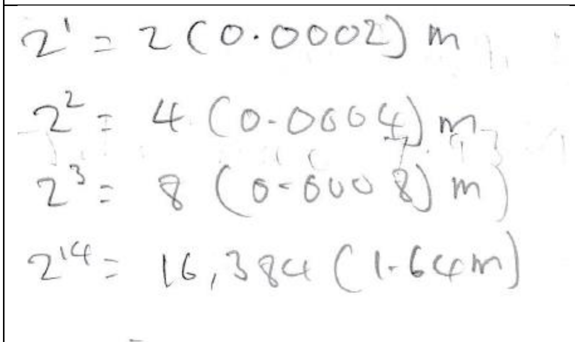
APPENDIX G

	How many tears do we need to reach average height of human	E21	Incomplete Response
3) What could you INTRODUCE to be able to express what you WANT concisely?	Fixed how much you need to fold the pape	E31	Incorrect Response
	Find how much you need to fold to find the height of a perso	E31	Incorrect Response
	Try to fold a piece of a paper as much as possible to get same height of a person	E31	Partial Response but it shows he is trying to relate the no of fold of a paper to the height of a person.
	how much you need to fold sheet of pap	E31	Incorrect Response
Attack Phase: • Do necessary experiments with paper. • Draw the diagram to help you visualize the scenario if needed.	~~ot~~ 0.1 mm = 1.0×10^{-4} m ~~Jus~~ One paper $1.75 \times 1 \times 10^{4} = 17500$ sheet paper 1 paper using tear method $2^{14} = 16384$ ~~cm~~ mm ≈ 1.64 m ≈ a person's height	Ai	The resolution shows the step by step procedure to find out the value of exponential operation but for the half of the problem, the other half i.e. no of tear to reach the moon is not attempted
		S→Cj→J	The solution shows that the student successfully articulated a sense of the underlying pattern from the entry phase showing the evidence of conjecture. He then tried to justify why 14 tears will be equal to the height of the person. The student used the table of exponentials provided with worksheet
		CE→OS	The solution shows the proceptual-symbolic world that grows out of the embodied world through action
	height of person → 1.75m Thickness of paper → 1×10^{-4} m $= \frac{1.75}{1 \times 10^{-4}}$ = 17 500 − 1 = 17 499 tears needed to reach the height of a person	Ai	The response shows the step-by-step procedure to carry out an operation but the mistake in terms of related no to tear to the total layers after tear shows the conceptual issue.
		S→Cj→J	The solution shows that the student tried to articulate a sense of the underlying case from the entry

APPENDIX G

			phase showing the incorrect conjecture. He then tried to wrongly justify that 17499 tears are needed to reach the height of a person
		CE→OS	Although the solution is incorrect, it shows the proceptual-symbolic world that grows out of the embodied world through action
		Aii	Several different representations to carry out the same operation. He used
		S→Cj→J	The solution shows that the student tried to articulate a sense of the underlying case from the entry phase showing the partially correct conjecture. He then tried to wrongly justify the answer whereas we can clearly see the careless mistake of using 16 folds instead of 14 folds. There must be some influence of overhear solution by others in the group.
		CE→OS	The solution is partially correct, it shows the proceptual-symbolic world that grows out of the conceptual embodied world through action
		Ai	The response shows the step by step procedure to carry out an operation
		S→Cj→J	The solution shows that the student tried to articulate a sense of the underlying case from the entry phase showing the correct conjecture. He then tried to justify the answer in terms of relating the exponential values with the height of the person
		CE→OS	The solution is partially correct, it shows the proceptual-symbolic world that grows out of the conceptual embodied world through action
Review Phase: Review 1: CHECK the resolution	We suppose that students have check their resolutions. In future, we recommend putting a check box for students to mark if they have done it.		

APPENDIX G

<table>
<tr><td rowspan="9">Review 2: REFLECT on the key ideas and key moments</td><td rowspan="3"></td><td>R21</td><td>The response shows the partial reflection to make clear what and how the concept is used but the response does not show any reflection on the key moments during the resolution</td></tr>
<tr><td>J</td><td>The response shows the justification of the no of tear reaching a certain value.</td></tr>
<tr><td>OS</td><td>The solution shows the response in symbolic world. This would be counted as an incomplete grasp of the concept.</td></tr>
<tr><td>No response</td><td>R20</td><td>No reflection</td></tr>
<tr><td rowspan="3"></td><td>R21</td><td>The response shows the partial reflection to make clear what and how the concept is used but the response does not show any reflection on the key moments during the resolution</td></tr>
<tr><td>S</td><td>The response shows that student tries to repeat the different cases to carry on with the specializing.</td></tr>
<tr><td>BES</td><td>The solution shows blended use of operational symbolism and conceptual embodiment.</td></tr>
<tr><td rowspan="3"></td><td>R21</td><td>Incomplete response</td></tr>
<tr><td>S</td><td>The response shows that student tries to repeat the different cases to carry on with the specializing.</td></tr>
<tr><td>OS</td><td>The response is written under symbolic world</td></tr>
<tr><td rowspan="4">Review 3: EXTEND to a wider context by introducing examples having the same concept and using a sheet of paper.</td><td>No response</td><td>R30</td><td>No Extending</td></tr>
<tr><td>No response</td><td>R30</td><td>No Extending</td></tr>
<tr><td>No response</td><td>R30</td><td>No Extending</td></tr>
<tr><td>No response</td><td>R30</td><td>No Extending</td></tr>
<tr><td colspan="4">Watch video segment#3
Scene 7: [Swati and Nataly explaining the solution of the problem posed in the previous video segment]
Swati: Just as we suspected, the pile grows taller in a hurry. Because we are doubling the number of sheets of paper every time after n tears, I guess we could say we have 2 to the n sheets of paper.
Nataly: So if I assume the height of a human being is 1.75 meters, and knowing that there are 10,000 sheets of paper in a meter, help me figure out that an average human being is about 17,500 sheets of paper tall, which is 1.75 times 10 to the power of four in scientific notation.
So checking my powers of 2 table, it looks like my stack will be roughly 1.64 times 10 to the fourth sheets high after 14 tears, which is not quite the height we need. However, after 15 tears, the stack will be 3.28 times 10 to the fourth sheets tall about the height of your classroom and certainly taller than anyone I know.
Swati: So 15 tears was not much to get to the ceiling of the classroom. Not bad, we go to the ceiling in just 15 tears.</td></tr>
</table>

APPENDIX G

Nataly: For sure, but when I looked at the moon, things started getting really crazy. The moon is around 3.84 times 10 to the eighth meters from Earth. And I again used the fact that there are 10 to the fourth sheets of paper in a meter to determine that the moon is about 3.84 trillion sheets of paper from the Earth.
Seems like a lot, but after checking my handy power table, I saw that after 41 tears, we will have 2.2 times 10 to the 12th sheets. And after 42, we will have 4.4 times 10 to the 12th, more than enough to reach the moon.
Swati: 42 tears, wow, John must have already made it to the moon. Let's go and check on him.
Scene 8: *[John is shown struggling hard to tear the stack of paper, then Swati and Nataly comes]*
John: Ah man, tearing paper is so hard. I have only torn it five times, and I just cannot tear it anymore. I will never make it to the moon at this rate.
[John leaves the scene]
Nataly: Boy, he gives up really fast.
Swati: Well, I think as the stack gets taller and taller, it becomes harder and harder to tear it in half.
Swati: Yeah, plus I guess when we tear the paper into half each time; the area the stack covers decreases exponentially. I wonder if we will be even able to see the stack after 42 tears.
Nataly: OH, now I see why we cannot tear it 42 times. Well, I am not a queen. We cannot make it to the moon. I wonder what exponentials are good for.
Swati: Hmm, well, let me show you a neat trick I learned when I was a little kid. Let us go.
Scene 9: *[Swati and Nataly brings a sag full of Tootsie roll to play a trick with John]*
Swati: Hey, John.
Nataly: Hey, John.
John: Oh hey. How are you guys? Geez. Tootsie rolls, I love these things.
Student 1: We got you 10,000 tootsie rolls.
John: 10,000 tootsie rolls.
Nataly: You know, because you could not make it to the moon.
John: Oh my goodness. Guys, tootsie rolls are my absolute favorites.
Student 1: Yeah, we will give you 10,000 every day of the month.
John: 10,000 tootsie rolls every day of the month. Swati, what do I have to do to get 10,000 tootsie rolls every day of the month?
Swati: You just have to give us one today, double tomorrow, double the next day, and so on, for a month. You know just so that we have a little bit for ourselves. Yeah, you are on. 10,000 a day. Thanks, guys. Here is your tootsie roll for today. Have a good one.
John: Thank you. *[John leaves the scene with the bag full of 10,000 Tootsie rolls*]
Swati: Geez, I really do think exponential growth is pretty quick. And I bet we're going to end up having more tootsie rolls than John by the end of the month.
Nataly: On what day of the month, would we have given each other the same number of tootsie rolls? And how many more tootsie rolls should John have asked us for in order to have the same number of tootsie rolls by the end of the month? We'll give you some time to think it through, and we will come back soon.
[Control now is again passed to the class teacher and class teacher used worksheet already designed to record students' responses]

Do Activity 3 [Detail of activity is given but written responses are not included for this part for verification here]

Scheme:
John gets paid 10,000 rolls a day. Swati gets paid twice the number from the previous day, starting with 1 on the first day.

Questions:
On what day of the month, would we have given each other the same number tootsie rolls? And how many more tootsie rolls should John have asked us for in order to have the same or more number of tootsie rolls by the end of the month?

Watch video segment#4
Scene 10: [Nataly and Swati explaining the solution]
Nataly: Wow, Swati, you really knew what you were doing. I actually drew a plot of the tootsie payments by day n versus John's tootsie roll total payments. And even though, originally, the gap between the payouts seemed to be increasing, it closed up, and you had broken even by day 18. *[The plot is shown on the screen]*
Just like with the grain payouts to the Indian queen we discussed earlier, you had been paid 2 to the power n minus one tootsie rolls by the nth day. And by the last day of the month, you had pulled in a whopping total of one billion tootsie rolls.
Dividing by 30, John should have asked for 35 million tootsie rolls a day to have broken even.
Swati: Exactly, now this is what you call a lifetime supply of tootsie rolls. We have over a 3,000 metric ton of tootsie rolls in our storage now.
Nataly: Well, it is clear from the graph that you ended up gaining tootsie rolls much faster than John. What would have had happened if the amount John had received by day n had been 10,000n squared, instead of 10,000n.
Surely, John would be getting more money overall, but does the function 10,000n squared grow faster or slower than our exponential payout.
Hmm. What do you think? On what day of the month would we have exchanged the same number of tootsie rolls as John? What would have happened if we had paid John 10,000n to the 50 by the nth day, which means by the second day, we would have paid him 11 quintillion, that is 11 billion billion. And by the third day, we would have paid him seven octillion. That is 7 billion billion billion by day three.
Do you think we would have ever caught up with the doubling for the scheme of ours? We will leave you to discuss now, and we will be back shortly.
[Control is transferred to the class teacher and class teacher used worksheet already designed to record students' responses]

Do Activity 4 [Detail of activity is given but written responses are not included for this part for verification here]

Scheme:
John gets paid $10{,}000n^2$ rolls by the n^{th} day. Swati is paid twice the number from the previous day, starting with a 1 on the 1^{st} day.

Questions:
What do you think? On what day of the month would they have exchanged the same number of tootsie rolls? What would have happened if we had paid John $10{,}000n^{50}$ by the n^{th} day, which means by the second day, we would have paid him 11 quintillion, that's 11 billion billion. And by the third day, we would have paid him seven octillion. That's 7 billion billion

APPENDIX G

billion by day three. Do you think we would have a point in future when swati has been paid more Tootsie Rolls than John?
Watch video segment#5 ***Scene 11:*** *[Nataly and Swati together are making sense of the concept]* **Nataly**: Well, nothing solves a problem like a good old fashioned graph. I plotted two curves, one of which is 2 to the power n minus 1, the number of tootsie rolls we have been paid by day n. The other is 10,000n squared, the number we paid John by day n. Sure enough, we still end up making more tootsie rolls by the end of the month, even though we are paying John a lot more tootsie rolls in this new scheme. Though paying John seven quintillion tootsie rolls by day three is a pretty scary concept, I made another plot to see if we end up with more tootsie rolls than him with the n to the 50 scheme. Here, I compare 2 to the power n minus 1 and 10,000n to the 50. And sure enough, we eventually end up getting paid more. It took over a year before we made as many tootsie rolls as we paid John. But as you can see, in the long run, the exponential is growing much faster than even the curve 10,000n to the 50. **Swati:** To be fair though by the time the two payouts meet, the amount of tootsie rolls we would have exchanged would be more than the weight of the total observable universe. It's a freaking 137 digit number. **Nataly**:True, but in general, exponentials with base more than one would eventually become bigger than polynomial functions in general. **Swati:** How do you figure that? **Nataly**: I noticed something interesting when dividing the payouts between consecutive days for each of the payout functions. When I have an exponential payout function like 2 to the power n, or more generally, like b to the power n with b strictly greater than one, and when I took the ratio between the nth and the nth plus first element, I always got the same value b no matter what the value of n was. **Swati:** That makes sense. It's like saying we doubled the number of tootsie rolls every day no matter what day we are on. **Nataly**: But when I look at a polynomial function, like to the power 50, or more generally, n to the power k, I noticed that when I took the ratio of the n plus 1 to the power of k and n to the power of k, I got n plus 1 divided by n, the total to the power of k, which can also be written as 1 plus 1 over n, the total to the power of k. As n approaches infinity, this goes to one. So once n gets big enough, we are sure that our exponential function is growing faster than our polynomial function regardless of how big the power is on the polynomial or how close to one the base is on the exponential. **Swati:** Intriguing, isn't it? Since in the long run, we multiply consecutive numbers by a larger value in an exponential function than we do in a polynomial function. We conclude that the exponential will eventually exceed the polynomial function, even if it takes a long time as in the case of our 10,000n to the power 50 example. In other words, exponential functions always grow faster than polynomial functions in the long run. **Nataly**:Exactly. *[John enters the scene*] **John**: Guys, guys, so first just going to say I may be a tiny bit annoyed that recently I had to pay you guys maybe like 30,000 metric tons of tootsie rolls, but I just learned the coolest thing in computer engineering class. **Swati:** Oh, what? **John**: Let me tell you. So apparently, there's a law called Moore's law, which states that the number of transistors on a square inch of a computer chip tends to double every two years. **Nataly**: Oh, I guess this means that the size of a computer with a given amount of computing power is expected to be halved after two years. **Swati:** You know recently, in my history of computing project, I was reading about this computer called Cray-1 supercomputer, which was released in 1976. Based on our knowledge of exponentials, if I halved it's size every two years, I'm sure it would fit on my desktop before too long. **Nataly**:But computers were so slow back then. **Swati:** Well, fine, let's pretend we have 25 Cray-1 supercomputers instead. **Nataly**: So what do you think? Do you think that something with the same computing power today could fit in your classroom? Work it out with your neighbors and your teacher, and we'll be back in a little bit. *[Due to the lack of time we could not do this activity in class but we have manged to show the rest of the video segment to leave students with a thought that whatever they are learning related to the functions of exponentials make a lot of sense in the world of mathematics and engineering]* **Nataly**: Hey, that was a pretty cool calculation. It was just like our rice, and the queen, and the tootsie rolls. Except that here for computer chips, we halved the size each time instead of doubling it. **John**: Whoa, let me tell you a crazy thing about this is that those 25 Cray-1 supercomputers have about the same computing power as a single iPhone 4 does today. **Nataly**:Wow. Which means Moore's law really did a pretty good job of predicting the size of that iPhone. **Swati:** Hmm. I must say I'm getting really wound up for exponential functions. Just like exponential growth exponential decay, as in the case of Moore's law, is extremely rapid. So it seems to me that exponential functions are just about the fastest growing and the fastest decaying functions in the world. **Nataly** Not really. I mean I'm sure I can write a function that grows faster than an exponential **Swati:** Really? **Nataly**:Yeah, consider factorials for example. N factorial is equal to n times n minus 1 times n minus 2 all the way down to 1. For instance, 4 factorial is 4 times 3 times 2 times 1, which is 24. The result is something that grows very quickly. 10 factorial, for example, is 3,628,800. 20 factorial is already more than 2.4 times 10 to the 18. Meanwhile, 2 to the 10 is just 1,024. And 2 to the 20 is just over a million, which are much smaller numbers. If you look at the ratio between two consecutive factorials, n plus one factorial is n plus 1 times larger than n factorial, while 2 to the n plus 1 is only two times larger than 2 to the n. So the ratio between factorials is increasing as n increases, while the ratio between exponential values is staying just the same. **John**: Well then, I guess it certainly stands to reason that the factorial function should grow a lot faster as n gets large. Anyway, can you guys think of any other functions that grow faster than the exponential? Might be something fun to talk about with your neighbor now, or maybe quiz your parents when you get home tonight. Anyway, thanks for watching, and we hope you had as much fun as we did. **Nataly:** I hope you learned as much as John did. ***[Module Ends]***

APPENDIX H

Table H(a) & H(b): Emerging methodologies in Engineering Education, their defining features extracted from Light and Case (2011) and Creswell (2007)

Defining Features of Case Study
• An in-depth study or examination of a distinct, single instance of a class of phenomena such as an event, an individual, a group, an activity, or a community • Choosing a set of cases with maximum variation in order to explore a range of different settings, or identifying unusual cases, which allow the researcher to probe particularly problematic situations, or using critical cases, which allow for logical deductions of the type "If this holds for this case, then it will hold for all other cases." • It is not possible to develop "general propositions and theories" from a single case study and, as such, a "case study cannot contribute to scientific development" • The concrete, context dependent nature of the knowledge which case studies unearth, on which these critiques focus, however, is precisely the source of its methodological strength • It can therefore be particularly appropriate to address research questions concerned with the specific application of initiatives or innovations to improve or enhance learning and teaching • The new knowledge here takes into consideration the particular idiosyncrasies of the institution, its resources, teachers and students, as well as its overall culture • This can be contrasted with a more positivist kind of study aimed at evaluating the general applicability of an educational intervention—characterized, for example, by a more traditional randomized controlled study—primarily focused on "proving" the general effectiveness of the intervention • "Sometimes we simply have to keep our eyes open and look carefully at individual cases—not in the hope of proving anything, but rather in the hope of learning something!" Hans Eysenck (1976, p. 9)
Defining Features of Grounded Theory
• Established in a seminal piece of work by Glaser and Strauss (1967) • One of the first methodological positions put forward that supported the use of qualitative data in social research. • A general methodology for developing theory that is grounded in data systematically gathered and analysed (Strauss and Corbin, 1994). At the heart of grounded theory is the idea that theory is generated from the data at hand, rather than already existing theory being used in the analysis as is generally common in education research. • Researchers are required to keep an open mind at the start of the research with minimal inspiration from the literature • The constant comparative method is a central data collection method in the grounded theory methodology • A clear systematic outline of a process for analysing qualitative data. 1. In the first stage of this procedure, also termed "open coding," initial categories are developed by grouping similar incidents together. During the coding of each incident, it must be carefully compared with other incidents previously coded in the same category. 2. In the next stage of the analysis, termed "axial coding," a further refinement is done on the categories and their properties by stepping back and testing all incidents coded in a category with the properties of that category. The categories are also compared for overlap and examined for possible relationships among categories. 3. The endpoint of data collection is reached through a process termed "theoretical saturation," which occurs when additional data collection and analysis does not substantially change the findings. Strauss and Corbin describe theory as "plausible relationships proposed among concepts and sets of concepts" (Strauss and Corbin, 1994, p. 277), and stress that theory building is very different to description. • Two foci in developing theory are discovering patterns and identifying processes. • Data collection and analysis are tightly interwoven. The initial theories emerging from the data are used to direct further data collection. One possibility in this respect is to use theoretical sampling, where additional research subjects are selected as the study proceeds in order to explore issues that have arisen. • Tends to find limited application in education research, where researchers often find it productive to use existing theoretical constructs in their analysis. However, as a mode of research for challenging preconceptions and allowing for alternative conceptualizations it has enormous strength.

APPENDIX H

Table H(c) & H(d): Emerging methodologies in Engineering Education, their defining features extracted from Light and Case (2011)

Defining Features of Ethnography

- This research method is generally attributed to work in anthropology and goes back at least a hundred years to the work of Malinowski who stressed the importance of "grasp(ing) the native's point of view" in his work (Stocking, 1983)
- traces its ancestry to early developments in sociology, in particular the Chicago School which argued that observation, later participant observation, was critical for developing a full understanding of an environment.
- Central to this early ethnographic research was the idea of closely studying firsthand how people live in particular social situations. Drawing on this seminal work, ethnography has, according to Hammersley and Atkinson (2007), more recently generally come to be understood as:

  > "a particular method or set of methods … (which) involves the ethnographer participating, overtly or covertly, in people's daily lives for an extended period of time, watching what happens, listening to what is said, asking questions-in fact, collecting whatever data are available to throw light on the issues that are the focus of the research." (Hammersley and Atkinson, 2007, p. 1)

- demands of coming to a rich understanding of people's day-to-day lives within a social environment presents substantive challenges to ethnographers, particularly with respect to gathering and interpreting the data in terms of the meanings which the members, themselves, attach to their own world (Bryman, 2001).
- Challenging given the demand to write up these meanings for readers—academics and policy makers— who live in different social cultures. In addition, the constraints of doing educational research do not afford today's ethnographer the luxury of living with the people they are studying, and certainly not for years. They are more likely only to be able to follow their lives closely and carry out their fieldwork over many months.
- Data collection methods during fieldwork are usually diverse and multiple. In addition to the researcher's own field notes, which may include records of discussions, chance conversations, interviews, overheard remarks, observational notes, the researcher may also employ audio and video recordings and quantitative data gathered from surveys or structured observation.
- The quality of the research may be difficult to judge given more traditional evaluation criteria. Richardson (2000) suggests five general criteria. Two criteria concern judgments of whether or not the research makes a substantial contribution to our "understanding of social- life"; and whether it presents a "credible account" of the reality, which it attempts to describe (p. 254). In addition, she raises criteria related to the work's "aesthetic merit" (is it aesthetically successful?) and its "emotional impact" (does it move me?). Finally, she draws attention to the importance of the researchers' self-awareness of their own role in the research.

Defining Features of Phenomenography

- First developed as a specific research methodology by researchers in Sweden in the 1970s.
- Primary focus is the investigation of the different ways in which phenomena, or aspects of a phenomenon (such as specific concepts), are experienced or understood within particular educational and learning contexts to search for a comprehensive record of the variation in the experiences of people in such contexts.
- Seeking "the totality of ways in which people experience … the object of interest and interpret it in terms of distinctly different categories that capture the essence of the variation…" (Marton and Booth, 1997).
- This totality of different ways is often referred to as the "outcome space." The outcome space is not a record of individuals, but a description at the collective level, of different categories of experience.
- Resembles other forms of inquiry such as phenomenology, for instance, with a focus on the individual's point of view rather than the researcher's or a community's, and on the use of data collection methods such as semi-structured interviews, and related textual and visual materials. It is, however, unique in two respects, which have implications for the particular sampling procedures and data analysis in this methodology. First, rather than looking for common shared experiences of a phenomenon, as is the case with phenomenology (Manen, 1990)), Phenomenography focuses on the ways in which learners differ (Marton, 1989). It is therefore important to maximize the potential variation of experience in the sample of individuals interviewed, ensuring the sample is fully representative of potential experience with respect to the phenomenon under consideration: not all the highest performing students, for instance, nor all the poorest performing students. The second key difference with phenomenology is that rather than making the phenomenon the subject of the research, as phenomenology does (Manen, 1990), Phenomenography takes experience of the phenomenon as its unit of analysis. The data collected from individuals thus needs to be pooled together and analyzed (in a careful iterative process) to identify a set of distinctive categories (and the critical dimensions of variation which differentiate these categories) by which the full collective experience can be described.
- The analysis aims at identifying the fewest, logically related, categories required to describe the totality of variation discerned in the pool of experience.
- The identification of different conceptions, moreover, makes it particularly well suited for the design of educational learning objectives, pedagogical strategies, assessments, and evaluations (Micari et al., 2007).
- Phenomenographic work provides program developers with a profile of the variation in experience across all of the participants in the program.
- Phenomenographic research has been used primarily in education, including engineering education, to investigate variations in the ways students understand important concepts such as energy in solution processes (Ebenezer and Fraser, 2001), size and scale in nanoscale science and technology contexts (Swarat et al., 2011), and transient responses in student problem solving contexts (Carstensen and Bernhard, 2009).
- It is also beginning to emerge in other engineering contexts as well. For example, it has been used to identify conceptions of competent work among engineers in an auto manufacturing company (Sandberg, 2000), and conceptions of the value of information-technology (IT) research among IT researchers and practitioners (Bruce et al., 2004).

APPENDIX H

Table H(e) & H(f): Emerging methodologies in Engineering Education, their defining features extracted from Light and Case (2011)

Defining Features of Discourse Analysis
• Emerged from the field of linguistics but now well established across a range of education research areas. • Some of the key approaches to discourse analysis are those by Fairclough (2003) and Gee (2011). • The data that forms the focus are actual instances of language in use, for example the transcript of a classroom discussion. • Hicks (1995) emphasizes that the term discourse always refers to communication that is socially situated. The significance of an analysis of discourse is that it allows us to get insights into the beliefs, values, and worldviews, which are held by participants since these, are always reflected in the use of discourse. • In the context of engineering education, it is important to note that discourse not only comprises written text; it also includes mathematical equations, graphs, figures, verbal exchanges, and so on. • The discourse of being an engineer will involve the practice of design to solve real world problems, and this includes collecting and analyzing data, using empirical laws and correlations, doing mathematical calculations and modeling, as well as presenting one's results to a range of different audiences. In short, discourse comprises everything that describes the academic and professional activities, which characterize engineering practice. Over time, this community has developed shared ways of talking about and understanding the issues and practices that matter to them. From this point of view, successful learning involves using a discourse in order to participate in this community (Northedge, 2003). • The way of describing learning is at the heart of the methodological justification on which discourse analysis is based. From this description of discourse analysis it can be seen that this methodology is not limited to studies of student reading and writing, as might have previously been assumed, but has wide applicability to a range of research questions in engineering education. A central question concerns the role that language plays in student learning as well as the role it plays in social interaction. • Importantly, a discourse perspective reminds us that the activities of academic discourse are never neutral and can pose particular difficulties when they clash with other discourses in which the student is engaged; learning a new discourse involves taking on a new identity (Gee, 2011). • Usually requires data in the form of transcriptions of utterances and practices—pieces of discourse—recorded during the event, for example, classroom discussions, student conversations, and so on. It can also productively be applied to analysis of interviews. • Has been extensively used across a range of disciplinary contexts in both school and university education, with highly productive applications in mathematics and science (Airey and Linder, 2009; Sfard, 2001). It is starting to be used in engineering education research and a recent paper by Allie et al. (2009) draws on discourse theory to put forward a position on how to improve student learning in engineering education. In research with third-year engineering students, Case and Marshall (2008) demonstrate an application of Gee's concept of discourse models to identify the implicit theories that students use to make sense of their learning experiences. • The study of discourse can be divided into three domains: the study of social interaction, the study of minds, selves, and sense-making, and the study of culture and social relations (Wetherell et al., 2001, p. 5).

Table Q(g): Emerging methodologies in Engineering Education, their defining features extracted from Light and Case (2011)

Defining Features of Action Research

• Kurt Lewin first used the term while he was at MIT in the 1940's with a special focus on social action.
• A critical educational research methodology looking to foster change in social practices in the social situations in which they take place "within every day, natural contexts rather than within controlled settings" (Cousin, 2009, p. 150).
• The aims and benefits are strategic improvement of practice. This critical focus on continuous improvement raises the second defining feature, which distinguishes action research from other educational methodologies: it is almost entirely determined and conducted by its various practitioners. Indeed, improvement occurs through the active engagement of the practitioners. As such, action research is research with subjects, not on them, an idea that, as Cousins notes, "reverses the conventional scientific understanding of objectivity" (Cousin, 2009, p. 151).
• Carr and Kemmis (1986) describe three types of participatory inquiry, which have been described as action research. They differ in terms of the degree to which the practitioners or participants are the principal researchers. In technical action research, the researcher facilitates the process and establishes and judges the standards for improving educational practice. The participants mainly contribute at a technical level. Practical action research also focuses on improving practice, but participants participate more fully and reflectively in the research to develop their practical understanding. Finally, emancipatory action research encourages the full participation of participant-as-researcher to critically explore the effectiveness of practice in the social and organizational constraints in which it is situated. In this mode, action research employs a critical research methodology involving issues of equity and power relationships.
• Reflective, systematic, and cyclical.
• In its design, methods, and realization, it consciously and deliberately sets out to improve, enhance, and realize practice through actions informed, but not constrained, by research and theory. It is flexible, open to change necessitated by experience and circumstance, and it is subject to the practitioner's critical and rational practical judgments. Kemmis and McTaggart (2014, p. 18) describe the implementation of this strategic action as a continuous cycle of four moments:
  ➢ A plan of action to change or improve what is already happening;
  ➢ Action to implement the plan;
  ➢ Observation of the effects of action in the context in which it is occurs;
  ➢ Reflection on these effects as a basis for further planning, subsequent action and so on, through a succession of cycles.
• Effective methodology for engineering faculty who are not only interested in systematically researching their own educational practices but also in implementing substantial personal and social change in their practice.
• The effective use of action research is to create "Collaborative Engineering Environments."

APPENDIX H

Table H(f): Emerging methodologies in Engineering Education, their defining features extracted from Light and Case (2011)

Defining Features of Narrative Analysis

- It is especially suited to focus on investigating the way people experience life and has its origins in the field of literature studies, but has found applicability across a range of social science fields including education. In fact, building on the work of John Dewey, which points to the deep interrelationships between experience, education, and life, some narrative researchers have argued that narrative methodology is particularly applicable to education research: “In its most general sense, when one asks what it means to study education, the answer is to study experience” Clandinin & Connelly (1998, p. 154).
- It has many forms, uses a variety of analytic practices, and is rooted in different social and humanities disciplines (Daiute and Lightfoot, 2004).
- "Narrative might be the term assigned to any text of discourse, or, it might be text used within the context of a mode of inquiry” in qualitative research (Chase, 2005) with a specific focus on the stories told by individuals (Polkinghorne, 1995).” (Creswell, 2007, p. 54)
- “As Pinnegar and Daynes (2006) suggest, narrative can be both a method and the phenomenon of study. As a method, it begins with the experiences as lived and told stories of individuals. Writers have provided ways for analyzing and understanding the stories lived and told.” (Creswell, 2007, p. 54)
- What do we mean by the term “narrative”? Polkinghorne (1995) notes that although the commonsense term “story” told by individuals is used by some researchers it carries with it connotations of falsehood or misrepresentation.
- Oliviera (2005) makes the important point that not all pieces of prose are automatically narratives. Hinchman and Hinchman (1997, p. xvi) define narratives as “discourses with a clear sequential order that connect events in a meaningful way for a definite audience, and thus offer insights about the world and/or people’s experiences of it.”
- An influential education scholar, Jerome Bruner (1986) has argued that “narrative cognition” is a particular form of human knowledge, distinct from what he terms “logical- scientific” knowledge. Telling stories is a fundamental human activity, a means by which we represent ourselves to others and make sense of our lives. Narrative methodology focuses on collecting and analyzing these stories in order to understand human experience. In the context of engineering education, narrative methodology can help us understand how students experience their education contexts.
- Polkinghorne (1995) develops a useful distinction between two modes of analysis: The more usual form of analysis he terms as “paradigmatic” analysis where the researcher attempts to identify common themes across the various narratives that have been collected as data. This has links to the method of constant comparison associated with grounded theory analysis mentioned earlier. Polkinghorne (1995), however, points to the utility of an alternative approach he terms “narrative analysis” where each narrative is considered on its own merits. A key activity in this approach is in organizing the various data elements present into a coherent account of that person’s development.
- Gergen (1994) has pointed out that there are different orientations to the study of people’s narrative accounts. Cognitivist approaches have tended to emphasize the individual, while Gergen advocates a position, which focuses on the links between the narrative and the broader culture. He describes self-narratives as “forms of social accounting or public discourse” (p. 249). This approach has some similarities to the methodology of discourse analysis discussed earlier where the study of instances of discourse is linked to the broader discourses operating in society.
- There has been considerable use of narrative inquiry in educational research, but mostly focusing on teacher education. Some recent examples are Volkmann and Zgagacz’s (2004) study of the experience of a graduate teaching assistant in an inquiry-based physics course, and Clark and Linder’s (2006) study of a South African township high school teacher’s experiences of introducing innovations in her physical science classroom.
- The procedures for implementing this research consist of focusing on studying one or two individuals, gathering data through the collection of their stories, reporting individual experiences and chronologically ordering (or using life course stages) the meaning of these experiences.
- In engineering education, it has been adopted in a recent study analyzing data from one South African engineering student in order to develop a more in-depth understanding of the experiences of students from educationally disadvantaged backgrounds by Marshall & Case (2010).

APPENDIX I

Screenshot of the logbook entries and associated table of contents

Ref. No.	Date	Day	Venue	Time	Subject	Instructor/Respondents	Activity	Video	Video Recorder	Photos	Observations	Observer
W1-C1	10th September, 2013	Thuesday	C25-419, UTM Skudai Campus	2:00PM -3:00PM	SSE1693 Engineering Mathematics I	Associate Professor for Main Course	Introductory Class, Course outline discussed, Assessment criteria Discussed, Intruduction to Week1 Topic discussed	Yes	N/A	Yes	Yes	Ash
W1-C2	11th September,2013	Wednesday	C24-108, UTM Skudai Campus	12:00Noon-1:00PM	SSE1693 Engineering Mathematics I	Associate Professor for Main Course	Grouping, Transcendental Functions	N/A	N/A	N/A	N/A	N/A
W1-C3	12th September,2013	Thursday	FKM-DK2,UTM Skudai Campus	8:00AM to 10:00AM	SSE1693 Engineering Mathematics I	Associate Professor for Main Course	Inverse Trignometric functions, Hyperbolic functions and its inverse in logrithmic form, Solving equations related to Functions	Yes	Ash	Yes	Yes	Ash
W2-L1	17th September,2013	Tuesday	C24-408, Computer Lab, UTM Skudai Campus	2:00PM -3:00PM	SSE1693 Engineering Mathematics I	Researcher	Introductory Class, Introduction to Mathematical thinking, Purpose, Objectives	Yes	Ash	Yes	Yes	Ash
W2-C4	18th September,2013	Wednesday	C24-108, UTM Skudai Campus	12:00Noon-1:00PM	SSE1693 Engineering Mathematics I	Associate Professor for Main Course	Trigonometric and Inverse Trigonometric Function, Inverse Exponential and Inverse Hyperbolic	Yes	Nur	Yes	Yes	Aina
W2-C5	19th September,2013	Thursday	FKM-DK2,UTM Skudai Campus	8:00AM to 10:00AM	SSE1693 Engineering Mathematics I	Associate Professor for Main Course	Inverse Hyperbolic	Yes	Nur	Yes	Yes	Aina
W3-L2	24th September, 2013	Tuesday	C24-408, Computer Lab, UTM Skudai Campus	2:00PM -3:00PM	SSE1693 Engineering Mathematics I	Researcher	Guide Provided for Mathematical Thinking, Presented MT rubric and started module named The Power of Exponentials .	Yes	Nur	Yes	Yes	Aina

APPENDIX J

The key activities undertaken by the researcher during Main Study

AR-Cycle	Ref. No.	Date	Day	Venue	Time	Activity	Videos	Photos	OBSV
Main Cycle 1	W2-MTL1	17th September, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Introductory Class, Introduction to Mathematical thinking, Purpose, Objectives	Yes	Yes	Yes
	W3-MTL2	24th September, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Guide Provided for Mathematical Thinking, Presented MT rubric and started module "The Power of Exponentials"	Yes	Yes	Yes
	W4-MTL3	1st October, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Guide Provided for Mathematical Thinking, Presented MT rubric and finished module "The Power of Exponentials"	Yes	Yes	Yes
	W5-MTL4	8th October, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Quiz 1 a & b (Charlie's delightful machine & function growing faster than exponentials)	Yes	Yes	Yes
	Week 6 (Hari Raya Holidays)								
	W7-MTL5	22nd October, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Worksheet on Movement of Sand dunes	No-Lost	Yes	Yes
	W8-MTL6	29th October, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Continue worksheet sand dune	Yes	Yes	Yes
Week-9 (Mid-semester break)									
Main Cycle 2	W10-MTL7	12th November, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Quiz 2	Yes	Yes	Yes
	W12-MTL8	26th November, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Fill the Perception form, watch video on Math, Video on what is engineering (Improving the Perception) video of elevator to the moon	Yes	Yes	Yes
	W13-MTL9	3rd December, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Flaws of Average	Yes	Yes	Yes
	W14-MTL10	10th December, 2013	Tuesday	**MTL Cancelled due to Researcher 's unavailability and transferred to next week**					
	W15-MTL10	17th December, 2013	Tuesday	C24-408, Computer Lab,	2:00PM - 3:00PM	Quiz 3,Assessment in ALEKSs	Yes	Yes	Yes

APPENDIX K

External Audit _ Expert Validation

Confirmation of Content validity by Expert

1. Title for the Study ✓
2. Problem Statement ✓
3. Review of the Literature
4. Purpose and Research Questions ✓
5. Data collection Techniques ✓
6. Worksheets based on BLOSSOMS Modules ✓
7. Code Book ✓
8. Rubric for activity response Analysis ✓
9. Process of Rubric improvement ✓
10. Data Analysis and Findings
11. Writing
12. Video Protocol
13. Observation Protocol

This is to confirm that I have reviewed thoroughly the content of the research based worksheets and provided the feedback on the above 13 components of the research conducted by Aisha Mahmood from Center of Engineering Education, SPS, UTM, JohorBahru. I also verify the rigor in the research work done by the researcher.

Signature Kashefi

Name Hamidreza Kashefi

Position Senior Lecturer

Institute

Degree PhD

Teaching Experience 16 years

Stamp

DR. HAMIDREZA KASHEFI
Department of Science and Mathematics Education
Universiti Teknologi Malaysia,
81310 UTM Skudai, Malaysia
Tel: +(6)07-553 3146
Email: hamidreza@utm.my

APPENDIX L

Log file for Pilot Cycles

Action Research Cycles	Date	Venue	Time	Course	Instructor/Respondents	Activity	Documents/Data Collected	Video	Photos
Introductory session	27th June, 2013	Lecturer Activity room (bilik aktiviti pensyarah), block Q, level 2, RU KL	9:00AM to 10:00AM	Foundation of Engineering Mathematics (DDPS 1012)	Senior Lecturer	Instructor-Researcher (First Interaction/meeting)	Consent Form, Course Outline	Yes	Yes
Cycle 1	27th June, 2013	DK5, RU, KL	10:00AM to 11:00AM	Foundation of Engineering Mathematics (DDPS 1012)	Diploma Students in Mechanical Engineering-First Year-Sec 17	Introduction, Consent, BLOSSOMS Module on Exponentials	Consent Form, blossom module worksheets Power of Exponentials	Yes	Yes
Cycle 2	29th July, 2013	Classroom 5.44 , Level 5, MJIIT, RU, KL	11:00AM to 1:00PM	Foundation of Engineering Mathematics (DDPS 1012)	Diploma Students in Mechanical Engineering-First Year-Sec 17	BLOSSOMS Module on Flaws of Average	blossom module worksheets Flaws of Average	Yes	Yes

APPENDIX M

BLOSSOMS MODULE_The Power of Exponentials, Big and Small (Worksheet)

Greetings and Introduction to the Module

Assalmualikum and welcome to our module on "The power of exponentials, big and small".

- Please record your responses in the space provided under each question.
- Do the necessary calculations
- Justify your answers.

G3

Activity 1:

REWARD:

1 grain of rice for the 1st square of the chessboard, two for the second, four for the third, and so on. Every square must have double the number of grains as the previous one.

QUES:

Do you think the award was very small? Was the treasurer able to pay off the mathematician with a single bag of rice? Was the queen able to pay off the mathematician at all?

DATA:

1. 64 squares on a chess board
2. Number of grains of the rice in a bag = 800,000

1	2	4
8	16	32
64	128	256

1	2	4	8
16	32	64	128
128	256		

Not able

APPENDIX M

BLOSSOMS MODULE_FLAW OF AVERAGES_Worksheet

Greetings and Introduction to the Module

Assalmualikum and welcome to our module on the flaws of averages.

Module Entry (required to raise the curiosity about the topic)_watch video

Activity 1: Flaw of Average #1

"the average is not always a good description of the actual situation."

Modeling:

Let's imagine that you're at the edge of a river that you want to cross. But, there's a sign. The sign says, "**Average river depth one meter.**" Now, given this sign, would you cross the river?

- Please record your responses in the space provided under each question.
- Prompts will help you to come out of any STUCK state.

Entry Phase:

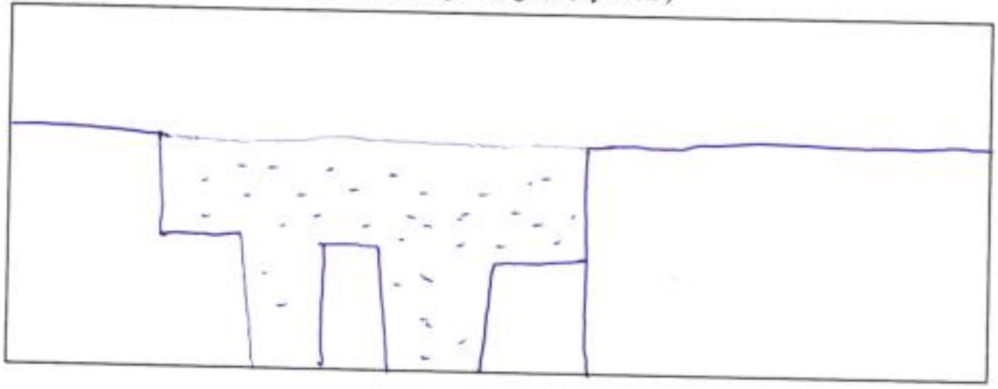
Entry 1: What do you KNOW from the question and also from your past experience?

No, I will not cross the river.

Entry 2: What do I WANT?

I ~~did~~ need to know the real depth for every ~~m~~ river before I cross the river.

Entry 3: What can I INTRODUCE? (e.g images, diagrams, symbols)



APPENDIX N

Excerpts from the conversation between EER Expert (E) and the researcher (R) on April 4, 2014

E: We usually talk too much as if the educator is not there but in reality, the educators are the gatekeepers. Therefore, we need to know the trajectory of the educators.

R: Hmmm.

E: Are you actively teaching?

R: Yes but as a part of my research, the action research, I taught during the last semester. Actually, things that emerged out of my research are very interesting, they helped me to understand my own practice as well as what should be the direction of improvement in my future practice. I have found some loopholes as well, which we usually do not present in front of others, but I very well understand them. So, I think, understanding the underlying issues are more important than proving the effectiveness of an innovative practice. Every time you have different groups of students and to be adaptable you need to know who your students are, then you have to adjust your teaching methods according to the group of students you are teaching. Knowing the issues of my previous practice and embracing them, and changing my philosophical assumptions helped me to make sense of my first ever experience as a practitioner cum researcher.

E: So let me ask you, when you were talking about the question about the effectiveness of the practices and you have taught what you were studying. Were you studying the students? Were you studying yourself? Were you studying the system that was you plus the students? What would you say you were studying when you did that research?

R: Actually, I wanted to capture a holistic picture. I was studying my own practice as well as students' learning, and I wanted to see how the whole system works in a new situation. Therefore, I started by knowing my own students and then slowly building up my understanding of the whole system. I had to be adaptive towards the group of students, because they were having different profiles, they were having different attitudes, some of the students were really at risk, and I found that there was no support system for them. Therefore, actually, there were multiple questions that emerged, like what is more important to highlight, the learning issues of students or to prove the effectiveness of a classroom innovation. Let me explain further, suppose someone wants to teach engineering mathematic I course, but he/she is not aware of the current knowledge state of the students, and there was no system available to check whether students have sufficient prior knowledge. The teacher starts building new knowledge without even knowing the weaknesses in the foundation of that knowledge. Therefore, what I did, I asked my students to practice on ALEKS that is web-based artificial intelligence system, and I discovered that the low achievers are those students, who do not have enough prior knowledge, and this was a straightforward issue, like why are we enrolling them into an engineering program without even knowing their current knowledge state? I think this is a basic issue we need to address first, in common practice the in-class teacher starts introducing new methods, new strategies, new ways of doing things, and then ending up having no good results or leaving the students by categorizing them into multiple groups and labeling them as good or bad, and it is frustrating for me. Therefore, I tried to change that. I got evidence that some of the students, who are not good in their prior knowledge, are still able to think mathematically, because my research was about supporting their mathematical thinking, and they were comfortable in solving the problems, by staying into the conceptual embodied world of mathematics. However, once we asked them to transfer that knowledge into the symbolic world of mathematics, they are unable to do that. So what is the main issue? The main issue is as if they have not been in practice of transferring that knowledge from the conceptual embodiment to operational symbolism. So this was the main issue, we are not addressing that at that moment. We are just trying to see which students are doing what. However, why they are doing what was more important to me.

E: Well, what's interesting about listening to your story in the context of engineering education, I think we have set and prioritized that we need to focus on learning and learning trajectories, but you're telling me a story about you as an educator coming to understand educational practice better, you're telling me a story about you doing action research, which is a kind of research we talk very little about, but is exactly what you were doing, research on your own practice. You're telling me a story that helps you see the larger system, about how we admit students and how we come to understand and how we deal with the fact that even though they had the prerequisites, they don't have exactly what you thought they were going to have. You know, you are telling me about this piece of research that you did that was action research that opened up the whole system to you. You're also telling me about how you got better as an educator, but then the question is, if you got better as an educator but then what you took out of your research, what if we would just fix this problem that I encountered, it wouldn't be a problem for everybody else, and I can say right now it's probably still going to be a problem for everybody else. So we're focusing on training students, but we need to focus on training the educators, so part of it is, what you just described is about the big educational system, and the struggle that we're having in engineering education, is sometimes we think if we just focusing on the learning and the learning trajectory, then we're going to change the educational system, which is no more true than if we focus on molecules, we're going to change the practice of medicine. In addition, a lot of the things that you just described, are really important pieces of your experience, would never really end up in the stories that we would tell formally in the engineering education research community, but they have incredible clues for why things are not changing and why they are staying the same, right? So I really appreciate you telling me the whole story, because I think that you can think about but its something that we don't talk about.

R: Yeah.

E: Well, I am not supposed to talk about things like that. Why don't we talk about those kind of things? Why are certain things that you learn from that experience the ones that you should share and why are there other things that you should not share or you would share less, right? Because all of them are part of the whole and you just have a chance to be exposed to the system and see the system in this holistic way. There is a larger goal that we are all trying to achieve, which is to make engineering education better. So then, you are torn a little bit. There's a part of you that has to tell a simple story to get it published, to make your accomplishment, to get your opportunity to stay in the game, and then this other part of you going, ah, there are these messy realities, and what do we do with them and how do I have to study them, and maybe I should study them. Why cannot we study them? Why are they off the table? You know, so what is it that we need to do in order to really try to figure out how to make engineering education better, and in the meantime, how do you continue to do pieces of research that you can get credit for that don't piss the people off that, you know, don't get rejected, and balance those two kind of competing commitments, the commitment to making sort of incremental progress and the commitment to keeping the big picture in mind about what we're all trying to accomplish together. And then at the same time, the interesting question is, so you have this experience yourself, this powerful, powerful experience, who's helping you reflect on that experience?

R: Myself. I used to reflect on what I did and I sometimes get feedback from others.

E: Right.

R: But every time I come back after a session with my students, I feel more concerned and more involved emotionally. I do not know whether being involved emotionally is a good thing or a bad thing, but I really felt bad when somebody has a potential to grow and learn, but doesn't have enough support, enough support from the system to let him survive here. So that makes me quite frustrated at that moment, because actually there were one or two cases where the students were really good at mathematical thinking in conceptual embodiment, but since they didn't have prior knowledge related to their engineering mathematic, they couldn't manage to survive the course. There should be a balance in assessing the computational skills and mathematical thinking skills. I am also struggling with the questions like why we are still doing what is obsolete, why we are still doing what is not being required by the workplace. At this level of technological advancement, we just need the basic computational skills, and then we want our students to grow as mathematical thinkers. We want them to be problem solvers in new situation. Having those skills, they should be able to transform those skills from one situation to another situation.

E: At the beginning of this conversation, you asked me about the effectiveness of innovative practice and whether we needed to know that. Now, what you just said, why are we still doing what's obsolete. It strikes me that we're a little preoccupied as a community with the effectiveness and innovations, and we should be a little bit more preoccupied with the exact question you just asked, which is why are we still doing things that are obsolete, not because we should be getting mad at each other, but we should be acknowledging the strong forcing functions that keep things in the status quo state, right? And it's as if we think we had enough effectiveness of innovations that status quo would become not the status quo, but the things suggest that the status quo stays the status quo for very powerful reasons. Until you get that, things are not going to change. So I would say, you know, over time what I'd like to be contributing even more is trying to figure out how stories like the one you just told become a little bit more the norm in our research community, and these simple stories about if we just had enough effectiveness research on innovation, everything would be fixed, and if we just shut our eyes to the complex realities, it will go away. It just seems like we are somewhat collectively ignoring things that we all know pretty well. So we have students having lots and lots of experiences, they go from messy team to messy team to messy team, but they're not sure kind of where they stop and make sense of the experience and figure out what they're going to do next. You know, and we go from messy teaching situation to messy teaching situation to messy teaching situation, waiting for the research to figure out what the innovation is that's going to fix it all. Meanwhile, you know, there is this chance to learn a lot from the experience like you just had. So I would say the advice is, just realizing that struggling is the evidence that you're doing it right, not evidence that you're doing it wrong, and to find somebody who can help you make sense of your struggle, you know, to figure out which part of this is, maybe you're struggling because you haven't learned how to make the arguments, but part of the struggle actually is a legitimate piece of the puzzle that we're trying to understand.

E: One advice for you is to study adult learning theory, particularly about adult development, about stepping outside of situations. And the other is I think our entire community needs to know much more about complexity theory, about phenomena that are not sort of linearly caused, but are the emergent effects of lots of little pieces interacting with each other. It's showing up a lot. The theoretical perspective shows up in management of change, it shows up in research on ecosystems, it shows up in things like discrimination and how little tiny bits of micro-aggressions accumulate in individuals and have them experience oppression, and I think that we're not talking about that kind of explanatory basis enough.

R: Thanks. There should also be a network platform for CoP, and there, we should share common objectives and goals. Being a researcher, I always feel a need to discuss, to reflect in the presence of someone, somebody to listen to and somebody to discuss with, somebody to even disagree with what I am saying.

E: So quick question, so who helps you provide feedback? So as you are working on explaining your research, who works alongside you to help you figure out how to better explain what you figured out you are trying to explain?

R: Actually, we used to have lots of international researchers who are coming and visiting us, and we got a chance to discuss our research face-to-face, one-to-one basis, so that is a very good and healthy practice. Moreover, whenever I am given a chance, I really felt great, because it is also an encouraging exercise for a newcomer, as if someone is agreeing with what you are saying. I am not saying that if somebody is differing with your idea, your way of doing things, is not important, that is also important, but at the same time, you know, someone should be there to encourage you, like keep it up with the constructive feedback, and, locally it's not that strong, but we have been given some opportunities on and off. That is the reason I'm very comfortable in talking to you right now, because I was the one who just felt like I need to excel in engineering education. I have to find my ways out to figure out the things by myself. So out of all this, you know, experience of my Ph.D., I think I was the one who was the most influenced out of my own research, because turned me into a totally new me.

E: Right, right.

R: It changed my perceptions. I am more concerned about health now, I am more concerned about my kids' way of learning new things, and there is a shift in the way I was thinking before actually I started my Ph.D. So I think one of the reason was the area I have selected for my research, so whenever there is thinking involved, whenever there is reflection involved, I think you learn a lot. It was really an enlightening experience for me.

E: Right.

R: I learn about my students. I have learned a lot. I have improved a lot. I mean my thinking, my perception, my way of approaching issues, difficulties, and I have learned how to be patient, I have learned how to listen to others, because I was very emotional as a person. So I think, my own research, helped me to slow down, I learned that slowing down is not a bad thing, doing mistakes, but smartly, correcting those mistakes, and timely getting out of your stuck situations are also very important.

E: Right.

R: This is what I have tried to transfer to my own students too, like, do not try to be perfect, try to understand your issues, challenges, whenever you are in a stuck situation, try to understand why and how can you get out of that, and make sense of your mistakes and try to learn out of your mistakes.

E: Right.

R: And strategize your way out, so that you cannot repeat those mistakes.

R: I think Ph.D. students are the most appropriate for addressing those issues, because once you are in the academic position, then your focus, is diverted towards other issues.

E: It's interesting then, so if I could play back your comment just now, so we're wondering why the system doesn't change, we know that practitioners don't get a lot of training for teaching, and even if they do, there's a lot of things that you learn through your experience with particular populations, but you said Ph.D. students have more time to make sense of it and publish.

R: Yeah.

E: And so then structurally, you think, well, maybe the reason we're not changing is because the time that it takes for practitioners to really think and learn is just not there, and no amount of proving that some practice is out there that's magically going to fix everything, is going to make up for the fact that we don't actually allocate enough time for practitioners to really learn and get smarter at their own practice. Anyway, you know, we have to struggle a lot to think about really, really deep things and try to figure out how to help people have really profound learning experiences as quickly as possible, and even if studying abroad is not really part of it, we're not going to find the perfect pedagogy that's going to fix all the problems, that's going to take away from educators just needing to learn really hard lessons and just becoming good educators, or graduate students really grappling with big things, so we just need to be brave and unafraid.

R: Yeah.

E: About the social piece, you need to have somebody holding your hand, not, you know, literally, but metaphorically, sort of there by your side helping to say this is important, you know, don't underestimate this particular experience you just had, or I want you to stop paying attention to X and pay attention to Y, so that you know it's worth reflecting on or what's interesting to take away.

R: Yeah, I think it's very important, because without discussing your own mental processes and what is going on into your head, it's very hard to digest, it's very hard to make sense why you are doing this, what is so important , what if we don't do like that or do like that. Nevertheless, I think we need a big support system.

E: Yeah, I would say I want to be there as a cheerleader helping people navigate the part-- and it is a real challenge. You want to think broadly about the big system and what we're trying to accomplish, better engineering education, better practicing engineers, whatever that means, more inclusive engineering education, more environmentally aware engineers, et cetera, and at the same time you still have to do something and write it up and publish to make contributions in order to be in the conversation, and that can be really hard to be thinking big thoughts, but, actually, you know, delivering at the same time. However, it is doable.

R: Yeah, that is doable.

E: I would say to you that your experience with action research is very like the kinds of things that matter to me, that you were out there in the middle of it, you were getting insights about students and insights about yourself, it involved a great deal of reflection, and the more reflection you could do, the more you'd get out of it all. and thank you for having done such an action research project.

R: Thank you so much for appreciating that.

APPENDIX O

Summary of mathematical thinking (Mason et al., 2010)

Questions	Answers
What is mathematical thinking?	A dynamic process which, by enabling us to increase the complexity of ideas we can handle, expands our understanding
What do we use to do this?	Specializing, generalizing, conjecturing and convincing
How does it proceed?	● In phases – Entry, Attack, Review. ● Associated with emotional states – getting started, getting involved, mulling, keeping going, insight, being sceptical, contemplating.
Which are the phases to underline?	● Entry – because it lays the foundations for Attack. ● Review – because it is the least acknowledged and most educational.
What improves mathematical thinking?	Practice with reflection.
What supports mathematical thinking?	An atmosphere of questioning, challenging, reflecting, with ample space and time.
What provokes mathematical thinking?	A challenge, a surprise, a contradiction, a perceived gap in understanding.
Where does mathematical thinking lead?	● To a deeper understanding of yourself. ● To a more coherent view of what you know. ● To a more effective investigation of what you want to know.● To a more critical assessment of what you hear and see.
What are the assumptions to cultivate mathematical thinking?	1) You can think mathematically! 2) Mathematical thinking can be improved by practice with reflection. 3) Mathematical thinking is provoked by contradiction, tension, and surprise. 4) Mathematical thinking is supported by an atmosphere of questioning, challenging and reflecting. 5) Mathematical thinking helps in understanding yourself and the world.

APPENDIX P

Data type considerations for the given research adapted after (Heinze, 2008). Additions are italicised.

	Quantitative	Qualitative	*This Research*
Method design	• Predetermined	• Ad hoc, • Opportunistic	• *Ad hoc, opportunistic while entertaining emergent needs*
Sampling	• Large, • Representative, • Random	• Small, • Strategic	• *Small* • *Opportunistic or Emergent Purposive sampling,* • *A batch of first year engineering students over a complete semester*
Data Analysis	• Standardized measures allow efficient data reduction. • Facilitates combining and comparing across cases.	• Volume of raw data overwhelming, often of unclear pertinence • Data reduction not straight- forward • Data not standardized across cases	• *Interpretive data analysis, drawing on emergent met-befores, profiles, and attitudes* • *Illustrating the emergent activation of mathematical thinking processes*
Evaluation of Quality	• Standards of quality exist, • looks objective, • degree of support for inferences, • open to scrutiny	• Inferences can seem to come from "invisible" intuitions, • hard to assess quality	• *Triangulation* • *Expert Audit* • *Prolonged engagement of researcher* • *Quality is based on participants' interpretation and* • *related with academic publications*
Focus	• Questions should be specified in advance based on theory • Must be narrowed, sometimes ridiculously, to isolate variables, or it takes "black box" approach	• Open to possibility you don't know the right questions to ask in advance • Broad, holistic, explanatory, tries to grasp complex interactions of • Factors	• *Exploratory with some emphasis on actions being investigated in particular research cycles*
Aimed at	• Understanding "What?" Numerical Abstractions Characterizing the population	• Understanding "How and why?" Realistic representations • Characterizing the "Design Space"	• *Research questions are focusing on understanding of "How" and "why?"*
Values	• Statistical validity	• Practical implications	• *Pragmatist emphasis on theory supported by practice*

APPENDIX Q

Action research risk management and mitigation adapted and modified from (Heinze, 2008)

Criticism	This Research
Lack of contribution to theory and practice	Several theoretical models and theories are identified, implemented and monitored through the action research cycles. These are tested in practice and subsequently amended. Where possible, findings are communicated to a wider research community. The practical contributions focused on collaboration with Associate Professor delivering the main course and bringing theoretical issues into practice, for example the development of mathematical thinking skills, knowing the prior knowledge state of students, implementing the problem solving strategy and learning to think mathematically using BLOSSOMS modules.
That action research is consultancy not research	The power lies with individual practitioner responsible for the delivery of the Mathematical Thinking Lab. This prevented the consultancy mode of action research in taking place. Essentially, due to academic freedom, unless people believed that a certain change has to take place they are not making it. Documentation of events and data collection is taped (where possible), transcribed, analyzed and presented to the fellow researchers. There are no deadlines apart from the Ph.D. completion period and there is no financial gain associated with this work. The theoretical rationalization is based on theories of learning, which are utilized in order to understand the theoretical effects in practice.
No clear finishing date	Despite not having a clear finishing date, the process of a full time Ph.D. dictated a timeframe limit. The finishing time was one and a half years after the proposal defense of the current research and that would be end of year 2014.
Inappropriate duration of projects	The individual cycles were followed by the academic semesters. The cycles during pilot study coincided with the semester 1/year1 of engineering diploma students at the International Campus of the RU whereas the cycles during main study coincided with the semester1/year1 of engineering program intake 2013/2014 of the RU
Lack of rigor	The research process was implemented as rigorously as the situation permitted. Where possible documentary evidence is collected and each research cycle is associated with observations, written activity responses and focus group transcripts. Preliminary findings is published in the conference and passed through a peer reviewed process. As a part of the Ph.D. process in the researcher's university, this research also successfully passed internal evaluations. Building on interpretive beliefs a detailed account of events will be provided (see chapters 4 and 5). This thesis aims to strike a balance between data overload and insufficient data to be able to see the development of argumentation

APPENDIX Q

Action research risk management and mitigation adapted and modified from (Heinze, 2008) (continued)

Criticism	This Research
No clear level of participation	There were a number of actors involved in the given research. These actors include two academic members of staff, two research assistants during pilot, three research assistants during the main study and an engineering education researcher. A number of measures were taken to allow democratic decision-making. These included staff interviews, co-authoring of academic papers that resulted from findings in the given research, and a number of informal discussions with colleagues involved. The participation levels were indicated in the documentation of each of the action research cycles, outlining the number of individuals involved in the process. All informants were observed in the class and monitored using ALEKS individually as well as during the MTL sessions while working in the mixed ability groups. There were a couple of individuals who were less willing to participate and were mentioned during the discussion of the findings. It is therefore important to note that the class teacher (main study) took on the ideas for improvement with varying degrees of interest and hence the actions were implemented to a greater or lesser degree in some cases, as described in the individual research cycles.
Potential for self-delusion/group think	Students' written activity responses and classroom observation data were used to evaluate the intervention on a cyclic basis and this was fed back to researcher in regular meetings and focus groups. The data were discussed collectively and the actions were agreed upon and communicated to peer researchers. Further feedback from research experts on the data analysis and findings was collected through presenting them on campus and online. The majority of critical reflection came from the experts actively involved in engineering education research. Some students were not shy of making critical comments on their learning experiences whereas some were reluctant to express their opinions. Similarly, internal group of researchers were constructively critical during the internal meetings and knowledge dissemination sessions.
Limited generalizations	Both generalization and validation are characteristics of the positivist paradigm and therefore are not expected in this research. To counteract any criticism of 'lack of rigor', a detailed account of all significant situations is provided in the spirit of interpretive research. The research aim, theory and method have been outlined in detail. This enables the reader to follow the planning of research criteria, activity and evaluation cycles. As to the quality or validity of action research, the 'one right' outcome is impossible to produce. The emphasis in this work is therefore on the exploration of alternative interpretations. The further details regarding the quality of this research are provided in Section 3.11
No agreed sequence of activities	Going back to the roots of action research, the researcher attempted to adhere to the process as outlined by Lewin's cyclic model (Burns, 2000) combined with Kemmis and McTaggart's (2014) simple model. However, the reality of action research in practice was non-sequential rather than sequential. The modified model in shown in Figure 3.6.

APPENDIX R

Definitions and deciding factors of emerging methodologies in engineering education extracted from Light and Case (2011) and Creswell (Creswell, 2012, 2007) along with a comparison with Action Research as Living Educational Theory adapted and inspired from Whitehead (2014).

Methodology	Deciding Factors	Comparison with Action Research (Living Educational Theory)
Case study: "A qualitative approach in which the investigator explores a bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information (e.g., observations, interviews, audio-visual material, and documents and reports), and reports a case description and case-based themes." (Creswell, 2007, p. 73)	• "Argues for the significance of knowledge generated in particular contexts" (Light and Case, 2011) • "Underpins much research in the interpretive and critical theoretical perspectives and is frequently combined with other methodologies" (Light and Case, 2011)	• "An Action Research (the living theory) may sometimes be mistaken as a case study. Stake (2005) refers to case study as a choice of what is to be studied within a bounded system." (Whitehead, 2014) • (Action research) "living theories generated from a perspective of inclusionality, as a relationally dynamic awareness of space and boundaries are aware of the experience and expression of a life-affirming and unbounded energy flowing through the cosmos." (Whitehead, 2014) • "The main difference between a case study and a living theory is that a case study is a study of a bounded system while the explanatory principles of living theories are not constrained by a bounded system. They articulate explanatory principles in terms of flows of life-affirming energy, values, and understandings that are transformatory and not contained within a bounded system." (Whitehead, 2014)
Grounded Theory: "A qualitative research design in which the inquiry generates a general explanation (a theory) of a process, action, or interaction shaped by the views of a large number of participants (Strauss & Corbin, 1998)". (Creswell, 2007, p. 63)	• "Focuses on the generation of knowledge from empirical data without the use of a priori theory" (Light and Case, 2011) • "Informs the design of studies in a wide variety of research situations" (Light and Case, 2011)	• "A living theory is similar to a grounded theory in that the intent of a living theory is to move beyond description and to generate a valid explanation for an individual's educational influence in his or her own learning and in the learning of others." (Whitehead, 2014) • "Living Theory differs from Grounded Theory in that the theory is not an abstract analytic scheme of a process. A living theory is an explanation for an individual's educational influence in learning where the explanatory principles are not abstract generalizations." (Whitehead, 2014) • "The explanatory principles are the energy flowing values and understandings the individual uses to give meaning and purpose to their life and to explain their educational influences in learning." (Whitehead, 2014)

APPENDIX R

Definitions and deciding factors of emerging methodologies in engineering education extracted from Light and Case (2011) and Creswell (Creswell, 2012, 2007) along with a comparison with Action Research as Living Educational Theory adapted and inspired from Whitehead (2014).

Methodology	Deciding Factors	Comparison with Action Research (Living Educational Theory)
Ethnography: "A qualitative design in which the researcher describes and interprets the shared and learned patterns of values, behaviors, beliefs and language of a culture-sharing group (Harris, 1968)" (Creswell, 2007, p. 68)	• "Favors long term engagement with the social context under investigation" (Light and Case, 2011) • "Aims for the generation of rich descriptions of the lives of the research participants" (Light and Case, 2011) • Employs in very specialized investigations in its pure form in education research • Applies to a range of research contexts.	• "A living theory is similar to ethnographic research in paying attention to the cultural norms within which the researcher is acting and researching." (Whitehead, 2014) • "It differs from ethnographic research in that it does not focus on an entire culture group." (Whitehead, 2014) • "A living theory is an explanation of an individual′s educational influence in his/her own learning, in the learning of others and in the social formations in which the researcher is living and working. (Whitehead, 2014) • "In engaging with the cultural influences in the individual′s learning, especially in the learning of social formations, living theorists include an understanding of cultural influences in the explanations of their educational influences in learning." (Whitehead, 2014)
Phenomenology: "Whereas a narrative study reports the life of a single individual, a phenomenological study describes the meaning for several individuals of their lived experiences of a concept of a phenomenon. Phenomenologists focus on describing what all participants have in common as they experience a phenomenon (e.g., grief is universally experienced). The basic purpose of phenomenology is to reduce individual experiences within a phenomenon to a description of the universal essence (a "grasp of the very nature of the thing," van Manen, 1990, p.177)." (Creswell, 2007, pp. 57–58)	• "Seeks to uncover the different ways in which people in a particular context experience a phenomenon" (Light and Case, 2011) • "Well-established in research on student learning in higher education" (Light and Case, 2011)	• "Living theories are phenomenological in that they begin from the experience of the phenomenon the researcher is seeking to understand." (Whitehead, 2014) • "The purpose of a living theory differs from the basic purpose of phenomenology in that the purpose of phenomenology is to produce a description of a universal essence whilst the purpose of a living theory is to produce a unique explanation of the individual′s educational influences in learning." (Whitehead, 2014)

APPENDIX R

Definitions and deciding factors of emerging methodologies in engineering education extracted from Light and Case (2011) and Creswell (Creswell, 2012, 2007) along with a comparison with Action Research as Living Educational Theory adapted and inspired from Whitehead (2014).

Methodology	Deciding Factors	Comparison with Action Research (Living Educational Theory)
Discourse analysis: "Under the broader definition, discourse analysis covers large areas of sociolinguistics and linguistics, much cognitive science concerned with language use, social semiotics, and work on educational interaction as well as areas of work such as discursive psychology, critical discourse analysis, Foucauldian discourse analysis, and simply discourse analysis." (Given, 2008, p. 217)	• A form of linguistic or text analysis "focused on instances of socially situated communication" (Light and Case, 2011) • "Seeks to link these to the underlying cultural ideas, which they represent" (Light and Case, 2011)	• A living theory is similar to discourse in a sense of analyzing the "actual instances of language in use, for example the transcript of a classroom discussion" or the written response analysis that "also includes mathematical equations, graphs, figures, verbal exchanges, and so on." (Light and Case, 2011) • It is different from discourse analysis because of its critical focus on continuous improvement of practice and its effective use to create "Collaborative Engineering Environments." (Light and Case, 2011) • Action research may or may not utilize discourse analysis.
Narrative analysis: "A specific type of qualitative design in which "narrative is understood as a spoken or written text giving an account of an event/action or series of events/actions, chronologically connected" (Czarniawska, 2004, p. 17)." (Creswell, 2007, p. 54)	• A form of linguistic analysis centered "on the stories which people generate as they seek to make sense of their experiences" (Light and Case, 2011)	• "A living theory, as an explanation by an individual of their educational influences in their own learning and in the learning of others can be understood as a form of narrative research in that it begins with the experiences as lived and told by the researcher."(Whitehead, 2014) • "Within the narrative, what distinguishes the story as a living theory is that it is an explanation of the educational influences of the individual in their own learning and in the learning of others." (Whitehead, 2014) • "Not all narratives are living theories, but all living theories are narratives." (Whitehead, 2014)

APPENDIX R

Definitions and deciding factors of emerging methodologies in engineering education extracted from Light and Case (2011) and Creswell (Creswell, 2012, 2007) along with a comparison with Action Research as Living Educational Theory adapted and inspired from Whitehead (2014).

Methodology	**Deciding Factors**	**Comparison with Action Research (Living Educational Theory)**
Action research: "Action research designs are systematic procedures done by teachers (or other individuals in an educational setting) to gather information about, and subsequently improve, the ways their particular educational setting operates, their teaching, and their student learning (Mills, 2011)." (Creswell, 2012, p. 577)	• Aims towards the improvement of practice. • Characterizes "by engaging the main participants in the research" (Light and Case, 2011) either at a technical level or as reflective participants or "as co-researchers active in the research design, implementation, and analysis" (Light and Case, 2011)	• In this study, the researcher planned for "an enquiry of the kind," "How do I improve" the way I learn about my students, their learning trajectories in a new environment and the way I tackle the on stage issues? How the transformation occurs during researcher's epistemological transition from a positivist practitioner to a reflective teacher, "with the intention of improving" researcher's "practice and generating knowledge" in her own "living educational theory." (Whitehead, 2014) • It is a form of self-reflective problem solving, which enables practitioners to better understand and solve pressing problems in educational and social settings. The action (what you do) aspect of action research is about improving practice. The research (how you learn about and explain what you do) aspect is about creating knowledge about practice. The knowledge created is the knowledge of one's practice (McNiff and Whitehead, 2010).